4b. Properties of the negative lightning discharge to ground - II
Attachment process

Vertical leader above flat ground

Fig. 4. Electric gradient below leader channel: (a) as function of horizontal distance and (b) as function of height of leader tip above ground. (Golde, 1977)

1- downward leader channel
2- corona sheath
3, 4, 5 - streamer zones
6, 7 - return stroke tip (25-110 m)
8 - return-stroke channel

An upward-moving connecting leader is initiated when

\[ \int \frac{E(z)}{V_{\text{BD}}} \geq V_{\text{BD}}(l_{\text{crit}}) \]

where \( V_{\text{BD}}(l_{\text{crit}}) \) is the breakdown voltage.

\[
E(z) = E_0 \left( 1 - \frac{\tan^{-1} \left( \frac{a}{z} \right) \sqrt{a}} {\tan^{-1} \frac{a}{z} - \frac{a}{z}} \right) + \frac{A}{z^2/a^2 + b^2/a^2 - 1} \left( \frac{\sqrt{a}}{\tan^{-1} \frac{a}{z} - \frac{a}{z}} \right)
\]

\[
A = 1 - \frac{b^2}{a^2}, \quad a = h \text{(structure height)}, \quad \text{semi-ellipsoid}
\]

\( E_0 \) is the undisturbed field (assumed to be independent on \( z \)) due to the downward leader.

For \( h = 50-300 \text{ m} \), the field is strongly enhanced over \( (z-a) \approx 15 \text{ m} \), \( \Rightarrow l_{\text{crit}} \approx 15 \text{ m} \)

\[ \rho_L = 10^{-3} \text{ Ohm m} \]

\[ K_p = \frac{V_{\text{up}}}{V_{\text{down}}} \]

(Bazelyan et al., 1978)
The stepped leader meets an upward-moving connecting leader from the tower at point \( A \). The connecting leader branches at point \( B \).

**Figure 6.2**

Junction point \( A \) is about 40 m above and 40 m horizontally away from the tower.
Fig. 42. Downward negative stroke to tower 2. a) Photograph from mountain peak on fast moving film. b) Current oscillogram (impulse current). (Berger, 1967)
Striking-Distance Concept

A reasonable charge distribution is assumed for the leader channel and the resultant fields on the ground or nearby objects are calculated. The leader is assumed to be at the striking distance when the field between the leader tip and some point exceeds a critical breakdown value (2-6 kV/cm) determined from the long-spark experiments.

\[ r = 10.1 \cdot I^{0.65} \text{ m} \]

\[ I = 20.0 \text{ kA} \] (Golde, 1977), \[ I = 10.6 \cdot Q^{0.2} \] (Berger)

\[ f = 6.0 \] 6.1

and \( \bullet \) are the estimates from, respectively, three-dimensional and two-dimensional photographs and direct current measurements (Eriksson, 1978).
Estimation of the striking distance using photographic records

Fig 8(a) Downward flash to the research mast photographed from two directions.

Fig 9(b) Reconstruction of flash progression (Erickson, 1978) striking distance of 950m.
Average negative return-stroke current waveshapes normalized to unity peak amplitude

The records were aligned in such a way that the 50% amplitude points coincided. A different technique using the cross-correlation function produced almost identical results.

\[
\bar{i}_k = \frac{1}{N} \sum_{j=1}^{N} i_{jk}
\]

where \(N\) is the total number of curves contributing to the average current value in the point \(k\).
Fig. 1.5b. Luminosity versus time at different heights above ground corresponding to the streak photograph in Fig. 1.5a.
Fig. 1.5c. Same as Fig. 1.5b, but for the bottom 480 m of the channel only.
Return-stroke peak current distributions

\[ P(I_p) = \frac{1}{\sqrt{2\pi} \delta_{I_p}^2} \int_{I_p}^\infty \exp\left[ -\frac{(\log I_p - \log I_p_0)^2}{2 \delta_{I_p}^2} \right] dI_p \]

(log-normal approximation)

Percentage of events exceeding a given value of the parameter on the horizontal axis.

