LIGHTNING

PROFESSOR RAKOV

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1. Types of lightning discharges and lightning terminology
Types of lightning discharge from cumulonimbus

Fig. 7.3. A local or convective thunderstorm at Socorro, New Mexico. (Courtesy, Marx Brook, New Mexico Institute of Mines and Technology) (Uman, 1986)

Cloud discharges (~67%)

- Intracoud
- Cloud-to-cloud
- Cloud-to-ground (~33%)
- Cloud-to-air
FIGURE 8.11 The apparent evolution of lightning with time in a thunderstorm, based on a variety of observations in different storms. See text for explanation. The dendritic structure of the lightning has been guessed in all cases except for the multicellular intracloud discharge of part c. The dotted region in the dissipating part of the storm in parts e and f represents the radar brightband from melting snowflakes.

(Krehbiel, 1986)
Fig. 11. Plan and elevation views of flash 139, using symbols A through Z to denote the coarse time scale shown in Figure 10. The U axis is indicated on the plan view. A total of 1486 sources were located. (Proctor et al., 1988)

Obscured occasionally by errors in the measurement of source heights, height error magnitudes were 1.4 km rms for sources near ground below A and 370 m rms at (10, 5, 3) km. These errors were not the cause of our locating several exceptional sources which became active at the end of the leader process, 1 - 1.5 km directly above the leader origin. The second stepped leader, EF, began 26 ms after the first return stroke of A. (The sequence of events is shown by Figure 10). The origin of the second leader was 780 m away from the origin of A. One of the leader branches, F, was 6 km long, and this branch progressed down before it turned through a wide loop to travel upward. It reached a height of 4.5 km agl and its terminus was 3 km
branch RSTUVW traveled toward A until time W, when it turned to progress away from A.

9.7. Seventh Case Study: Flash 62, Recorded at 1834:43.5 on March 27, 1979

Although this unusual flash is not representative of the majority, it has several features that are worthy of mention. One of these concerned some anomalous changes in electric field which were not accompanied by radio noise. (This happened in three of ISO flashes). The auxiliary record, Figure 24, shows that the field changed measurably during the second interstroke interval, despite the almost complete absence of radio noise during the latter part of this interval. Small and gradual changes in electric field also occurred immediately after the first return stroke and again during the latter part of the third interstroke interval, without radio noise accompanying either event. These anomalous changes in electric field were not continuations of a gradual change that had been in progress when the flash began.

The electric field change produced by the first stepped leader is shown magnified fivefold.
Categorization of cloud-to-ground lightning

1 (≥ 90%)

2

3 (≤ 10%)

4
**Fig. 1.4** A still photograph of a typical cloud-to-ground flash. Courtesy J. Rodney Hastings. *(Uman, 1987)*

**Fig. 6.1a.** Lightning initiated by an upward-moving leader from a tower on Mt. San Salvatore near Lugano, Switzerland. The spot directly beneath the bottom of the lightning channel is a tower light. *Upward-initiated lightning is branched upward in contrast to the downward branching of the usual cloud-to-ground lightning flash.* (Courtesy, Richard E. Orville, State University of New York at Albany) *(Uman, 1986)*
Object-initiated lightning

Objects electrically connected to ground

"Classical" triggering (LRSG)

"Altitude" triggering (LRSA)

Figure 8 General features of triggered lightning pictures: (a) classical; (b) anomalous; (c) TIPSy (Hubert, 1984)

LRSAG technique: a short length of grounded wire below the non-conducting segment.
Cloud-to-stratosphere discharge

Fig. 1. This video frame was captured during the shuttle STS-31 mission at 03:35:50 UTC 28 April 1990 using a shuttle payload-bay low-light-level TV camera while the shuttle was on its 55th orbit over Mauritania, northwest Africa. The payload-bay TV camera was pointed to the southeast of the orbital ground track so that thunderstorm complexes near the earth's limb could be observed. Seen in this image is an arc of the earth's airglow, a vertical line, which is the shuttle's rudder, five clouds that are illuminated by lightning in the foreground, and a single cloud located on the horizon with a vertical discharge; various stars can be seen above the arc of the earth's airglow. The storm, which had a vertical discharge, was located at approximately 7.5°N, 40°E and was about 2000 km from the shuttle. The length of the discharge is estimated to be at least 31 km. (Vaughan et al., 1992)

Figure 5. Schematic which suggests that the form of the CS discharge may change from channel to fan-like plume with decreasing pressure. (Lyons and Williams, 1993)

Hundreds of images were obtained in 1993:
Average duration ~ 100 ms
Maximum height ~ 50-80 km
Significant horizontal extent (up to tens of kilometers)

Low-light-level TV image of the upward discharge in Minnesota (Franz et al., 1990)
Distance - 250 km
Duration - < 30 ms
Length - 20 km
Natural (downward) lightning

Fig. 1.6  (a) A drawing of the luminous features of a lightning flash below a 3-km cloud base as would be recorded by a streak camera (Section C.4: Fig. C.8). Increasing time is to the right. For clarity the time scale has been distorted. (b) The same lightning flash as would be recorded by a camera with stationary film. Adapted from Uman (1969). (Uman, 1987)

Tower-initiated lightning

Fig. 12.2  Drawing of streak-camera photograph illustrating usual lightning from the Empire State Building. Adapted from McEachron (1939). (Uman, 1987)

Rocket-and-wire triggered lightning

Fig. 1  Schematic streak photograph of a typical, classical, triggered flash of negative polarity. (Willett, 1992)
Global circuit of atmospheric electricity

To maintain about 300 kV between the Earth and the electrosphere, the Earth has about $10^6 \text{C}$ of negative charge on its surface, and an equal positive charge is distributed throughout the atmosphere. The charge of the Earth is continuously leaking off into the conducting atmosphere and would disappear (if not re-supplied) in less than an hour. The Earth is apparently recharged by the action of thunderstorms (roughly 2000 are in progress at any one time over about $10^6 \text{km}^2$).

Total current $\sim 1500 \text{A}$ (of the order of $10^{-12} \text{A/m}^2$)
Total resistance $\sim 200 \Omega$
2. Incidence of lightning to areas and structures
Climatology of Lightning Incidence

1. Annual number of thunderstorm days ($T_D$)

The thunderstorm day is the only parameter related to lightning incidence for which worldwide data extending over many years are available.

The thunderstorm-day parameter is normally defined as a local calendar day on which thunder is heard (practical limit of audibility is about 15 km). Thunderstorm-day data are recorded at most weather stations and often presented cartographically (global, national, or regional maps).

Relation of annual ground flash density to annual number of thunderdays

Many relationships have been proposed, most being of the form

$$N_g = a T_D^b, \text{ km}^{-2} \text{ yr}^{-1}$$

The most reliable relation is probably that based on the lightning flash counter observations in South Africa:

$$N_g = 0.023 T_D^{1.3}$$

(60 stations, 2 years)

Recommended by CIGRE (International Conference on Large High Voltage Electric Systems)

(Anderson and Eriksson, '10)

<table>
<thead>
<tr>
<th>$T_D$</th>
<th>$N_g$, km$^{-2}$ yr$^{-1}$</th>
<th>Calculated from equation (1)</th>
<th>Observed scatter</th>
</tr>
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<td>0.3 - 3</td>
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<td>0.6 - 5</td>
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<td>50</td>
<td>3.2</td>
<td>1.2 - 10</td>
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<td>60</td>
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<td>1.8 - 12</td>
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<td>80</td>
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<td>3 - 17</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>9.2</td>
<td>4 - 20</td>
<td></td>
</tr>
</tbody>
</table>
A world thunderday map (World Meteorological Organization Publication 21, 1956)
450 weather stations having continuous records for 10 to 30 years
(MacGorman et al., 1984)
Fig. 1. Relationship between ground flash density ($N_g$) and keraunic level ($T_D$) (based upon five-year averages as opposed to one-year averages). Mean data from 310 annual registrations. 1976-1980. (62 recording stations)

$$N_g = 0.04 T_D^{1.25}$$

$r = 0.85$  

(Eriksson, 1981)
(2) Annual number of thunderstorm hours \( (T_H) \)

This is potentially more informative parameter (as compared to the annual number of thunderydays) related to lightning incidence and routinely recorded at some weather stations. MacGorman et al. (1984), using ground flash data from magnetic direction finding systems operated in Florida and Oklahoma, have compared relations between \( N_g \) and \( T_D \) and between \( N_g \) and \( T_H \). In both locations, correlation with \( T_H \) was better than with \( T_D \). MacGorman et al. (1984) constructed equation

\[
N_g = 0.054 T_H^{1.1}
\]

which they used to obtain \( N_g \) map for the contiguous United States from the corresponding \( T_H \) map.

However, lightning-related outages of the power lines do not show a better correlation with \( T_H \) compared to \( T_D \).

All power lines are of similar design.

Perhaps, \( T_H \) records are more susceptible to errors than \( T_D \) records. For instance, an observer may be hesitant in regarding a given thunders as the last when non-audible (out of thunder range) but visible lightning activity is present. This error is more probable after dark.
450 weather stations having continuous records for 10 to 30 years
(MacGorman et al., 1984)
(3) Annual ground flash density ($N_g$)

This parameter is a base for any estimate of lightning strike incidence to a ground-based structure.

$N_g$ can be determined using:

a) lightning flash counters
b) lightning locating systems (MDF or TOA)
c) satellite observations (there is presently no way to distinguish between ground and cloud flashes from a satellite)

**Lightning flash counters**

This is an instrument that produces a registration if the electric field change from the lightning, after being appropriately filtered, exceeds a fixed threshold level.

---

**CIGRE 10 kHz**

Center frequency of the filter

10 kHz

3-dB limits

2 – 50 kHz

Sensitivity threshold level

$\sim 20 \text{ V/m}$

---

Fig. 1  Frequency-response characteristic of RSA 10 lightning-flash-counter input circuit (Anderson et al., 1973)
Operation characteristics of lightning flash counters

Since (1) the probability of counting a flash gradually decreases with distance from the counter and (2) the total counter registrations \( K \) inevitably include both ground and cloud flashes, \( N_g \) is determined as

\[
N_g = \frac{Y_g K}{\pi R_g^2}
\]

where \( R_g \) is the effective range of the counter to ground flashes ("misses" within this range are exactly compensated for by the "hits" outside of it); \( Y_g \) is a correction factor to exclude undesired cloud flashes from \( K \).

For \( N_g = \text{const} \)

\[
R_g = \sqrt{2 \int_0^\infty P(r) \cdot r \, dr}
\]

\( P(r) \) depends upon:
- signal strength distribution at the source;
- signal attenuation with distance;
- counter threshold level.

\[
Y_g = \left[ 1 + \pi \frac{(R_c)^2}{(R_g)^2} \right]^{-1}
\]

where \( \pi = N_c / N_g \) is the ratio of the densities of cloud and ground flashes, and \( R_c \) is the effective range of the counter to cloud flashes. \( R_c / R_g \) is different for different filters and threshold levels.

Fig 24: Probability functions for the RSA 10 counter on ground and inter-cloud flashes showing good discrimination against the latter.

**TABLE 4** Pertinent characteristics of three lightning flash counters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>CIGRE</th>
<th>RSA 10</th>
<th>RSA 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response frequencies</td>
<td>kHz</td>
<td>kHz</td>
<td>kHz</td>
</tr>
<tr>
<td>3 dB limits - lower</td>
<td>0.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>centre</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>upper</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective ranges</td>
<td>km</td>
<td>km</td>
<td>km</td>
</tr>
<tr>
<td>ground flashes</td>
<td>36.8</td>
<td>19.9</td>
<td>15.7</td>
</tr>
<tr>
<td>cloud flashes</td>
<td>16.7</td>
<td>6.9</td>
<td>7.6</td>
</tr>
</tbody>
</table>

*Correction factor \( Y_g \) is the proportion of total registration of counters which are ground flashes. The value for the RSA 10 counter was observed while the values for the CIGRE and RSA 5 counters were deduced from comparative measurements.

(Anderson, 1980)
LIGHTNING RECORDING SCHEME
WEERLIGREGISTRASIESKEMA

LIGHTNING GROUND FLASH DENSITY
(nearly 400 CIGRE 10 kHz lightning flash counters were used)

(1975 - 1986) 11 years

GRONDWEERLIGDIGTIEID

DIVISION OF ENERGY TECHNOLOGY
CSIR P O BOX 395 PRETORIA 0001 TÉLÉPHONE 841-3963/4482

DIVISIE VIR ENERGITEKNOLOGIE
WNWR POSBUS 395 PRETORIA 0001 TELEFOON 841-3963/4482
the peak Florida value. This observation is consistent with the enhanced winter lightning activity over the Gulf Stream reported previously (Orville 1990; Biswas and Hobbs 1990). A broad relative flash density maximum is apparent inland along the Carolina Coast that extends into eastern Virginia. Other maxima occur in eastern Texas, Kansas, and along the Illinois–Indiana border.

Some annual maxima in Fig. 4 can be associated with only a few storms. For example, the peak in the Illinois–Indiana area is associated with thunderstorms in late May and early June. In other cases, only one storm may have a significant effect on the annual flash density. Note the northwest to southeast orientation of the 1–2 flashes km⁻² contour region extending from Lake Ontario southeast across New York to Connecticut, where the annual flash density reaches 2–3 flashes km⁻². This direction is the path followed by the October 14 storm that produced approximately one-third of the annual flash count for this area.