Cumulative probability distribution graph paper on which the Gaussian (normal) cumulative distribution function appears as a straight line.
Direct measurements of lightning currents in South Africa

(lightning strikes to a 60-m research mast)

3dB Frequency bandwidth: 1 Hz to 10 MHz

Maximum di/dt is 180 kA/ms

Fig 6 Multiple stroke lightning current wave shapes.

First strokes (N = 11)

Fig 7 Cumulative frequency distribution of lightning current amplitudes for first negative downward strokes - research mast. (Eriksson, 1972)
Table 2-1. Lightning current parameters for negative flashes from Berger et al. (1975).

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameters</th>
<th>Unit</th>
<th>95%</th>
<th>50%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>First strokes</td>
<td>kA</td>
<td>14</td>
<td>30</td>
<td>80</td>
</tr>
<tr>
<td>135</td>
<td>Subsequent strokes</td>
<td>kA</td>
<td>4.6</td>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td>93</td>
<td>First strokes (total stroke charge)</td>
<td>C</td>
<td>1.1</td>
<td>5.2</td>
<td>24</td>
</tr>
<tr>
<td>122</td>
<td>Subsequent strokes</td>
<td>C</td>
<td>0.2</td>
<td>1.4</td>
<td>11</td>
</tr>
<tr>
<td>94</td>
<td>Complete flash</td>
<td>C</td>
<td>1.3</td>
<td>7.5</td>
<td>40</td>
</tr>
<tr>
<td>90</td>
<td>First strokes (excluding c.c.)</td>
<td>C</td>
<td>1.1</td>
<td>4.5</td>
<td>20</td>
</tr>
<tr>
<td>117</td>
<td>Subsequent strokes</td>
<td>C</td>
<td>.22</td>
<td>0.95</td>
<td>4</td>
</tr>
<tr>
<td>89</td>
<td>Front duration (2 kA to peak)</td>
<td>μs</td>
<td>1.8</td>
<td>5.5</td>
<td>18</td>
</tr>
<tr>
<td>118</td>
<td>First strokes</td>
<td>μs</td>
<td>.22</td>
<td>1.1</td>
<td>4.5</td>
</tr>
<tr>
<td>92</td>
<td>Maximum di/dt</td>
<td>kA/μs</td>
<td>5.5</td>
<td>12</td>
<td>32</td>
</tr>
<tr>
<td>122</td>
<td>Subsequent strokes</td>
<td>kA/μs</td>
<td>12</td>
<td>40</td>
<td>120</td>
</tr>
<tr>
<td>90</td>
<td>Stroke duration (2 kA to half value) on the tail</td>
<td>μs</td>
<td>30</td>
<td>75</td>
<td>200</td>
</tr>
<tr>
<td>115</td>
<td>Subsequent strokes</td>
<td>μs</td>
<td>6.5</td>
<td>32</td>
<td>140</td>
</tr>
<tr>
<td>91</td>
<td>Integral (i^2dt)</td>
<td>A^2s</td>
<td>6.0x10^3</td>
<td>5.5x10^4</td>
<td>5.5x10^4</td>
</tr>
<tr>
<td>88</td>
<td>Subsequent strokes</td>
<td>A^2s</td>
<td>5.5x10^3</td>
<td>6.0x10^3</td>
<td>5.2x10^4</td>
</tr>
<tr>
<td>133</td>
<td>Time interval</td>
<td>ms</td>
<td>7</td>
<td>33</td>
<td>150</td>
</tr>
</tbody>
</table>

The energy that would be dissipated in a 1-Ω resistor if the stroke current were to flow through it.

The shortest measurable time in the oscillograms was 0.5 ms.

The percentages (95%, 50%, and 5%) of cases exceeding the tabulated values are based on the lognormal approximations.
Distribution of measured return-stroke speed (two-dimensional)

Data include 17 first and 46 subsequent strokes. The speed was averaged over the visible channel section (within 1.3 km of ground). There was a tendency for the speed to decrease with height for both first and subsequent strokes.

\[ \text{Mean} = 1.1 \times 10^8 \text{ m/s} \]
\[ N = 63 \]

First
\[ \text{Mean} = 9.6 \times 10^7 \text{ m/s} \]

Subsequent
\[ \text{Mean} = 1.2 \times 10^8 \text{ m/s} \]

(Idone and Orville, 1982)
The average two-dimensional speed between the two selected levels of the channel is
\[ V = \frac{2D}{T} \]  
where \( T \) is the net horizontal displacement between the levels \((s_3 - s_1)\).

\( M \) is print-to-negative magnification factor for the time-resolved images.
\( W \) is film movement rate.
\[ L = \frac{DL'}{FM'} \]  
(determined from the still picture)

\( D \) is the distance to the channel;
\( F \) is focal length of the still camera;
\( L' \) is channel length measured from the print;
\( M' \) is print-to-negative magnification factor for the still photograph.

Fig. 1. Qualitative representation of the stepped leader and dart leader image tracks resulting from the exposure to two leader-return stroke sequences. The leaders propagate from cloud to ground while the film is rapidly swept through the image plane. The actual channel geometry is represented by the dashed lines labeled "Still," which are superimposed for illustrative purposes. The stepped leader, propagating downward relatively slowly and brightening every ~41 μs, produces the intermittent track shown on the left. The propagation time between a given height and ground is \( T_S = \frac{S}{W} \). The dart leader, in contrast, propagates downward at a generally greater speed with constant emission of light and records a continuous track. The propagation time of the dart leader between the same height and ground is \( T_D = \frac{D}{W} \). (Orville and Idone, 1982)

Fig. 2. Schematic diagram of the principles involved in streaking photography used for the time resolution of lightning return strokes. (Idone and Orville, 1982)
<table>
<thead>
<tr>
<th>Reference</th>
<th>Min. Speed, m/s</th>
<th>Max. Speed, m/s</th>
<th>Mean Speed, m/s</th>
<th>s.d., m/s</th>
<th>Sample Size</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Triggered Lightning</td>
</tr>
<tr>
<td>Hubert and Mouget [1981]</td>
<td>4.5x10^7</td>
<td>1.7x10^8</td>
<td>9.9x10^7</td>
<td>4.1x10^7</td>
<td>13</td>
<td>Photoelectric, 3-D speed</td>
</tr>
<tr>
<td>Idone et al. [1984]</td>
<td>6.7x10^7</td>
<td>1.7x10^8</td>
<td>1.2x10^8</td>
<td>2.7x10^7</td>
<td>56</td>
<td>Streak camera, 3-D speed</td>
</tr>
<tr>
<td>Willett et al. [1988]</td>
<td>1.0x10^8</td>
<td>1.5x10^8</td>
<td>1.2x10^8</td>
<td>1.6x10^7</td>
<td>9</td>
<td>Streak camera, 2-D speed</td>
</tr>
<tr>
<td>Willett et al. [1989]</td>
<td>1.2x10^8</td>
<td>1.9x10^8</td>
<td>1.5x10^8</td>
<td>1.7x10^7</td>
<td>18</td>
<td>Streak camera, 2-D speed</td>
</tr>
<tr>
<td>Mach and Rust [1989, Figure 8]</td>
<td>6.0x10^7</td>
<td>1.6x10^8</td>
<td>1.2x10^8</td>
<td>2x10^7</td>
<td>40</td>
<td>Long channel</td>
</tr>
<tr>
<td></td>
<td>6.0x10^7</td>
<td>2.0x10^8</td>
<td>1.4x10^8</td>
<td>4x10^7</td>
<td>39</td>
<td>Short channel</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Photoelectric, 2-D)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Natural Lightning</td>
</tr>
<tr>
<td>Boyle and Orville [1976]</td>
<td>2.0x10^7</td>
<td>1.2x10^8</td>
<td>0.7x10^8</td>
<td>2.6x10^7</td>
<td>12</td>
<td>Streak camera, 2-D speed</td>
</tr>
<tr>
<td>Idone and Orville [1982]</td>
<td>2.9x10^7</td>
<td>2.4x10^8</td>
<td>1.1x10^8</td>
<td>4.7x10^7</td>
<td>63</td>
<td>Streak camera, 2-D speed</td>
</tr>
<tr>
<td>Mach and Rust [1989, Figure 7]</td>
<td>2.0x10^7</td>
<td>2.6x10^8</td>
<td>1.3x10^8</td>
<td>5x10^7</td>
<td>54</td>
<td>Long channel</td>
</tr>
<tr>
<td></td>
<td>8.0x10^7</td>
<td>&gt;2.8x10^8</td>
<td>1.9x10^8</td>
<td>7x10^7</td>
<td>43</td>
<td>Short channel</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Photoelectric, 2-D)</td>
</tr>
</tbody>
</table>
Magnetic field due to a vertical current-carrying line (static approximation)