Variations in the observed flash density values in Fig. 4 are believed to reflect the natural variations in flash density that occurred in 1989. There may, however, be artifacts in the dataset. One example is the relatively lower flash density value of 2–3 flashes km⁻² extending into Louisiana that may be real or, in part, the result of not having a direction finder in Louisiana (See Fig. 2). Caution should also be followed in interpreting the flash density in the west using the BLM direction finders. The true flash density may be higher because the BLM flash detection efficiency may be less than the 70% assumed for the NSSL and SUNYA networks. Certainly, Fig. 1 suggests that relatively higher flash densities should be expected in northern Arizona.

Another problem exists in the dataset that does not seem to be apparent or important in Figure 4. This problem occurs along the interface of the BLM network with the NSSL and SUNYA networks, or roughly the 104th meridian paralleling the New Mexico–Texas border.
114 direction finders; 70% detection efficiency assumed; 13.4 mln flashes recorded

FIG. 4: Annual lightning flash density contours for 1989 are drawn on a grid of 120 horizontal points and 100 vertical points. This provides for a spatial resolution of 30 km in the vertical and 50 km in the horizontal. The highest lightning flash density is northeast of Tampa, Florida, with secondary maxima occurring over the Gulf Stream in the Atlantic Ocean and in eastern Kansas. The lightning flash density values have been multiplied by a factor of 1.4, derived from assuming a detection efficiency of 70% throughout the network.
Lightning incidence vs. structure height

Table 3: Incidence of flashes to tall structures

<table>
<thead>
<tr>
<th>Source</th>
<th>Structure height in m</th>
<th>Regional keramic level T_D (Note 1)</th>
<th>Annual frequency of recorded flashes, normalised to a T_D base of 30</th>
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<tr>
<td>Kudoyanskaya(1)**</td>
<td>25</td>
<td>25</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>...</td>
<td>0.08</td>
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<td></td>
<td>45</td>
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<td></td>
<td>55</td>
<td>...</td>
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<td></td>
<td>65</td>
<td>...</td>
<td>0.15</td>
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<tr>
<td></td>
<td>75</td>
<td>...</td>
<td>0.18</td>
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<tr>
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<td>...</td>
<td>0.22</td>
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<td></td>
<td>95</td>
<td>...</td>
<td>0.20</td>
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<tr>
<td></td>
<td>115</td>
<td>...</td>
<td>0.33</td>
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<tr>
<td></td>
<td>540</td>
<td>...</td>
<td>41.00</td>
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<tr>
<td>Muller Hillsbrand(1)**</td>
<td>38</td>
<td>10</td>
<td>0.16</td>
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<td>...</td>
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<td>Supor et al(1)**</td>
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<td></td>
<td>225</td>
<td>...</td>
<td>2.13</td>
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<tr>
<td>Beck — as summarised in Ciano and Pierce (1)**</td>
<td>24</td>
<td>32</td>
<td>0.18</td>
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<td>...</td>
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<td>340</td>
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<td>3.00</td>
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<td>McCandliss(1)**</td>
<td>400</td>
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<td>23.00</td>
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<tr>
<td>Anderson and Jenner(1)**</td>
<td>20</td>
<td>54</td>
<td>0.03</td>
</tr>
<tr>
<td>Berger(1) (negative flashes only) (Note 2)</td>
<td>350</td>
<td>36</td>
<td>31.00</td>
</tr>
</tbody>
</table>

Note 1 In many instances, the values quoted for regional T_D are averages for many different structures distributed over one geographically similar area.

Note 2 A structure 'effective' height of 300 m has been assigned to this mountain-top station, as explained in 4.1.

(Eriksson, 1978)

\[ N_S = \frac{N_{obs}}{N_G} \]

\[ N_G = 0.04 T_D^{1.25} \]

Fig. 2 Observed incidence of strikes to structures of various heights.
(Eriksson, 1978)

Fig. 6. Proportions of conventional and structure-initiated (triggered) lightning as a function of structure height.
(Pierce, 1974)
Table 4
The relative incidence of upward flashes from tall structures

<table>
<thead>
<tr>
<th>Source</th>
<th>Structure height in m</th>
<th>Relative frequency of occurrence of upward flashes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pierce(*)</td>
<td>150</td>
<td>23%</td>
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<tr>
<td></td>
<td>200</td>
<td>35%</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>80%</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>91%</td>
</tr>
<tr>
<td>McCane(*)</td>
<td>110</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>24%</td>
</tr>
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<td></td>
<td>400</td>
<td>94%</td>
</tr>
<tr>
<td>Berger(*)</td>
<td>350 m*</td>
<td>84%</td>
</tr>
<tr>
<td>Gorin(m, m)</td>
<td>540 m</td>
<td>93%**</td>
</tr>
<tr>
<td>Garagash(m)</td>
<td>500 m</td>
<td>98%</td>
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</table>

Notes: *An effective height of 350 m has again been assigned to Berger's statistics. (See 4.1).
**50 per cent of the flashes recorded in this study were classified as unidentified. The relative incidence of upward flashes is based upon analysis of only the identified data.

(Eriksson, 1978)

Fig. 2
Electric field distribution about tall earthed structure
(Golde, 1978)

(k_d = 1 - \frac{P_u.\%}{100})

(Fraction of downward flashes)

(Eriksson and Meal, 1984)

Structure "slenderness" (ratio of height to radius)

Fig. 4. Ratio of resultant field strength E_o at the top of a structure to the inducing field E_0 for given ratios of structure dimensions L/R.

(Golde, 1977) \quad (E_o = \text{electric field at ground level})

(Anderson, 1985)
Number of lightning strikes to a structure

When the incidence of only downward lightning strikes is concerned, it is common to ascribe the so-called "equivalent attractive (or exposure) area" to a ground-based object.

For a free-standing structure, such as a mast or chimney, this area is given by

\[ A = \pi R_a^2 \]

where \( R_a \) is the equivalent attractive radius.

For stretched structures (such as a power line) the equivalent attractive area is termed the "shadow zone" or "attractive swath" and expressed as

\[ A = L(b + 2R_a) \]

where, for power lines, \( L \) is the line length, \( b \) is the line effective width, and \( R_a \) is the equivalent attractive distance commonly thought to be about equal to the equivalent attractive radius for a free-standing structure of the same height.

\( b \) is usually taken as the distance between overhead shield wires or between outer phase conductors.

\( R_a \) is usually assumed to be a function of structure height \( H_s \):

\[ R_a = \alpha H_s^\beta \]

where \( \alpha \) and \( \beta \) can be estimated using actual data on lightning incidence to structures of different heights.

\[ R_a = \sqrt{N_a/N_g / \pi} \]

Some data, but different relationships between \( N_g \) and \( T_D \) were used.

Fig. 6 Equivalent attractive radius for lightning on structures depending on their height \( H_s \). (Anderson, 1985)
Interception of flashes by the line

(IEEE WG on Lightning Performance

\[ W = b + 4 \left( h^{1.09} \right) \]

\[ N_L = 0.004 \ T^{1.25} \left( b + 4 \left( h^{1.09} \right) \right) \]

**Example:**

\[ h = 26 \ m \quad b = 6.7 \ m \quad T = 30 \]

\[ N_L = 0.004 \times 30^{1.25} \left( 6.7 + 4 \times 26^{1.09} \right) \approx 41 \ \text{per 100 km per year} \]
### Table 3

<table>
<thead>
<tr>
<th>Number of years</th>
<th>Probability</th>
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<td>0</td>
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<tr>
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<td>2</td>
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<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

The body of the table indicates the percentiles of the number of times a 200 m tower will be struck by lightning (Andersson, 1985).

### Table 2

<table>
<thead>
<tr>
<th>Number of times struck by lightning</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

The probability of a structure represented by its attractive area to be struck exactly 0, 1, 2, 3, ... in 10 years is given by the Poisson probability distribution:

\[
P(n) = \frac{e^{-A} A^n}{n!}
\]

where

\[
N = A \cdot r, \quad r = 1.5 \text{ yr}^{-1}
\]

The probability of a structure can be estimated as a structure can be estimated as the sum of the probabilities of it being struck by lightning in 0, 1, 2, ..., 10 years:

\[
N = A \cdot N, \quad (\text{on average})
\]

For power lines:

\[
H_s = H - \frac{2d}{3}
\]

where

- \(H_s\) is the height of the shield wire
- \(H\) is the height of the top conductor
- \(d\) is the shield wire diameter

For T = 1 yr:

\[
A = \pi \left(\frac{2H_s}{5}\right)^2 0.5 \text{ km}^2
\]

The probability of a structure is less than 60%.
3. Electrical structure of thunderstorm clouds
Electrical structure of thundercloud

A model for the charge structure of a thundercloud was developed by the early 1930s. The model was derived from ground-based measurements of both the electric fields associated with the cloud charges and the electric field changes associated with the effective neutralization of a portion of those cloud charges by lightning. In that model, the primary thundercloud charges form a positive electric dipole (positive charge region above, negative charge region).

![Diagram of thundercloud charge distribution](image)

**Fig. 3.2** Probable distribution of the thundercloud charges, P, N, and p for a South African thundercloud according to Malan (1952, 1963). Solid black circles indicate locations of effective point charges, typically P = +40 coul, N = −40 coul, and p = +10 coul, to give observed electric field intensity in the vicinity of the thundercloud. (Uman, 1984)

Simpson and Scrase (1937) and Simpson and Robinson (1941) verified this dipole structure from in-cloud measurements made with instrumented balloons, and, additionally, identified a localized region of positive charge at the base of the cloud (tripole charge structure). The tripolar model has been widely accepted and quoted, although 42 out of 69 (~60%) soundings of Simpson et al. fit neither tripolar nor dipolar model.
Fig. 2. Variation of fields and field-changes with distance.
(Pierce; in Golde, 1977)

Cloud electric field at ground level

Types of electric field changes in cloud discharges

28 September 1961

Fig. 3. Types of cloud discharge field-change and their occurrence throughout various states of the storm shown in Fig. 2. The electric field curve is also included for reference to show the time of occurrence of each type (from Ogawa and Brook, 1964).
Cloud charge structure from in-situ measurements

Figure 1. Balloon measurements of corona current and the inferred vertical electric field (E) versus altitude and air temperature inside a small thunderstorm in New Mexico. Charge regions are labeled positive (pos) or negative (neg) on the right. The total time to acquire the record above cloud base was about 11 min. [Adapted from reference 12.]

(Byrne et al., 1983) No lightning was produced by the storm.

\[ I_c = a(E^2 - M^2) \]

\[ \rho = \varepsilon_0 \frac{\Delta E_z}{\Delta Z} \] - one-dimensional approximation to Gauss' law
Electric field from a stationary point charge

\[ E = \frac{(Q/4\pi \varepsilon_0 R^2)}{(4\pi \varepsilon_0)^{-1}} = 9 \times 10^9 \text{ m/F} \]

\[ \sin \alpha = \frac{H}{R} \]
\[ \frac{\sin \alpha}{R^2} = \frac{H}{R^3} = \frac{H}{(H^2 + D^2)^{3/2}} \]

Fig. A.1 Diagram for the calculation of the electric field intensity at a due to a positive point charge \(-Q\) at height \(H\) above a conducting plane. (Uman, 1987)

\[ E_{+Q} = E_{-Q} = \frac{|Q|}{[4\pi \varepsilon_0 (H^2 + D^2)]} \text{ - magnitudes of the field vectors} \]

The tangential field components \((E_{+Q} \cos \alpha = -E_{-Q} \cos \alpha)\) add to zero.

The normal field components \((E_{+Q} \sin \alpha = E_{-Q} \sin \alpha)\) add to

\[ E_{\text{total}} = 2 \frac{Q}{4\pi \varepsilon_0 R^2} \sin \alpha = \frac{2QH}{4\pi \varepsilon_0 (H^2 + D^2)^{3/2}} = \frac{2Q}{4\pi \varepsilon_0 D^2} \left[ \frac{H/D}{1 + (H/D)^2} \right]^{3/2} \]

Fig. A.2 Electric field intensity for the charge configuration given in Fig. A.1 as a function of \(H/D\). To obtain electric field in volts per meter for a given \(H/D\) multiply the ordinate by \(Q \times 10^7 D^2\). (Uman, 1987)

If \((H/D)^2 \ll 1\)

\[ E_{\text{total}} \approx \frac{2QH}{4\pi \varepsilon_0 D^3} = \frac{M}{4\pi \varepsilon_0 D^3} \]

\[ \text{electric dipole moment of the charge } Q \text{ and its image} \]

\[ R^2 \text{ increases faster than } \sin \alpha \text{ increases} \]
Cloud electric field at the ground as a function of distance

\[ E = E_{Q_P} + E_{Q_N} + E_{Q_P} \text{ for } Q_P = 0, 5C, \text{ and } 10C \]

If \( Q_P = 0 \)

\[ D_o = \left[ \frac{(H_P H_N)^{2/3} (H_P^{2/3} + H_N^{2/3})}{2} \right]^{1/2} \]

\[ D < D_o \quad \text{○} \]

\[ \text{("R² is more important")} \]

\[ D > D_o \quad \text{●} \]

\[ \text{("sinh is more important")} \]

\( Q_P \) at 10 km
\( Q_N \) at 5 km
\( Q_P \) at 2 km

Effective Charge Height, km

Electric Field Intensity, kV/m

Distance \( D \), kilometers

\[ E_P = 2Q_H / [4\pi\varepsilon_0 (H^2 + D^2)^{3/2}] \]

\[ E = 1.8 \times 10^{10} \left[ \frac{2 \times 10^5 Q_P}{(4 \times 10^5 + D^2)^{3/2}} - \frac{2 \times 10^5}{(2.5 \times 10^7 + D^2)^{3/2}} + \frac{4 \times 10^5}{(10^8 + D^2)^{3/2}} \right] \text{ V/m} \]

\[ \frac{2}{(4\pi\varepsilon_0)} \]

\( Q_P \) (D in m)

(The effects of space charges are neglected)
Charge locations for strokes and continuing current in four multiple-stroke flashes in New Mexico

Measurements of the electrostatic field changes at 8 stations and the point charge model were used

\[ \Delta E_i = \frac{1}{4\pi\varepsilon_0} \frac{2Qz}{[ (x-x_i)^2 + (y-y_i)^2 + z^2 ]^{3/2}} = \frac{1}{4\pi\varepsilon_0} \frac{2Qz}{R_i^3} \]

---

**FLASH 9**
Q TOTAL = 30 COUL

---

**FLASH 14**
Q TOTAL = 66 COUL

---

**FLASH 16**
Q TOTAL = 40 COUL

---

**FLASH 17**
Q TOTAL = 46 COUL

---

The size of the spheres shown surrounding the charge location points was arbitrarily determined by assuming the charge density to be 20 nC/m³. Charge volumes for the continuing current of flash 14 are cumulative from the beginning of the discharge, except for the last volume.