The Bio-Savart law
\[ d\vec{H} = \frac{I(d\vec{z}' \times \vec{a}_R)}{4\pi R^2}, \text{A/m} \]
\[ d\vec{B} = \frac{M_0 I dz'}{4\pi R^2} (\vec{a}_I \times \vec{a}_R), \text{Wb/m}^2 \]

\[ |(\vec{a}_I \times \vec{a}_R)| = \sin \theta = \frac{D}{R} \]

since \( \sin \theta = \sin (180^\circ - \theta) \)

\[ dB = \frac{M_0 I dz'}{4\pi} \frac{D}{(z'^2 + D^2)^{3/2}} \]

(magnitude)

The vector field points into the page.

\[ \vec{B} \] due to image current is identical to \( \vec{dB} \) due to actual current.

To take account of the image current.

\[ B = 2 \int_{H_B}^{H_T} \frac{M_0 I}{4\pi} \frac{D}{(z'^2 + D^2)^{3/2}} dz' = \frac{M_0 I}{2\pi D} \left[ \frac{H_T}{(H_T^2 + D^2)^{1/2}} - \frac{H_B}{(H_B^2 + D^2)^{1/2}} \right] \]

For \( H_T = H, H_B = 0 \)

\[ B = \frac{M_0 I}{2\pi D} \frac{H}{(H^2 + D^2)^{1/2}} \]

Spatially uniform but slowly time-varying current flows from a charge center to ground. (close)

\[ \text{If } (H/D)^2 \gg 1 \quad B \approx \frac{M_0 I}{2\pi D} \]

Fla. A-6

\[ \text{If } (H/D)^2 \ll 1 \quad B \approx \frac{M_0 I H}{2\pi D^2} \quad (\text{far}) \rightarrow B = \frac{M_0}{4\pi D^2} \frac{dM}{d\sigma} \]

87
Electric and magnetic fields due to a vertical dipole radiator

The electric dipole moment of the charge \( Q \) located a distance \( H \) above a conducting plane and its image is

\[ M = 2QH \quad \text{(in general, } M = 2 \sum_{i=1}^{N} Q_i H_i) \]

\[
\frac{dM}{dt} = 2IH
\]

\[
\frac{d^2M}{dt^2} = 2H \frac{dI}{dt}
\]

Has a non-zero value before and after lightning

\[ E_z = \frac{1}{4\pi \varepsilon_0} \frac{M}{D^3} + \frac{1}{4\pi \varepsilon_0 cD^2} \frac{dM}{dt} + \frac{1}{4\pi \varepsilon_0 c^2 D} \frac{d^2M}{dt^2} \]

\[ B_\phi = M_0 \frac{dM}{dt} + \frac{M_0}{4\pi c D} \frac{d^2M}{dt^2} \]

M and its derivatives should be determined at time \( t-D/c \) (are retarded values)

The dipole approximation is valid if (1) \( D \gg H \), (2) \( I(z,t) = I(0,t) \) for all \( z \) from 0 to \( H \), and (3) \( H = \text{const} \).
Two-station measurements of vertical electric and horizontal magnetic fields from a two-stroke flash

$200 \text{ V/M}$

$5 \times 10^{-7} \text{ Wb/m}^2$

OCF, $D = 1.9 \text{ km}$

$B_{2\text{NS}}$

$B_{2\text{EW}}$

$B_{1\text{NS}}$

$B_{1\text{EW}}$

$5 \text{ V/M}$

$2 \times 10^{-8} \text{ Wb/m}^2$

GNV, $D = 50 \text{ km}$
**ELECTRIC FIELD INTENSITY**

Initial peak 50 V/m

D = 10 km

20 V/m Ramp starting time

Value at 170 μs

D = 15 km

5 V/m Zero crossing

D = 50 km

2 V/m

D = 200 km

**MAGNETIC FLUX DENSITY**

Hump

2 x 10^-7 Wb/m²

D = 10 km

Half value

1 x 10^-7 Wb/m²

D = 15 km

2 x 10^-8 Wb/m²

D = 50 km

1 x 10^-8 Wb/m²

D = 200 km

Fig 7.1b
Fine structure of the radiation fields produced by return strokes

-20 -15 -10 -5 0 5 10 15 20

(\sim 20\% \text{ of the initial peak; } \sim 0.5 - 1 \text{ ms})