(Krehbiel et al., 1979)

Elevation 4.5 - 6 km (one exception 3.6 km)
Temperature -9 to -17°C
The overall charge is apparently localized in pockets of relatively high space-charge concentration.
The main negative charge is found in a relatively narrow range of altitudes at temperatures that vary between 0 and -25°C in different kinds of storms.

Figure 6. Sketches of the negative charge regions inferred from analyses of field changes during cloud-to-ground lightning. Note that the temperature levels in summer storms in New Mexico and Florida are similar, even though the Florida storms have much more liquid water and precipitation below the 0°C level. Analyses of winter lightning in Japan suggest that the negative charge is at a lower altitude but a similar temperature level as the summer storms. [Adapted from reference 25.] (Krehbiel, 1986)

The main negative charge appears to remain at approximately constant altitude or temperature as the storm grows.

FIGURE 8.6 The altitude of the lightning charge centers for the first 15 discharges in the small Florida storm of Figure 6.3. The upper positive-charge centers of the intracloud flashes increased in altitude as the storm grew, while the negative-charge centers remained at constant altitude. Two cloud-to-ground discharges occurred toward the end of the sequence. (Krehbiel, 1986)
Cloud charge structure from the electric field soundings through thunderstorms

The charge distribution in the cloud is modeled as charge layers of infinite horizontal extent stacked one above another.

\[ \rho = \varepsilon_0 \frac{\Delta E_z}{\Delta Z} \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{F/m} \]

**Figure 2.** Balloon measurements of the vertical electric field inside a small Alabama thunderstorm. The values of inferred average charge density, assuming that the field is steady with time, are shown on the right. [Adapted from reference 13.]

(Marshall and Rust, 1991)

None of a total of 12 studied storms in Alabama, New Mexico, and Oklahoma had charge distributions that fit the "classical" (tripolar) model. Four to ten charge regions were inferred, using the one-dimensional approximation to Gauss' law.
Strong electrification does not occur until the cloud and precipitation develop above a threshold altitude that is 7–8 km above MSL in summer.

(Figure 8.7 The radar reflectivity of precipitation versus height and time in a small storm near Langmuir Laboratory on August 3, 1984, and a record of the electric field at the ground 25 km from the storm. The electrification was associated with convective growth above 8-km altitude (about -20°C) and with the development of moderately strong precipitation up to this altitude. An initial convective surge between 12:20 and 12:25 produced only weak electrification, as measured by instrumented aircraft inside the storm. (Unpublished data from J. E. Dye [National Center for Atmospheric Research] and C. B. Moore and W. P. Winn [New Mexico Institute of Mining and Technology].) (Krehbiel, 1986)

The threshold altitude corresponds to an air temperature of -15 to -20°C. At these temperatures the clouds contain both supercooled water and ice.)
Mechanisms of cloud electrification

Cloud electrification can be viewed as acting on two spatial scales: first there must be a small-scale process that electrifies individual cloud particles and then there must be another process that effectively separates these charges, preferentially according to their polarity, by distances on the order of kilometers.

Basically, there are two types of models for the generation of the main cloud charge dipole:

**Precipitation model**

The electric charges are produced by collisions between falling precipitation particles. The large-scale separation is provided by the action of gravity.

**Convection model**

The electric charges are supplied initially by two external sources: corona discharge and cosmic rays. The organized convection provides the large-scale separation.

Two models attempt to explain the electrical structure of thunderclouds. The precipitation model (left) suggests that gravity pulls heavy raindrops, hailstones and millimeter-size ice particles called graupel past smaller water droplets and ice crystals, which remain suspended. Collisions between the falling particles and the suspended mist are conjectured to transfer positive charge to the mist and negative charge to the heavier particles. As these heavier particles fall, the lower part of the cloud becomes negatively charged and the upper part becomes positively charged—a structure known as a positive dipole. The convection hypothesis (right) proposes that warm air currents carry positive charges released from the earth’s surface to the top of the cloud. Negative charges, produced by cosmic rays above the cloud, are attracted to the cloud’s surface by the positive charges within it. The negative charges attach themselves to cloud particles to form a negative “screening layer.” Downdrafts are assumed to carry the negative charges downward; this process again results in a positive dipole. Note that the convection model invokes no precipitation and the precipitation model no convection.

The precipitation model can account for more aspects of cloud electrification than the convection model. However, the latter can play an important role at some stage of the cloud lifecycle.
The droplets remain in a supercooled liquid state until they contact an ice surface whereupon they freeze and stick to the surface in a process called riming.

It is believed that the polarity of the charge that is separated in ice-graupel collisions is determined by the rates at which these ice surfaces are growing. The surface that is growing fastest acquires positive charge. There is no consensus on the detailed physics involved.

**Graupel** = soft hail (millimeter-size ice particles) (Williams, 1988)

**Microphysics of Charge Transfer** involves collisions between graupel particles and ice crystals. The heavy graupel particles fall through a suspension of smaller ice crystals (hexagons) and supercooled water droplets (dots). Laboratory experiments show that when the temperature is below a critical value called the charge-reversal temperature, $T_R$, the falling graupel particles acquire a negative charge in collisions with the ice crystals. At temperatures above $T_R$ they acquire a positive charge. $T_R$ is thought to be about −15 degrees C, the temperature of the main negative region found in thunderclouds; thus graupel picks up a positive charge when it falls below this altitude to higher temperatures. There is now evidence that these positively charged graupel particles form the lower positive region of the thundercloud tripoles.

Other microscale mechanisms are possible.

**Figure 7.** The charge acquired by a riming hail particle during collisions with ice crystals is a function of the temperature of the rime (and the cloud liquid water content).  [Adapted from reference 31.] (Jayaratne et. al., 1993)
4. Properties of the negative-downward lightning discharge to ground
Illustration of the various processes comprising a negative C-G flash

- Cloud Charge Distribution
- Preliminary Breakdown
- Stepped Leader
- Attachment Process
- First Return Stroke
- K and J Processes
- Dart Leader
- Second Return Stroke

Time Stamps:
- t = 0
- 1.00 ms
- 1.10 ms
- 1.15 ms
- 1.20 ms
- 19.00 ms
- 20.00 ms
- 20.10 ms
- 20.15 ms
- 20.20 ms
- 40.00 ms
- 60.00 ms
- 61.00 ms
- 62.00 ms
- 62.05 ms
FIGURE 5  SCHEMATIC ILLUSTRATION OF PROCESSES AND CURRENTS OCCURRING DURING A FLASH TO GROUND (Cianos and Pierce, 1972)
516 Interstroke Intervals in 132 Flashes in Florida and New Mexico. Intervals Preceding Long Continuing Current are Shown Shaded.
Fig. 1  Example of the current record of a 10 stroke flash.

Fig. 2  Portion of the flash between the arrows shown in Fig. 1

Fig. 3  Example of a return-stroke current pulse waveshape

Fig. 4  Example of an M-current pulse waveshape
Photographic record

\begin{itemize}
\item \( R_1 R_2 \) M Components
\item \( R_3 R_4 R_5 \) \( R_6 R_7 R_8 \)
\item "Electric field record"
\item Continuing current
\item \( C \) field change \( I_1 \) \( \frac{1}{10} \) volts/cm
\item \( R \) change \( J \) change
\item \( \tau = 4 \text{ s} \)
\item \( E \)-field
\item \( \tau = 70 \mu \text{s} \)
\item \( M \) change
\item \( K \) change
\item "Electric field change" record
\item Flash with continuing current interval (flash no. 106, 20 km distant)
\item 0 50 100
\item Time, msec
\end{itemize}

Photographic record

\begin{itemize}
\item \( R_1 R_2 R_3 \) \( R_4 R_5 R_6 R_7 R_8 \) \( R_9 \)
\item "Electric field record"
\item \( I_1 \) \( \frac{1}{10} \) volts/cm
\item "Electric field change" record
\item Flash without continuing current (flash no. 109, 19 km distant)
\item 0 50 100
\item Time, msec
\end{itemize}

\( \tau = 4 \text{ s} \)

\( \tau = 70 \mu \text{s} \)
Preliminary Breakdown

This is the process within the cloud initiating or leading to the initiation of the downward-moving stepped leader of a cloud-to-ground flash. The existence of preliminary breakdown as a unique lightning process was inferred from (1) the observation that clouds often produce luminosity for a hundred or more milliseconds before the emergence of the stepped leader from the cloud base and (2) the observation of relatively long (over 100 ms) pre-first-stroke field changes.

Typical (according to Clarence and Malan, 1957 and Schouland, 1956) electric field changes due to processes preceding the first return stroke of a ground flash:

The B field change exhibited the polarity reversal in the distance range from 2 to 5 km. This was interpreted by Clarence and Malan (1957) as indicative of a vertical discharge between the main negative charge center (located at 3.6 km above ground or 5.4 km above MSL) and the positive charge center at the cloud base (at 1.4 km above ground or 3.2 km above MSL). The I field change was suggested to be due to negative charging of the vertical breakdown channel until the field at the bottom of the channel is high enough to initiate a stepped leader, the conjecture not confirmed by more recent studies.
From multiple-station electric field measurements, preliminary breakdown appeared as a series of vertical and horizontal charge motions within the cloud, one of which finally resulted in the launching of a leader toward ground. The activity effectively transported negative charge away from the first-stroke charge volume.

Figure 1.5: Multiple station electric field observations of cloud-to-ground flash at New Mexico. (a). Field changes observed at different locations produced by a 5-stroke cloud-to-ground discharge. (b). Plane and vertical projection views of effective charge centers for strokes and dipole moments for interstroke intervals. Adapted from Krehbiel et al. (1979).

<table>
<thead>
<tr>
<th>Event</th>
<th>$\Delta t, \text{ms}$</th>
<th>$M, \text{Ckm}$</th>
<th>$Q, \text{C}$</th>
<th>$\Delta R, \text{km}$</th>
<th>$I_{av}, \text{A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>46</td>
<td>3.0</td>
<td>2.0</td>
<td>1.5</td>
<td>43</td>
</tr>
<tr>
<td>B</td>
<td>51</td>
<td>13.4</td>
<td>3.6</td>
<td>3.8</td>
<td>71</td>
</tr>
</tbody>
</table>
Figure 4.1: Overall time waveforms of the electrostatic field change, fast electric field change, and logarithmic HF radiation amplitude of Flash 222235225.
Figure 4.2: Composite overviews of selected radiation sources of Flash 222.235225. (a), Phase projection format. (b). Azimuth-elevation format.
Figure 4.4: Preliminary breakdown inside the cloud. (a). Time waveforms from the beginning of the flash to the initial stroke with the preliminary breakdown indicated by PB. (b). Radiation sources of PB.
Figure 4.5: Time waveforms and radiation sources of the initial leader to ground.
Fig. 1. Electric fields radiated by lightning discharges at distances of 50-100 km. The arrows indicate the same event on time scales of 2 ms/div, 0.4 ms/div, and 40 μs/div. Positive field is shown as a downward deflection on all records.
Large bipolar electric fields radiated by lightning discharges at end of preliminary breakdown or beginning of stepped leader at distances of 30 to 50 km. Each waveform is shown on both a slow (20 μsec/div) and fast (4 μsec/div) time scale, the former indented with respect to the latter. The polarity of the waveforms is consistent with the lowering of negative charge or the raising of positive. The time interval between the return stroke R and the discharge preceding it in (c) and (d) is shown on the scale at left. Adapted from Weidman and Krider (1979).
Leader mechanisms from the laboratory experiments

(Bazelyan et al., 1978)

**Positive leader**

- Initial phase
- Break-through phase
- High-temperature plasma \( (T \approx 20 \times 10^3 \text{K}; \ i \approx 10^2 \text{S/cm}) \)
- \( d = 1 \text{mm} \)
- \( \Delta t \approx 1 \text{cm}^2 \)
- Break-through phase

\[ \frac{dU}{dt} > 5 \text{kV/\mu s} \] (continuous leader)

**Negative leader**

Cathode \( \rightarrow \) Anode

6 (leader step) - High potential is rapidly transferred to the bottom of the secondary channel launching a burst of the anode-directed streamers. As a result, a large charge is moved into the gap reducing the potential of the old channel sections and causing a "return stroke" (an upward-propagating luminosity wave).