SUBSEQUENT

(90 \pm 40 \text{ ns}) \rightarrow \text{Fast transition}

(40-50\% \text{ of the initial peak; } \sim 4 \text{ ms}) \rightarrow \text{Slow front}

FIRST

MICROSECONDS

E

VOLTS / METER

10

5

0

b

Subsequent with dart-stepped leader

10

\text{Initial peak}

\text{Secondary peak}

\text{Subsidiary peak}
a) 76 First Strokes (GEOMETRIC MEAN = 5.9 V/m)

b) 155 Second, Third, and Fourth Strokes (GEOMETRIC MEAN = 3.3 V/m)

c) 62 Fifth, Sixth, and Seventh Strokes (GEOMETRIC MEAN = 2.3 V/m)

d) 53 Strokes of the Order 8 through 18 (GEOMETRIC MEAN = 2.4 V/m)

INITIAL ELECTRIC FIELD PEAK NORMALIZED TO 100 km (V/m)
Return-stroke initial electric field peak (GM) as a function of stroke order

\[ I_{(-)} = 1.5 - 0.037 \cdot D \cdot E_{(+)} \]

\[ [E] = V/m \]
\[ [D] = km \]
\[ [I] = kA \]
Table 2-3. Statistics on return stroke electric field waveforms of negative cloud-to-ground lightning.

<table>
<thead>
<tr>
<th></th>
<th>First strokes</th>
<th></th>
<th>Subsequent strokes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No.</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td><strong>Initial peak (V/m)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(normalized to 100 km)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rakov and Uman (1990a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[All cases]</td>
<td>76</td>
<td>5.9</td>
<td>GM</td>
</tr>
<tr>
<td>[Multiple-stroke flashes]</td>
<td>63</td>
<td>6.2</td>
<td>GM</td>
</tr>
<tr>
<td>[Single-stroke flashes]</td>
<td>13</td>
<td>4.7</td>
<td>GM</td>
</tr>
<tr>
<td>[Strokes creating new termination]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Strokes following previous channel]</td>
<td></td>
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<tr>
<td>Cooray and Lundquist (1982)</td>
<td>553</td>
<td>5.3</td>
<td>2.7</td>
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<tr>
<td>Lin et al. (1979)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>[KSC]</td>
<td>51</td>
<td>6.7</td>
<td>3.8</td>
</tr>
<tr>
<td>[Ocala]</td>
<td>29</td>
<td>5.8</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>Zero-crossing time (μs)</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Cooray and Lundquist (1985)</td>
<td></td>
<td></td>
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</tr>
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<td>[Sweden]</td>
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<td>49</td>
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<td>[Sri Lanka]</td>
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<td>89</td>
<td>30</td>
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<tr>
<td>Lin and Uman (1973)</td>
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<td>18</td>
</tr>
<tr>
<td><strong>Zero-to-peak rise time (μs)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Master et al. (1984)</td>
<td>105</td>
<td>4.4</td>
<td>1.8</td>
</tr>
<tr>
<td>Cooray and Lundquist (1982)</td>
<td>140</td>
<td>7.0</td>
<td>2.0</td>
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<tr>
<td>Lin et al. (1979)</td>
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<td></td>
</tr>
<tr>
<td>[KSC]</td>
<td>51</td>
<td>2.4</td>
<td>1.2</td>
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<tr>
<td>[Ocala]</td>
<td>29</td>
<td>2.7</td>
<td>1.3</td>
</tr>
<tr>
<td><strong>10-90% rise time (μs)</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Master et al. (1984)</td>
<td>105</td>
<td>2.6</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Note: The numbers with GM in parenthesis are the Geometric Mean values.