Leader-channel conductivity is about 2 orders of magnitude higher than the conductivity of streamer channel.

- Mid-gap streamers
  - 1 (leader tip)
  - 2 (leader channel)
  - 3 (cathode-directed streamers)
  - 4 (plasma nucleus) \( \sim 1 \text{cm}^3 \)
  - 5 (anode-directed streamers) \( E \approx 6 \text{kV/cm} \)
  - \( E \approx 13 \text{kV/cm} \)
Electric field change due to positively charged leader

\[ \rho_L = \text{const} \]

\[
\sin \alpha_S = \frac{H}{R_S} \\
R_S = \left( H^2 + D^2 \right)^{\frac{1}{2}}
\]

\[
\sin \alpha = \frac{Z'}{R} \\
R = \left( Z'^2 + D^2 \right)^{\frac{1}{2}}
\]

\[
(4\pi \varepsilon_0)^{-1} \approx 9 \times 10^9 \frac{m}{F}
\]

\[
dE_S = -2 \frac{\rho_L dZ'}{4\pi \varepsilon_0 R_S^2} \sin \alpha_S \quad \text{(due to the decrease in the source charge)}
\]

\[
dE_\text{total} = 2 \frac{\rho_L dZ'}{4\pi \varepsilon_0 R^2} \sin \alpha \quad \text{(due to the charge within } dz')
\]

\[
E_\text{channel} = \int_{H_0}^{H} dE_\text{total} = 2 \rho_L \int_{H_0}^{H} \frac{Z' dZ'}{(Z'^2 + D^2)^{\frac{3}{2}}} = \frac{2 \rho_L}{4\pi \varepsilon_0} \left[ \frac{1}{(H_0^2 + D^2)^{\frac{1}{2}}} - \frac{1}{(H^2 + D^2)^{\frac{1}{2}}} \right]
\]

\[
\frac{E_S}{\Delta E} = -2 \frac{\sin \alpha_S}{4\pi \varepsilon_0 R_S^2} \int_{H_0}^{H} \rho_L dZ' = -\frac{2 \rho_L l}{4\pi \varepsilon_0} \frac{H}{(H^2 + D^2)^{\frac{3}{2}}} \quad \text{where } l = H - H_0 \quad \left( l = \int V(t) dt \right)
\]

Field change \[ \frac{H_0}{\varepsilon_0} c \exp \left( -\frac{x^2}{\varepsilon_0} \right) \]
Electric field change for a negative leader ($\rho_L =$ const)

$$\Delta E = -\frac{2\rho_L}{4\pi\varepsilon_0} \left[ \frac{1}{(H_B^2 + D^2)^{3/2}} - \frac{1}{(H^2 + D^2)^{3/2}} - \frac{(H-H_B)H}{(H^2 + D^2)^{3/2}} \right]$$

$$= -\frac{2\rho_L}{4\pi\varepsilon_0 D} \left[ \frac{1}{(1 + H_B^2/D^2)^{3/2}} - \frac{1}{(1 + H^2/D^2)^{3/2}} - \frac{H-H_B}{D} \cdot \frac{H}{D} \cdot \frac{1}{(1 + H^2/D)^{3/2}} \right]$$

Multiply by $\rho_L / 2\pi\varepsilon_0 D$ in SI units to obtain $V/m$

Net leader field change

$$\Delta E (H_B/H = 0) = 0 \quad \text{for } H/D = 1.27 \quad (\alpha_s \approx 52^\circ)$$

(leader touches the ground)

\[\text{FIGS 2}\]
Fig. 9a. Electric field change for the first stroke in a three-stroke flash that occurred at 1916:31 UT on July 15, 1979, at a distance of 12.4 km. Originally recorded on the FN channel of an Ampex FR 1900 tape recorder and 10 µs moving-averaged. Positive field change deflects downward. Asterisk marks starting point of the leader field change. Shown is an example of the leader duration measurement.

Fig. 9b. Electric field change for the fifth stroke in a five-stroke flash whose overall field change is shown in Figure 1. See also comments for Figure 9a.

of the relevant data reported previously (see section 1). The geometric mean duration for first leaders was found to be 35 ms, in agreement with most of the data available (see Table 1), but about twice as long as reported for Florida lightning by Beasley et al. [1982]. About 35% of the first leader durations analyzed by Beasley et al. [1982, Figure 20] appeared to be shorter than 12.5 ms, while in our data such relatively short durations were measured only for one of 71 first
Net leader field change versus distance
(first leaders)

PLOT SYMBOL STORM YEAR DAY
△ 76181
▼ 76195
▼ 76201
▼ 76203
△ 77203
■ 77211
▲ 77212a
▲ 77212b
★ 77220

'DENOTES TWO EVENTS AT SAME COORDINATES

near intermediate
far intermediate
distant

classes of shapes

R.S. L
TABLE 3. Occurrence (Percent) of Three Different Categories of Leader Wave Shapes

<table>
<thead>
<tr>
<th>Distance Range</th>
<th>Closer Than 5 km</th>
<th>From 5 to 6 km (including 6 km)</th>
<th>From 6 to 9 km (including 9 km)</th>
<th>From 9 to 10 km</th>
<th>10 km and Farther</th>
<th>All Distances</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HN</td>
<td>HP</td>
<td>MP</td>
<td>HN</td>
<td>HP</td>
<td>MP</td>
</tr>
<tr>
<td>First strokes in a flash</td>
<td>91</td>
<td>9</td>
<td>0</td>
<td>45</td>
<td>55</td>
<td>0</td>
</tr>
<tr>
<td>Percentage</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>24</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Total number in the distance range</td>
<td>92</td>
<td>8</td>
<td>0</td>
<td>27</td>
<td>73</td>
<td>0</td>
</tr>
<tr>
<td>Second, third, and fourth strokes</td>
<td>92</td>
<td>8</td>
<td>0</td>
<td>27</td>
<td>73</td>
<td>0</td>
</tr>
<tr>
<td>Percentage</td>
<td>12</td>
<td>15</td>
<td>15</td>
<td>46</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Total number in the distance range</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>78</td>
<td>17</td>
<td>5</td>
</tr>
<tr>
<td>Fifth, sixth, and seventh strokes</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>78</td>
<td>17</td>
<td>5</td>
</tr>
<tr>
<td>Percentage</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Total number in the distance range</td>
<td>88</td>
<td>12</td>
<td>0</td>
<td>94</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Strokes from 8 through 18</td>
<td>88</td>
<td>12</td>
<td>0</td>
<td>94</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Percentage</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>16</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Total number in the distance range</td>
<td>92</td>
<td>8</td>
<td>0</td>
<td>35</td>
<td>65</td>
<td>0</td>
</tr>
<tr>
<td>All strokes from 1 through 18</td>
<td>92</td>
<td>8</td>
<td>0</td>
<td>35</td>
<td>65</td>
<td>0</td>
</tr>
</tbody>
</table>

- HN, hook-shaped, net negative; HP, hook-shaped, net positive; MP, monotonic, positive versus distance and stroke order.
- See also Figures 2a-2e.
Leader to return-stroke field change ratio \((\beta_L = \text{const})\)

\[
\Delta E_L = -\frac{2\beta_L}{4\pi\varepsilon_0 D} \left[ 1 - \frac{1}{(1 + H^2/D^2)^{1/2}} \right] + \frac{2\beta_L}{4\pi\varepsilon_0 D} \frac{H^2}{D^2} \frac{1}{(1 + H^2/D^2)^{3/2}} \quad (H_B = 0)
\]

due to channel charge
due to decrease in source charge

\[|\Delta Q| = |\beta_L| H\]

When \(D \to 0\)

\(\Delta E_L/\Delta E_R \to -1\)

\[
\Delta E_R = \frac{2\beta_L}{4\pi\varepsilon_0 D} \left[ 1 - \frac{1}{(1 + H^2/D^2)^{1/2}} \right] - \text{due to removal of channel charge}
\]
Fig. 3. Ratio of the leader-to-return stroke field change $\Delta E_L/\Delta E_R$ as a function of distance and thunderstorm day for five different sets of strokes: (a) first strokes; (b) second, third, and fourth strokes creating a new termination on ground; (c) second, third, and fourth strokes following the same channel as the preceding stroke; (d) fifth, sixth, and seventh strokes; and (e) strokes of order 8 through 18. Both solid and dashed curves show the ratio versus distance predicted by the simple model with vertical, uniformly charged channel and with cloud charge source centered at 7.5 km.
Leader to return-stroke field change ratio (inverted "L" channel shape)

\[ \frac{\Delta E_L}{\Delta E_R} \]

Toward observer

Away from observer

C - Charge Source
S - Strike Point
O - Observer

\( \phi = 90^\circ \)
\( \phi = 0^\circ \)
\( \phi = 180^\circ \)

DISTANCE, km

0 2 4 6 8 10 12 14 16 18 20 22

-2.8 -2.4 -2.0 -1.6 -1.2 -0.8 -0.4 0 0.4 0.8 1.2 1.6 2.0 2.4 2.8

Diagram showing the relationship between distance and field change ratio with labels for charge source, strike point, and observer.
a) 71 First Strokes

b) 28 Second, Third, and Fourth Strokes Creating a New Termination on Ground

c) 71 Second, Third, and Fourth Strokes Following the Same Channel as Preceding Stroke

d) 39 Fifth, Sixth, and Seventh Strokes

e) 44 Strokes of Order 8 Through 10

LEADER DURATION, MS
Electric fields radiated by individual leader steps (within 200 ms prior to return stroke)

Mean interval between pulses is 16 ms

0.4-0.5 ms

< 0.3 ms (10-90%)

80-μs waveforms are inverted

(D = 20-100 km)
Attachment process

Vertical
leader above
flat ground

Fig. 4. Electric gradient below leader channel: (a) as function of horizontal
distance and (b) as function of height of leader tip above ground. (Gold, 1977)

1. Downward leader channel
2. Corona sheath
3, 4, 5 - Streamer zones
6, 7 - Return stroke tip
(25 - 110 m)
8 - Return-stroke channel

An upward-moving connecting leader is initiated when

\[ \int_{h}^{l_{crit}} E(z) \geq V_{BD}(l_{crit}) \]

where \( V_{BD}(l_{crit}) \) is the breakdown voltage.

Total length of an upward-going connecting leader

\[ \rho_{L} = 10^{-3} \text{ C/m} \]

For \( h = 50 - 300 \text{ m} \) the field is strongly enhanced
over \( (z - \alpha) \approx 15 \text{ m} \), \( l_{crit} \approx 15 \text{ m} \),

\[ K_{V} = \frac{V_{up}}{V_{down}} \]
The stepped leader meets an upward-moving connecting leader from the tower at point A. The connecting leader branches at point B.

Streaked image

Another flash terminating below the tower top.

Fig. 6.2

Junction point A is about 40 m above and 40 m horizontally away from the tower.
Fig. 42. Downward negative stroke to tower 2. a) Photograph from mountain peak on fast moving film. b) Current oscillogram (impulse current). (Berger, 1967)
Striking-Distance Concept

A reasonable charge distribution is assumed for the leader channel, and the resultant fields on the ground or nearby objects are calculated. The leader is assumed to be at the striking distance when the field between the leader tip and some point exceeds a critical breakdown value (2-6 kV/cm) determined from the long-spark experiments.

![Diagram showing striking distance vs. current with data points and equations.]

- □ and ○ are the estimates from, respectively, three-dimensional and two-dimensional photographs and direct current measurements (Eriksson, 1978).
- (3.3C → 25 kA)
- I = 20Q, kA (Golde, 1977), P = 10.6Q^{0.7} (Berger)

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Estimation of the striking distance using photographic records

Fig. 2(a) Downward flash to the research mast photographed from two directions.

Fig. 2(b) Reconstruction of flash progression (flash striking distance of 798 m).

(Eriksson, 1978)
The records were aligned in such a way that the 50% amplitude points coincided. A different technique using the cross-correlation function produced almost identical results.

\[
\overline{i_k} = \frac{1}{N} \sum_{j=1}^{N} i_{jk}
\]

where \( N \) is the total number of curves contributing to the average current value in the point \( k \).
Fig. 1.5b. Luminosity versus time at different heights above ground corresponding to the streak photograph in Fig. 1.5a.
Fig. 1.5c. Same as Fig. 1.5b, but for the bottom 480 m of the channel only.
Return-stroke peak current distributions

\[ P(I_p) = \frac{1}{\sqrt{2\pi} \sigma_{\text{log}}} \int_{I_p}^{\infty} \exp\left[-\frac{(\log I_p - \log \mu_p)^2}{2 \sigma_{\text{log}}^2}\right] dI_p \] (log-normal approximation)

(Percentage of events exceeding a given value of the parameter on the horizontal axis.)

Cumulative probability distribution graph paper on which the Gaussian (normal) cumulative distribution function appears as a straight line.
Direct measurements of lightning currents in South Africa

(lightning strikes to a 60-m research mast)

3.68 Frequency bandwidth:
1 Hz to 10 MHz

Maximum \( \frac{di}{dt} \) is 180 kA/ms

First strokes \((N = 11)\)

Fig 6 Multiple stroke lightning current wave shapes.

Fig 7 Cumulative frequency distribution of lightning current amplitudes for first negative downward strokes — research mast.

(Eriksson, 1972)
Table 2-1. Lightning current parameters for negative flashes from Berger et al. (1975).

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameters</th>
<th>Unit</th>
<th>95%</th>
<th>50%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>First strokes</td>
<td>kA</td>
<td>14</td>
<td>30</td>
<td>80</td>
</tr>
<tr>
<td>135</td>
<td>Subsequent strokes</td>
<td>kA</td>
<td>4.6</td>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td>93</td>
<td>First strokes</td>
<td>C</td>
<td>1.1</td>
<td>5.2</td>
<td>24</td>
</tr>
<tr>
<td>122</td>
<td>Subsequent strokes</td>
<td>C</td>
<td>0.2</td>
<td>1.4</td>
<td>11</td>
</tr>
<tr>
<td>94</td>
<td>Complete flash</td>
<td>C</td>
<td>1.3</td>
<td>7.5</td>
<td>40</td>
</tr>
<tr>
<td>90</td>
<td>First strokes</td>
<td>C</td>
<td>1.1</td>
<td>4.5</td>
<td>20</td>
</tr>
<tr>
<td>117</td>
<td>Subsequent strokes</td>
<td>C</td>
<td>.22</td>
<td>0.95</td>
<td>4</td>
</tr>
<tr>
<td>89</td>
<td>First strokes</td>
<td>µs</td>
<td>1.8</td>
<td>5.5</td>
<td>18</td>
</tr>
<tr>
<td>118</td>
<td>Subsequent strokes</td>
<td>µs</td>
<td>.22</td>
<td>1.1</td>
<td>4.5</td>
</tr>
<tr>
<td>92</td>
<td>First strokes</td>
<td>kA/µs</td>
<td>5.5</td>
<td>12</td>
<td>32</td>
</tr>
<tr>
<td>122</td>
<td>Subsequent strokes</td>
<td>kA/µs</td>
<td>12</td>
<td>40</td>
<td>120</td>
</tr>
<tr>
<td>90</td>
<td>First strokes</td>
<td>µs</td>
<td>30</td>
<td>75</td>
<td>200</td>
</tr>
<tr>
<td>115</td>
<td>Subsequent strokes</td>
<td>µs</td>
<td>6.5</td>
<td>32</td>
<td>140</td>
</tr>
<tr>
<td>91</td>
<td>First strokes</td>
<td>A²s</td>
<td>$6.0 \times 10^3$</td>
<td>$5.5 \times 10^4$</td>
<td>$5.5 \times 10^4$</td>
</tr>
<tr>
<td>88</td>
<td>Subsequent strokes</td>
<td>A²s</td>
<td>$5.5 \times 10^3$</td>
<td>$6.0 \times 10^3$</td>
<td>$5.2 \times 10^4$</td>
</tr>
<tr>
<td>133</td>
<td>Between negative stroke</td>
<td>ms</td>
<td>7</td>
<td>33</td>
<td>150</td>
</tr>
</tbody>
</table>

The energy that would be dissipated in a 1-Ω resistor if the stroke current were to flow through it.

The shortest measurable time in the oscillograms was 0.5 ms.

The percentages (95%, 50%, and 5%) of cases exceeding the tabulated values are based on the lognormal approximations.
Data include 17 first and 46 subsequent strokes. The speed was averaged over the visible channel section (within 1.3 km of ground). There was a tendency for the speed to decrease with height for both first and subsequent strokes.

\[
\begin{align*}
\text{First} & : & \text{MEAN} = 9.6 \times 10^7 \text{ m/s} \\
\text{Subsequent} & : & \text{MEAN} = 1.2 \times 10^8 \text{ m/s} \\
\end{align*}
\]

\[N = 63\]

(Idone and Orville, 1982)
Measurement of lightning speed using Streak-camera records

The average two-dimensional speed between the two selected levels of the channel is

\[ V = \frac{L}{T} \] 2-D length of channel

\[ T = \frac{S}{2MW} \] net horizontal displacement between the levels \((S_1 - S_2)\)

\(M\) is print-to-negative magnification factor for the time-resolved images;

\(W\) is film movement rate.

\[ L = \frac{DL'}{FM'} \] (determined from the still picture)

\(D\) is the distance to the channel;

\(F\) is focal length of the still camera;

\(L'\) is channel length measured from the print;

\(M'\) is print-to-negative magnification factor for the still photograph.

Fig. 1. Qualitative representation of the stepped leader and dart leader image tracks, resulting from the exposure to two leader-return stroke sequences. The leaders propagate from cloud to ground while the film is rapidly swept through the image plane. The actual channel geometry is represented by the dashed lines labeled “Still,” which are superimposed for illustrative purposes. The stepped leader, propagating downward relatively slowly and brightening every \(\sim 5\) \(\mu\)s, produces the intermittent track shown on the left. The propagation time between a given height and ground is \(T_L = S/W\). The dart leader, in contrast, propagates downward at a generally greater speed with constant emission of light and records a continuous track. The propagation time of the dart leader between the same height and ground is \(T_R = D/W\). (Orville and Idone, 1982)

Fig. 2. Schematic diagram of the principles involved in streaking photography used for the time resolution of lightning return strokes. (Idone and Orville, 1982)

R.S.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Min. Speed, m/s</th>
<th>Max. Speed, m/s</th>
<th>Mean Speed, m/s</th>
<th>s.d., m/s</th>
<th>Sample Size</th>
<th>Comments</th>
</tr>
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<tr>
<td><strong>Triggered Lightning</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hubert and Mouget [1981]</td>
<td>4.5x10^7</td>
<td>1.7x10^8</td>
<td>9.9x10^7</td>
<td>4.1x10^7</td>
<td>13</td>
<td>Photoelectric, 3-D speed</td>
</tr>
<tr>
<td>Idone et al. [1984]</td>
<td>6.7x10^7</td>
<td>1.7x10^8</td>
<td>1.2x10^8</td>
<td>2.7x10^7</td>
<td>56</td>
<td>Streak camera, 3-D speed</td>
</tr>
<tr>
<td>Willett et al. [1988]</td>
<td>1.0x10^8</td>
<td>1.5x10^8</td>
<td>1.2x10^8</td>
<td>1.6x10^7</td>
<td>9</td>
<td>Streak camera, 2-D speed</td>
</tr>
<tr>
<td>Willett et al. [1989]</td>
<td>1.2x10^8</td>
<td>1.9x10^8</td>
<td>1.5x10^8</td>
<td>1.7x10^7</td>
<td>18</td>
<td>Streak camera, 2-D speed</td>
</tr>
<tr>
<td>Mach and Rust [1989, Figure 8]</td>
<td>6.0x10^7</td>
<td>1.6x10^8</td>
<td>1.2±0.3x10^8</td>
<td>2x10^7</td>
<td>40</td>
<td>Long channel</td>
</tr>
<tr>
<td></td>
<td>6.0x10^7</td>
<td>2.0x10^8</td>
<td>1.4±0.4x10^8</td>
<td>4x10^7</td>
<td>39</td>
<td>Short channel (Photoelectric, 2-D)</td>
</tr>
<tr>
<td><strong>Natural Lightning</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boyle and Orville [1976]</td>
<td>2.0x10^7</td>
<td>1.2x10^8</td>
<td>0.71x10^8</td>
<td>2.6x10^7</td>
<td>12</td>
<td>Streak camera, 2-D speed</td>
</tr>
<tr>
<td>Idone and Orville [1982]</td>
<td>2.9x10^7</td>
<td>2.4x10^8</td>
<td>1.1x10^8</td>
<td>4.7x10^7</td>
<td>63</td>
<td>Streak camera, 2-D speed</td>
</tr>
<tr>
<td>Mach and Rust [1989, Figure 7]</td>
<td>2.0x10^7</td>
<td>2.6x10^8</td>
<td>1.3±0.3x10^8</td>
<td>5x10^7</td>
<td>54</td>
<td>Long channel</td>
</tr>
<tr>
<td></td>
<td>8.0x10^7</td>
<td>&gt;2.8x10^8</td>
<td>1.9±0.7x10^8</td>
<td>7x10^7</td>
<td>43</td>
<td>Short channel (Photoelectric, 2-D)</td>
</tr>
</tbody>
</table>
Magnetic field due to a vertical current-carrying line (static approximation)

The Bio-Savart law
\[ d\vec{B} = \frac{M_0 I \, dz'}{4\pi R^2} \left( \vec{a}_I \times \vec{a}_R \right), \text{Wb/m}^2 \]

\[ |(\vec{a}_I \times \vec{a}_R)| = \sin \theta = \frac{D}{R} \]

since \( \sin \theta = \sin (180^\circ - \theta) \)

\[ dB = \frac{M_0 I \, dz'}{4\pi} \frac{D}{(z'^2 + D^2)^{3/2}} \] (magnitude)

The vector field points into the page.

To take account of the image current.

\[ B = 2 \int_{-H_B}^{H_T} \frac{M_0 I}{4\pi} \frac{D}{(z'^2 + D^2)^{3/2}} \, dz' = \frac{M_0 I}{2\pi D} \left[ \frac{H_T}{(H_T^2 + D^2)^{1/2}} - \frac{H_B}{(H_B^2 + D^2)^{1/2}} \right] \]

For \( H_T = H, H_B = 0 \)

\[ B = \frac{M_0 I}{2\pi D} \frac{H}{(H^2 + D^2)^{1/2}} \]

Spatially uniform but slowly time-varying current flows from a charge center to ground. (See Fig. 1-6)

If \( (H/D)^2 \gg 1 \)

\[ B \approx \frac{M_0 I H}{2\pi D^2} \] (close)

If \( (H/D)^2 \ll 1 \)

\[ B \approx \frac{M_0 I H}{2\pi D^2} \] (far) \( \Rightarrow B = \frac{M_0}{4\pi D^2} \frac{dM}{dz} \)

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Electric and magnetic fields due to a vertical dipole radiator

The electric dipole moment of the charge $Q$ located a distance $H$ above a conducting plane and its image is

$$ M = 2QH \quad \text{(in general, } M = 2 \sum_{i=1}^{N} Q_i H_i) $$

$$ \frac{dM}{dt} = 2IH $$

$$ \frac{d^2M}{dt^2} = 2H \frac{dI}{dt} $$

Has a nonzero value before and after lightning

Electrostatic field is dominant

Total electric field intensity vs. time at several distances for a return stroke. (Uman, 1984)

$$ E_z = \frac{1}{4\pi\varepsilon_0 D^3} M + \frac{1}{4\pi\varepsilon_0 c D^2} \frac{dM}{dt} + \frac{1}{4\pi\varepsilon_0 c^2 D} \frac{d^2M}{dt^2} $$

Magnetostatic

$$ B_\phi = \frac{M_0}{4\pi D^2} \frac{dM}{dt} + \frac{M_0}{4\pi c D} \frac{d^2M}{dt^2} $$

Radiation

The dipole approximation is valid if (1) $D \gg H$, (2) $I(z,t) = I(0,t)$ for all $z$ from 0 to $H$, and (3) $H = \text{const}$.  

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Two-station measurements of vertical electric and horizontal magnetic fields from a two-stroke flash

200 V/m

N

S

E

W

5 \times 10^{-7} \text{Wb/m}^2

OCF, D = 1.9 km

B2_{NS}

B2_{EW}

B1_{NS}

B1_{EW}

20 \mu s

5 V/m

N

S

2 \times 10^{-8} \text{Wb/m}^2

GNV, D = 50 km
ELECTRIC FIELD INTENSITY

Initial peak 50 V/m

- Value at 170 μs

- Ramp starting time

MAGNETIC FLUX DENSITY

- Hump

- Half value

- Zero crossing

- 0 50 100 150 170 μs

D = 10 km

D = 15 km

D = 50 km

D = 200 km

D = 10 km

D = 15 km

D = 50 km

D = 200 km

Fig 7.1b
Fine structure of the radiation fields produced by return strokes

(\sim 20\% \text{ of the initial peak; } \sim 0.5 - 1 \text{ ms})

\text{SUBSEQUENT}

(E)

\text{SUBSEQUENT WITH DART-STEPPED LEADER}

(90 \pm 40 \text{ ns}) \rightarrow \text{Fast transition}

(40-50\% \text{ of the initial peak; } \sim 4 \text{ ms}) \rightarrow \text{Slow front}

\text{FIRST}

\text{SECONDARY PEAK}

\text{SUBSIDIARY PEAK}

\text{INITIAL PEAK}

\text{MICROSECONDS}
63 First Strokes in Multiple-Stroke Flashes (GEOMETRIC MEAN = 6.2 V/m)

13 Strokes in Single-Stroke Flashes (GEOMETRIC MEAN = 4.7 V/m)

a) 76 First Strokes (GEOMETRIC MEAN = 5.9 V/m)

117 Strokes Following the Same Channel as Preceding Stroke (GEOMETRIC MEAN = 3.1 V/m)

38 Strokes Creating a New Termination on Ground (GEOMETRIC MEAN = 4.1 V/m)

b) 155 Second, Third, and Fourth Strokes (GEOMETRIC MEAN = 3.3 V/m)

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c) 62 Fifth, Sixth, and Seventh Strokes (GEOMETRIC MEAN = 2.3 V/m)

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d) 53 Strokes of the Order 8 through 18 (GEOMETRIC MEAN = 2.4 V/m)

INITIAL ELECTRIC FIELD PEAK NORMALIZED TO 100 km (V/m)
Return-stroke initial electric field peak (GM) as a function of stroke order

\[ I_{(-)} = 1.5 - 0.037 \cdot D \cdot E_{(+)} \]

\[
\begin{align*}
[ E ] &= \text{V/m} \\
[ D ] &= \text{km} \\
[ I ] &= \text{kA}
\end{align*}
\]
Table 2-3. Statistics on return stroke electric field waveforms of negative cloud-to-ground lightning.

<table>
<thead>
<tr>
<th></th>
<th>First strokes</th>
<th>Subsequent strokes</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>No.</td>
<td>Mean</td>
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<td><strong>Initial peak (V/m)</strong></td>
<td></td>
<td></td>
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<tr>
<td>(normalized to 100 km)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rakov and Uman (1990a)</td>
<td>76</td>
<td>5.9(GM)</td>
</tr>
<tr>
<td>[All cases]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Multiple-stroke flashes]</td>
<td>63</td>
<td>6.2(GM)</td>
</tr>
<tr>
<td>[Single-stroke flashes]</td>
<td>13</td>
<td>4.7(GM)</td>
</tr>
<tr>
<td>[Strokes creating new</td>
<td></td>
<td></td>
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<tr>
<td>termination]</td>
<td></td>
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<tr>
<td>[Strokes following</td>
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<td></td>
</tr>
<tr>
<td>previous channel]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cooray and Lundquist (1982)</td>
<td>553</td>
<td>5.3</td>
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<tr>
<td>Lin et al. (1979)</td>
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<td></td>
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<tr>
<td>[KSC]</td>
<td>51</td>
<td>6.7</td>
</tr>
<tr>
<td>[Ocala]</td>
<td>29</td>
<td>5.8</td>
</tr>
<tr>
<td><strong>Zero-crossing time (μs)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cooray and Lundquist (1985)</td>
<td>102</td>
<td>49</td>
</tr>
<tr>
<td>[Sweden]</td>
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<td></td>
</tr>
<tr>
<td>[Sri Lanka]</td>
<td>91</td>
<td>89</td>
</tr>
<tr>
<td>Lin and Uman (1973)</td>
<td>46</td>
<td>54</td>
</tr>
<tr>
<td><strong>Zero-to-peak rise time (μs)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Master et al. (1984)</td>
<td>105</td>
<td>4.4</td>
</tr>
<tr>
<td>Cooray and Lundquist (1982)</td>
<td>140</td>
<td>7.0</td>
</tr>
<tr>
<td>Lin et al. (1979)</td>
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<td></td>
</tr>
<tr>
<td>[KSC]</td>
<td>51</td>
<td>2.4</td>
</tr>
<tr>
<td>[Ocala]</td>
<td>29.</td>
<td>2.7</td>
</tr>
<tr>
<td><strong>10-90 % rise time (μs)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Master et al. (1984)</td>
<td>105</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Note: The numbers with GM in parenthesis are the Geometric Mean values.
5. Calculation of lightning electromagnetic fields
Maxwell's Equations

Differential form

\[ \nabla \cdot \mathbf{D} = \rho_v \]

\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]

Integral form

\[ \oint_{\mathcal{S}} \mathbf{D} \cdot d\mathbf{S} = \int_{\mathcal{V}} \rho_v \, dv \]

\[ \oint_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{S} = 0 \]

\[ \oint_{\mathcal{L}} \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \oint_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{S} \]

\[ \oint_{\mathcal{L}} \mathbf{H} \cdot d\mathbf{l} = \int_{\mathcal{S}} (\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}) \cdot d\mathbf{S} \]

Remarks

- Gauss's law. Electric charge is a source (or sink) of electric field (but not the only source).
- Magnetic charges (monopoles) are not known to exist. Magnetic field lines always form closed loops.
- Faraday's law. A time-varying magnetic field creates an electric field.
- Ampere's circuit law. An electric current (including a displacement current due to a time-varying electric field) creates a magnetic field.

In free space \( \mathbf{D} = \varepsilon_0 \mathbf{E} \) and \( \mathbf{B} = \mu_0 \mathbf{H} \) \( (c = \frac{1}{\mu_0 \varepsilon_0}) \)

The integral form of Maxwell's equations is obtained by integrating over an arbitrary volume and applying the divergence theorem (the first two equations), or by integrating over an arbitrary surface and applying Stokes' theorem (the last two equations).

\[ \oint_{\mathcal{S}} \mathbf{D} \cdot d\mathbf{S} = \int_{\mathcal{V}} \nabla \cdot \mathbf{D} \, dv \] - the divergence theorem for \( \mathbf{D} \)

\[ \oint_{\mathcal{L}} \mathbf{H} \cdot d\mathbf{l} = \int_{\mathcal{S}} (\nabla \times \mathbf{H}) \cdot d\mathbf{S} \] - Stokes' theorem for \( \mathbf{H} \)
General solution of the time-dependent Maxwell's equations.

To facilitate the solving of Maxwell's equations, a change of variables from \( E \) to \( \vec{E} \) and \( \vec{E} \) to \( \vec{A} \) and \( \phi \).

\[
\phi(\vec{r}, t) = \frac{1}{4\pi} \int_P \frac{\vec{J}(\vec{r}', t-R/c)}{R} \cdot \hat{n} dV'
\]

\[
\vec{A}(\vec{r}, t) = \frac{\vec{J}(\vec{r}', t-R/c)}{4\pi} \cdot \hat{n} dV'
\]

\( \vec{J} \) is the current density, \( \phi \) is the scalar potential, and \( \vec{A} \) is the vector potential.

Lorentz condition:

\[
\nabla \cdot \vec{E} = \frac{1}{c^2} \frac{\partial \phi}{\partial t}
\]

\[
\nabla \times \vec{B} = \mu_0 \frac{\partial \vec{E}}{\partial t}
\]

\( \vec{E} \) and \( \vec{B} \) are the electric and magnetic fields, respectively.

A vector field is defined completely when both its curl and divergence are given.

\[
\phi = -\frac{1}{c^2} \int_{\infty}^{t} \frac{1}{c^2} \frac{\partial \phi}{\partial t'} dr'
\]

\[
\vec{A} \cdot \vec{r} = \frac{1}{2c^2} \frac{\partial \phi}{\partial t} + \phi(\vec{r}, \infty)
\]
\[ d\mathbf{\bar{A}}(\mathbf{r}, t) = \frac{M_0}{4\pi} \frac{i(z', t - R/c)}{R} d\mathbf{z}' \hat{a}_z \quad \text{(only } z\text{-component; } \oint d\mathbf{V}' = i d\mathbf{z}' \text{)} \]

\[ d\mathbf{\bar{A}}(\mathbf{R}, t) = \frac{M_0}{4\pi} \frac{i(0, t - R/c)}{R} d\mathbf{z}' \hat{a}_z = A_z(\mathbf{R}, t) \hat{a}_z \quad \text{if } \mathbf{z}' = 0 \ (\mathbf{r}_s = \mathbf{R}) \]

For \( z' = 0 \)
\[ R = (r^2 + z^2)^{1/2} \]
\[ r = (x^2 + y^2)^{1/2} \]
\[ \frac{\partial}{\partial r}(\frac{1}{R}) = -\frac{r}{R^3} \]

\[ d\mathbf{\bar{B}}(r, \phi, z, t) = \nabla \times d\mathbf{\bar{A}} = \left( \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{a}_r + \left( \frac{\partial A_r}{\partial \phi} - \frac{\partial A_\phi}{\partial r} \right) \hat{a}_\phi + \frac{i}{r} \left( \frac{\partial}{\partial r} (rA_\phi) - \frac{\partial A_r}{\partial \phi} \right) \hat{a}_z \]

\[ = -\frac{\partial A_z}{\partial r} \hat{a}_\phi = \frac{M_0 d\mathbf{z}'}{4\pi} \frac{\partial}{\partial r} \left( \frac{i(t - R/c)}{R} \right) \hat{a}_\phi \]

\[ = \frac{M_0 d\mathbf{z}'}{4\pi} \left[ \frac{1}{R} \frac{\partial}{\partial r} \left( \frac{i(t - R/c)}{R} \right) + i(t - R/c) \frac{\partial}{\partial r} \left( \frac{1}{R^3} \right) \right] \hat{a}_\phi \]

"d" is omitted

\[ \frac{\partial}{\partial t} \rightarrow \nabla \times \text{ (convert the spatial derivative to time derivative)} \]

\[ \frac{\partial}{\partial z} \rightarrow -\frac{r}{R^3} \]

\[ \text{Fl} 7.12 \]
\[ \frac{\partial i(t-R/c)}{\partial r} = \frac{\partial i(t-R/c)}{\partial (t-R/c)} \cdot \frac{\partial (t-R/c)}{\partial r} = -\frac{r}{cr} \cdot i'(t-R/c) \]
\[ i'(t-R/c) - \frac{1}{c} \frac{\partial \sqrt{r^2+z^2}}{\partial r} \]

\[ \frac{\partial i(t-R/c)}{\partial t} = \frac{\partial i(t-R/c)}{\partial (t-R/c)} \cdot \frac{\partial (t-R/c)}{\partial t} - i'(t-R/c) \]

\[ \frac{\partial i}{\partial r} = -\frac{r}{cr} \frac{\partial i}{\partial t} \]

\[ d\vec{B}(r,\phi,z,t) = \frac{M_0 d^2 \gamma'}{4\pi} \left[ \frac{r}{cr^2} \frac{\partial i(t-R/c)}{\partial t} + \frac{1}{R^3} i(t-R/c) \right] \vec{A}_\phi \text{ (cylindrical)} \]

\[ d\vec{B}(r,s,\phi,\theta,t) = \frac{M_0 d^2 \gamma'}{4\pi} \sin \theta \left[ \frac{1}{cr} \frac{\partial i(t-R/c)}{\partial t} + \frac{i(t-R/c)}{R^2} \right] \vec{A}_\phi \text{ (spherical)} \]

\[ \sin \theta = \frac{r}{R} \]

Electrical field intensity from the source dipole
\[ \vec{E}(\vec{R},t) = -\nabla \phi - \frac{\partial \vec{A}}{\partial t} \text{ where } \phi = -c^2 \int_{-\infty}^{t} \nabla \cdot \vec{A} \, dt \]

\[ \nabla \cdot \vec{A} = \frac{\partial \vec{A}}{\partial z} = \frac{M_0 d^2 \gamma'}{4\pi} \frac{\partial}{\partial z} \left( \frac{i(t-R/c)}{R} \right) - \frac{\partial}{\partial z} \left( \frac{1}{R} \right) = -\frac{1}{R^2} \frac{\partial \sqrt{r^2+z^2}}{\partial z} \]

\[ = \frac{M_0 d^2 \gamma'}{4\pi} \left[ \frac{1}{R} \frac{\partial i(t-R/c)}{\partial z} - \frac{r}{R^2} i(t-R/c) \right] \]

Since there is no \( \phi \) variation, the gradient operator is
\[ \nabla = \vec{a}_r \frac{\partial}{\partial r} + \vec{a}_z \frac{\partial}{\partial z} \]

\[ \nabla \phi = \vec{a}_r \frac{\partial}{\partial r} \left( -c^2 \int_{-\infty}^{t} \nabla \cdot \vec{A} \, dt \right) + \vec{a}_z \frac{\partial}{\partial z} \left( -c^2 \int_{-\infty}^{t} \nabla \cdot \vec{A} \, dt \right) \]

\[ = -c^2 \int_{-\infty}^{t} \nabla (\nabla \cdot \vec{A}) \, dt = -c^2 \int_{-\infty}^{t} \left[ \vec{a}_r \frac{\partial (\nabla \cdot \vec{A})}{\partial r} + \vec{a}_z \frac{\partial (\nabla \cdot \vec{A})}{\partial z} \right] \, dt \]
\[
\frac{\partial i(t-R/c)}{\partial r} = \frac{\partial i(t-R/c)}{\partial (t-R/c)} \cdot \frac{\partial (t-R/c)}{\partial r} = -\frac{r}{cR} \cdot i'(t-R/c)
\]

\[
\frac{\partial i(t-R/c)}{\partial t} = \frac{\partial i(t-R/c)}{\partial (t-R/c)} \cdot \frac{\partial (t-R/c)}{\partial t} = i'(t-R/c)
\]

\[
\frac{\partial i}{\partial r} = -\frac{r}{cR} \frac{\partial i}{\partial t}
\]

\[
d\vec{B}(r,\phi,z,t) = \frac{M_0 dz'}{4\pi} \left[ \frac{r}{cR^2} \frac{\partial i(t-R/c)}{\partial t} + \frac{r}{R^3} i(t-R/c) \right] \vec{a}_\phi \text{ (cylindrical)}
\]

\[
d\vec{B}(r_s,\phi_s,\theta_s,t) = \frac{M_0 dz'}{4\pi} \sin \theta \left[ \frac{1}{cR} \frac{\partial i(t-R/c)}{\partial t} + \frac{i(t-R/c)}{R^2} \right] \vec{a}_\phi \text{ (spherical)}
\]

\[
\sin \theta = \frac{r}{R}
\]

**Electric field intensity from the source dipole**

\[
\vec{E}(R,t) = -\nabla \phi - \nabla \vec{A}/\partial t \quad \text{where} \quad \phi = -c^2 \int_\infty^t \nabla \cdot \vec{A} \, d\tau
\]

\[
\nabla \cdot \vec{A} = \frac{\partial \vec{A}}{\partial z} = \frac{M_0 dz'}{4\pi} \frac{\partial i(t-R/c)}{\partial z} \frac{\partial}{\partial z} \left( \frac{i(t-R/c)}{R} \right) = \frac{M_0 dz'}{4\pi} \left[ \frac{1}{R} \frac{\partial i(t-R/c)}{\partial z} - \frac{R}{R^2} i(t-R/c) \right]
\]

Since there is no \( \phi \) variation, the gradient operator is

\[
\nabla = \vec{a}_r \frac{\partial}{\partial r} + \vec{a}_\phi \frac{\partial}{\partial \phi} + \vec{a}_z \frac{\partial}{\partial z}
\]

\[
\nabla \phi = \vec{a}_r \frac{\partial}{\partial r} \left( -c^2 \int_\infty^t \nabla \cdot \vec{A} \, d\tau \right) + \vec{a}_\phi \frac{\partial}{\partial \phi} \left( -c^2 \int_\infty^t \nabla \cdot \vec{A} \, d\tau \right)
\]

\[
= -c^2 \int_\infty^t \nabla \left( \nabla \cdot \vec{A} \right) \, d\tau = -c^2 \int_\infty^t \left[ \vec{a}_r \frac{\partial}{\partial r} \left( \nabla \cdot \vec{A} \right) \right] d\tau - c^2 \int_\infty^t \left[ \vec{a}_z \frac{\partial}{\partial z} \left( \nabla \cdot \vec{A} \right) \right] d\tau
\]

\[
\nabla \phi_r \quad \nabla \phi_z
\]
Expand the integrand:

\[ \hat{a}_r \frac{\partial}{\partial r} (\nabla \cdot \hat{A}) = \frac{M_0 d_z' e^i}{4\pi} \frac{1}{R^3} \left[ \frac{1}{R} \frac{\partial i(t-R/c)}{\partial z} - \frac{2}{R^3} \frac{d i(t-R/c)}{d t} \right] \frac{\partial}{\partial z} \]

\[ \left\{ \right. \]

\[ \frac{\partial}{\partial r} \left[ \frac{1}{R} \frac{\partial i(t-R/c)}{\partial z} \right] = \frac{1}{R} \frac{\partial^2 i(t-R/c)}{\partial r^2} + \left( -\frac{r}{R^3} \right) \frac{\partial i(t-R/c)}{\partial z} \]

\[ \frac{\partial}{\partial r} \left[ -\frac{2}{R^3} i(t-R/c) \right] = -\frac{2}{R^3} \frac{\partial i(t-R/c)}{\partial r} - \frac{2}{R^3} \frac{d i(t-R/c)}{d t} \]

\[ \left. \right\} \]

\[ \hat{a}_z \frac{\partial}{\partial z} (\nabla \cdot \hat{A}) = \frac{M_0 d_z' e^i}{4\pi} \frac{1}{R} \left[ \frac{1}{R} \frac{\partial i(t-R/c)}{\partial z} - \frac{2}{R^3} \frac{d i(t-R/c)}{d t} \right] \frac{\partial}{\partial z} \]

\[ \left\{ \right. \]

\[ \frac{\partial}{\partial z} \left[ \frac{1}{R} \frac{\partial i(t-R/c)}{\partial z} \right] = \frac{1}{R} \frac{\partial^2 i(t-R/c)}{\partial z^2} + \left( -\frac{z}{R^3} \right) \frac{\partial i(t-R/c)}{\partial z} \]

\[ \frac{\partial}{\partial z} \left[ -\frac{2}{R^3} i(t-R/c) \right] = \frac{2}{R^3} \frac{\partial i(t-R/c)}{\partial r} + \frac{2}{R^3} \left( -\frac{z}{R^3} \right) i(t-R/c) + \frac{\partial i(t-R/c)}{\partial z} \]

\[ \left. \right\} \]

\[ = -\frac{1}{R^4} i(t-R/c) - \frac{2}{R^3} \frac{d i(t-R/c)}{d t} - \frac{2}{R^3} \frac{\partial i(t-R/c)}{\partial z} \]

Convert the spatial derivatives to time derivatives:

\[ \left\{ \right. \]

\[ \frac{\partial}{\partial r} \left[ i(t-R/c) \right] = -\frac{r}{cR} \frac{\partial}{\partial t} \left[ i(t-R/c) \right] \]

\[ \frac{\partial}{\partial z} \left[ i(t-R/c) \right] = -\frac{z}{cR} \frac{\partial}{\partial t} \left[ i(t-R/c) \right] \]

\[ \left. \right\} \]

Expand the second order derivatives:

\[ \frac{\partial^2 i(t-R/c)}{\partial z^2} = \frac{\partial}{\partial z} \left[ -\frac{z}{cR} \frac{\partial i(t-R/c)}{\partial t} \right] \]

\[ \left. \right\} \]

\[ \frac{\partial^2 i(t-R/c)}{\partial r^2} = \frac{\partial}{\partial r} \left[ \frac{1}{cR^3} \frac{\partial i(t-R/c)}{\partial t} \right] - \frac{2}{cR^3} \frac{\partial i(t-R/c)}{\partial t} \]

\[ \frac{\partial^2 i(t-R/c)}{\partial r \partial z} = \frac{\partial}{\partial r} \left[ -\frac{z}{cR^3} \frac{\partial i(t-R/c)}{\partial t} \right] - \frac{2}{cR^3} \frac{\partial i(t-R/c)}{\partial t} \]

\[ \left. \right\} \]
\[ \vec{a}_r \frac{\partial}{\partial r} (\nabla \cdot \vec{A}) = \frac{M_0}{4\pi} \frac{d^2 z'}{dt^2} \left\{ \frac{i}{R} \left[ \frac{2r}{cR^3} \frac{\partial i(t-R/c)}{\partial t} + \frac{r^2}{c^2R^2} \frac{\partial^2 i(t-R/c)}{\partial t^2} \right] + \right. \\
+ \left. \left( -\frac{r}{R^3} \right) \left[ -\frac{r^2}{cR} \frac{\partial i(t-R/c)}{\partial t} \right] + \left( -\frac{r^2}{R^3} \right) \left[ -\frac{r}{cR} \frac{\partial i(t-R/c)}{\partial t} \right] + \frac{3r^2}{R^5} i(t-R/c) \right\} \vec{a}_r \]

\[ \vec{a}_z \frac{\partial}{\partial z} (\nabla \cdot \vec{A}) = \frac{M_0}{4\pi} \frac{d^2 z'}{dt^2} \left\{ \frac{i}{R} \left[ \left( -\frac{1}{cR} + \frac{r^2}{cR^3} \right) \frac{\partial i(t-R/c)}{\partial t} + \frac{r^2}{c^2R^2} \frac{\partial^2 i(t-R/c)}{\partial t^2} \right] - \right. \\
- \left. \frac{r^2}{R^3} \left[ -\frac{r}{cR} \frac{\partial i(t-R/c)}{\partial t} \right] - \frac{r^3}{R^5} i(t-R/c) + \frac{3r^2}{R^5} i(t-R/c) - \right. \\
- \left. \frac{3r^2}{R^5} \left[ -\frac{r}{cR} \frac{\partial i(t-R/c)}{\partial t} \right] \right\} \vec{a}_z \]

\[ \nabla \phi_r = -C^2 \int_{-\infty}^{t} \vec{a}_r \frac{\partial}{\partial r} (\nabla \cdot \vec{A}) \, dt = -\frac{d^2 z'}{4\pi \varepsilon_0} \left[ \frac{3r^2}{R^5} \int_{0}^{t} i(t-R/c) \, dt + \right. \\
+ \left. \frac{3r^2}{cR^4} i(t-R/c) + \frac{r^2}{c^2R^3} \frac{\partial i(t-R/c)}{\partial t} \right] \]

\[ \nabla \phi_z = -C^2 \int_{-\infty}^{t} \vec{a}_z \frac{\partial}{\partial z} (\nabla \cdot \vec{A}) \, dt = -\frac{d^2 z'}{4\pi \varepsilon_0} \left[ \left( \frac{3r^2}{R^5} - \frac{r^3}{R^5} \right) \int_{0}^{t} i(t-R/c) \, dt + \right. \\
+ \left. \left( \frac{3r^2}{cR^4} - \frac{r^2}{c^2R^3} \right) i(t-R/c) + \frac{r^2}{c^2R^3} \frac{\partial i(t-R/c)}{\partial t} \right] \]

\[ \frac{1}{C^2 \varepsilon_0} \]

\[ \frac{\partial \vec{A}}{\partial t} = \frac{\partial}{\partial t} \left[ \frac{M_0 d^2 z'}{4\pi} \frac{i(t-R/c)}{R} \vec{a}_z \right] = \frac{d^2 z'}{4\pi \varepsilon_0} \left( \frac{1}{cR} \frac{\partial i(t-R/c)}{\partial t} \right) \vec{a}_z \]
\[ d\vec{E} = dE_r \hat{a}_r + dE_\varphi \hat{a}_\varphi + dE_z \hat{a}_z = (-\nabla \varphi) \hat{a}_r + (-\nabla \varphi_z - \partial A/\partial t) \hat{a}_z \]

\[ dE_r = \frac{dz'}{4\pi \varepsilon_0} \left[ \frac{3z_0^2}{R^5} \int_0^t i(T-R/c) dt + \frac{3z^2}{cR^4} i(t-R/c) + \frac{z_0^2}{c^2R^3} \frac{\partial i(t-R/c)}{\partial t} \right] \]

\[ dE_z = \frac{dz'}{4\pi \varepsilon_0} \left[ \frac{2z^2-R^2}{R^5} \int_0^t i(T-R/c) dt + \frac{2z^2-R^2}{cR^4} i(t-R/c) - \frac{z^2}{c^2R^3} \frac{\partial i(t-R/c)}{\partial t} \right] \]

\[
\begin{aligned}
\frac{3z_0^2}{R^5} - \frac{1}{R^3} &= \frac{3z^2-R^2}{R^5} = \frac{3z^2-z_0^2-R^2}{R^5} = \frac{2z^2-R^2}{R^5} \\
\frac{3z^2}{cR^4} - \frac{1}{cR^2} &= \frac{3z^2-R^2}{cR^4} = \frac{3z^2-2z^2-R^2}{cR^4} = \frac{2z^2-R^2}{cR^4} \\
\frac{z^2}{c^2R^3} - \frac{1}{c^2R} &= \frac{2z^2-R^2}{c^2R^3} = \frac{z^2-x^2-R^2}{c^2R^3} = -\frac{r^2}{c^2R^3}
\end{aligned}
\]

If the dipole lies along the z axis at an arbitrary source coordinate \( z' \), the fields may be obtained by substituting \((z-z')\) for \( z' \) in the above equations.

**Fields at the surface of a perfectly conducting Earth**

In order to determine the electric and magnetic fields above a perfectly conducting Earth, the fields from the real dipole above the conducting plane must be added to the fields from the image dipole beneath, and this result integrated over all channel segments.

Since distance \( R \) from both the source and image dipoles to a point on the plane is the same, the expression for the magnetic field from the image dipole will be the same as that for the real dipole:

\[ d\vec{B}(r,\varphi,z,t) = \frac{M_0 dz'}{4\pi} \left[ \frac{r}{cR^2} \frac{\partial i(z',t-R/c)}{\partial t} + \frac{r^2}{R^2} i(z',t-R/c) \right] \hat{a}_\varphi \]

Thus, the effect of the image is to double the magnetic flux density given by the above formula.
The total magnetic flux density at ground surface \((z=0)\) due to the entire channel of length \(L\) is

\[
\vec{B}(r, \phi, 0, t) = \frac{M_0}{2\pi} \left[ \int_0^H \frac{r}{R^3} i(z', t-R/c) \, dz' + \int_0^H \frac{1}{cR^2} \frac{\partial i(z', t-R/c)}{\partial t} \, dz' \right] \vec{a}.
\]

- **Magnetostatic field**
- **Radiation field**

\[
\vec{B}(r_s, \phi, \theta, t) = \frac{M_0}{2\pi} \left[ \int_0^H \frac{\sin \theta}{R^2} i(z', t-R/c) \, dz' + \int_0^H \frac{\sin \theta}{cR} \frac{\partial i(z', t-R/c)}{\partial t} \, dz' \right]
\]

\((r/R = \sin(\theta) = \sin(180° - \theta) = \sin(\theta) ) \quad r = D\)

**Electric field intensity**

\[
d\vec{E}_r = \frac{dz'}{4\pi \varepsilon_0} \left[ \frac{3r(z-z')}{R^5} \int_0^t i(z', t-R/c) \, dt + \frac{3r(z-z')}{cR^4} \frac{\partial i(z', t-R/c)}{\partial t} \right] \vec{a}_r
\]

For \(z=0\), the contributions from the real dipole (at \(z'\)) and from the image dipole (at \(-z'\)) will cancel each other. Thus, on the ground surface \(\vec{E}_r = 0\).

\[
d\vec{E}_z = \frac{dz'}{4\pi \varepsilon_0} \left[ \frac{2(z-z')^2 - r^2}{R^5} \int_0^t i(z', t-R/c) \, dt + \frac{2(z-z')^2}{cR^4} \frac{\partial i(z', t-R/c)}{\partial t} \right] \vec{a}_z
\]

For \(z=0\), the contributions from the real and image dipoles are equal. Thus

\[
\vec{E}(r, \phi, 0, t) = \frac{1}{2\pi \varepsilon_0} \left[ \int_0^H \frac{2(z'-z)^2 - r^2}{R^5} \int_0^t i(z', t-R/c) \, dt \, dz' \right. \\
\left. + \int_0^H \frac{r^2}{cR^4} \frac{\partial i(z', t-R/c)}{\partial t} \, dz' \right] \vec{a}_z
\]

- **Cylindrical**
- **Induction field**
- **Radiation field**
\[
\begin{align*}
\frac{r^2}{R^2} &= \sin^2(180^\circ - \theta) = \sin^2 \theta \\
\frac{z^2}{R^2} &= \cos^2(180^\circ - \theta) = (-\cos \theta)^2 = \cos^2 \theta \\
\frac{2z'^2 - r^2}{R^2} &= 2 \cos^2 \theta - \sin^2 \theta + 2 \sin^2 \theta - 2 \sin^2 \theta = 2 - 3 \sin^2 \theta \\
\end{align*}
\]

\[
\vec{E}(z, t) = \frac{1}{2\pi\varepsilon_0} \left[ \int_0^H (2 - 3 \sin^2 \theta) \frac{i(z', t - R/c)}{R^3} dz' dt + \right. \\
+ \left. \int_0^H \frac{(2 - 3 \sin^2 \theta)}{c^2 R} i(z', t - R/c) dz' - \right. \\
- \left. \int_0^H \frac{\sin^2 \theta}{c^2 R} \frac{\partial i(z', t - R/c)}{\partial t} dz' \right] \vec{a}_z
\]

Consider the case where \(D \gg H\). Then \(\theta \approx 90^\circ\), \(R \approx D\), and the radiation field term dominates:

\[
E_z(D, t) \approx E^{\text{RAD}}_z(D, t) = -\frac{1}{2\pi\varepsilon_0} \frac{1}{c^2 D} \int_0^H \frac{\partial i(z', t - D/c)}{\partial t} dz' 
\]

If a current pulse propagates up the channel at a constant speed \(v\) and without either distortion or attenuation (the transmission line model)

\[
i(z', t) = i(0, t - z'/v)
\]

\[
\frac{\partial i(t - z'/v)}{\partial z'} = \frac{\partial i(t - z'/v)}{\partial (t - z'/v)} \cdot \frac{\partial (t - z'/v)}{\partial z'} = -\frac{1}{v} i(t - z'/v)
\]

\[
\frac{\partial i(t - z'/v)}{\partial t} = \frac{\partial i(t - z'/v)}{\partial (t - z'/v)} \cdot \frac{\partial (t - z'/v)}{\partial t} = i'(t - z'/v) = -\sqrt{2} \frac{\partial i(t - z'/v)}{\partial z'}
\]

(convert time derivative to spatial derivative).
Substituting \( i(0, t - \frac{z'}{v} - \frac{D}{c}) \) for \( i(z', t - \frac{D}{c}) \) and \\
\(-v(\partial i/\partial z')\) for \( \partial i/\partial t \) we have

\[
E_{z}^{RAD} (D, t) = \frac{V}{2\pi \varepsilon_0 c^2 D} \int_{0}^{H} \frac{\partial i(0, t - \frac{z'}{v} - \frac{D}{c})}{\partial z'} \, dz'.
\]

The upper limit to the integral can be replaced by the maximum height from which radiation can be seen at distance \( D \) at time \( t \). This maximum height \( z'_{\text{max}} \) is found from the equation

\[
t = \frac{z'_{\text{max}}}{v} + \frac{R}{c} \approx \frac{z'_{\text{max}}}{v} + \frac{D}{c}
\]

\[
\Rightarrow \quad z'_{\text{max}} = v(\text{t} - \frac{D}{c})
\]

\[
E_{z}^{RAD} (D, t) = \frac{V}{2\pi \varepsilon_0 c^2 D} \int_{z'=0}^{z'=z'_{\text{max}}} \frac{\partial i(0, t - \frac{z'}{v} - \frac{D}{c})}{\partial z'} \, dz'
\]

\[
= -\frac{V}{2\pi \varepsilon_0 c^2 D} \, i(D, t - D/c)
\]

\[
= -\frac{M_0 V}{2\pi D} \, i(0, t - D/c)
\]

Similarly,

\[
B_{\phi}^{RAD} (D, t) = \frac{M_0 V}{2\pi c D} \, i(0, t - D/c)
\]

Thus the shape of the electric and magnetic field pulses at large distances is the same as the shape of the current pulse (assuming that the transmission line model is valid and \( v = \text{const} \)).
VECTOR OPERATIONS

RECTANGULAR COORDINATES

\[ \nabla u = \hat{x} \frac{\partial u}{\partial x} + \hat{y} \frac{\partial u}{\partial y} + \hat{z} \frac{\partial u}{\partial z} \]  

(1-37)

\[ \nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \]  

(1-42)

\[ \nabla \times A = \hat{x} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \]  

(1-43)

\[ \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \]  

(1-46)

CYLINDRICAL COORDINATES

\[ \nabla u = \hat{\rho} \frac{\partial u}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial u}{\partial \phi} + \hat{z} \frac{\partial u}{\partial z} \]  

(1-85)

\[ \nabla \cdot A = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \]  

(1-87)

\[ \nabla \times A = \hat{\rho} \left( \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\rho}{\partial \rho} \right) + \hat{z} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\phi) - \frac{1}{\rho} \frac{\partial A_\rho}{\partial \phi} \right] \]  

(1-88)

\[ \nabla^2 u = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial u}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} \]  

(1-89)

SPHERICAL COORDINATES

\[ \nabla u = \hat{r} \frac{\partial u}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial u}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial u}{\partial \phi} \]  

(1-101)

\[ \nabla \cdot A = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} \]  

(1-103)

\[ \nabla \times A = \frac{\hat{r}}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right] + \frac{\hat{\theta}}{r} \left[ \frac{1}{\sin \theta} \frac{\partial A_r}{\partial r} - \frac{\partial A_\phi}{\partial \phi} \right] \]  

\[ + \frac{\hat{\phi}}{r} \left[ \frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right] \]  

(1-104)

\[ \nabla^2 u = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial u}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u}{\partial \phi^2} \]  

(1-105)
VECTOR FORMULAS

\[ A \cdot (B \times C) = (A \times B) \cdot C \]  
(1-29)

\[ A \times (B \times C) = B(A \cdot C) - C(A \cdot B) \]  
(1-30)

\[ \nabla \times \nabla u = 0 \]  
(1-48)

\[ \nabla \cdot (\nabla \times A) = 0 \]  
(1-49)

\[ (A \times B) \cdot (C \times D) = (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C) \]  
(1-106)

\[ \frac{d}{d\sigma}(uA) = \frac{du}{d\sigma}A + u \frac{dA}{d\sigma} \]  
(1-107)

\[ \frac{d}{d\sigma}(A \cdot B) = \frac{dA}{d\sigma} \cdot B + A \frac{dB}{d\sigma} \]  
(1-108)

\[ \frac{d}{d\sigma}(A \times B) = \frac{dA}{d\sigma} \times B + A \times \frac{dB}{d\sigma} \]  
(1-109)

\[ \nabla(u + v) = \nabla u + \nabla v \]  
(1-110)

\[ \nabla(uc) = u\nabla c + c\nabla u \]  
(1-111)

\[ \nabla(A \cdot B) = B \times (\nabla \times A) + A \times (\nabla \times B) + (B \cdot \nabla)A + (A \cdot \nabla)B \]  
(1-112)

\[ \nabla(C \cdot r) = C \quad \text{where } C = \text{const.} \]  
(1-113)

\[ \nabla \cdot (A + B) = \nabla \cdot A + \nabla \cdot B \]  
(1-114)

\[ \nabla \cdot (uA) = A \cdot (\nabla u) + u(\nabla \cdot A) \]  
(1-115)

\[ \nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B) \]  
(1-116)

\[ \nabla \times (A + B) = \nabla \times A + \nabla \times B \]  
(1-117)

\[ \nabla \times (uA) = (\nabla u) \times A + u(\nabla \times A) \]  
(1-118)

\[ \nabla \times (A \times B) = (\nabla \cdot B)A - (\nabla \cdot A)B + (B \cdot \nabla)A - (A \cdot \nabla)B \]  
(1-119)

\[ \nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A \]  
(1-120)

where

\[ (A \cdot \nabla)B = \frac{1}{2} \left( A_x \frac{\partial B_x}{\partial x} + A_y \frac{\partial B_x}{\partial y} + A_z \frac{\partial B_x}{\partial z} \right) \]

\[ + \frac{1}{2} \left( A_x \frac{\partial B_y}{\partial x} + A_y \frac{\partial B_y}{\partial y} + A_z \frac{\partial B_y}{\partial z} \right) \]

\[ + \frac{1}{2} \left( A_x \frac{\partial B_z}{\partial x} + A_y \frac{\partial B_z}{\partial y} + A_z \frac{\partial B_z}{\partial z} \right) \]  
(1-121)
6. Modeling of lightning processes
7. Measurement of lightning electric and magnetic fields
Electrostatic fluxmeter (field mill)

(converts a slowly varying electric field into an amplitude-modulated ac voltage $V$ across the resistor)

![Diagram of a fluxmeter](image)

Time resolution depends on the rotation speed and the number of segments (typically of the order of tens of milliseconds).

Upper frequency response is up to 1 kHz or so.
not placed flush with the ground (meaning the bottom plate will be buried), the electric field will be distorted (enhanced). The charge induced on the top (sensor) plate is

\[ Q = \varepsilon_0 EA \]  

Eq. 2.1

since the boundary condition on the normal component of the electric flux density \( D = \varepsilon_0 E \) is that it is numerically equal to the surface charge density, where \( A \) is the area of the top (sensor) plate. If the charge is allowed to flow from the top plate to ground then a change in the incident electric field will cause a current. Taking the derivative of Eq. 2.1,

\[ I = \frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt} \]  

Eq. 2.2

The current-source (Norton) equivalent circuit of the antenna and integrator is shown below.

![Figure 2.1](image-url)

Figure 2.1
The Norton equivalent can be understood as the short circuit current of the source in parallel with its impedance.

The output voltage in Figure 2.1 is

\[ V_{\text{out}} = \frac{1}{C_s + C} \int I \, dt = \frac{1}{C_s + C} \varepsilon_o A \int \frac{dE}{dt} \, dt = \frac{\varepsilon_o A E}{C_s + C} \]

Eq. 2.3

The voltage-source (Thevenin) equivalent circuit model for the antenna can be derived from the Norton equivalent as its open-circuit voltage \( V_{\text{th}} \) in series with the same impedance,

\[ V_{\text{th}} = \frac{1}{C_s} \int I \, dt = \frac{1}{C_s} \int \varepsilon_o A \frac{dE}{dt} \, dt = \frac{\varepsilon_o A}{C_s} E, \]

where \( \frac{\varepsilon_o A}{C_s} \) is called the effective height of the antenna. The Thevenin equivalent circuit is shown in Figure 2.2.

The complete circuit is a capacitive divider operating on the open circuit voltage so that

\[ V_{\text{out}} = V_{\text{th}} \frac{C_s}{(C_s + C)} = \frac{\varepsilon_o A E}{(C_s + C)} \]

, the same as Equation 2.3.

Usually \( C_s \ll C \) and hence
\[ V_{\text{out}} = \frac{\varepsilon_0 AE}{C} \quad \text{Eq. 2.4} \]

The output voltage is proportional to the electric field from the lightning, and the decay time \( \tau \) of this system will be dependent on \( C \) and the input resistance and capacitance of the recorder used to measure the output voltage.

Usually \( C \) is much greater than the input capacitance of the recorder and we can write

\[ \tau = R_{\text{in}} C \quad \text{Eq. 2.5} \]

where \( R_{\text{in}} \) is the input resistance of the recorder.

2.4 Magnetic Field Measuring Techniques

To measure the magnetic field from lightning a loop of wire can be used as an antenna.

According to Faraday’s Law a changing magnetic field passing through an open circuited loop of wire will induce a voltage at the terminals of the wire. The voltage at the terminals of the wire is

\[ V_{\text{out}} = A \frac{dB}{dt} \quad \text{Eq. 2.6} \]

where \( A \) is the area of the antenna and \( B \) is the magnetic flux density passing through the loop, perpendicular to the plane of the loop. Since the output voltage is proportional to the time derivative of the magnetic field, this voltage will have to be integrated to obtain the signal proportional to the field.
The Thevenin equivalent circuit of a loop antenna is the open circuit source voltage $A\frac{dB}{dt}$ in series with the source impedance (primarily inductive). The Thevenin equivalent is shown in Figure 2.3.

![Figure 2.3](image)

A Norton equivalent circuit model for the antenna can be derived from the Thevenin equivalent with $I = V_{th} = A\frac{dB}{dt}$.

The Norton equivalent circuit is shown in Figure 2.4.

![Figure 2.4](image)
Since the loop antenna has an inductance $L$ associated with it, the impedance of the antenna will change with frequency. In the frequency domain that impedance is $\omega L$ where $\omega$ is the angular frequency. This frequency-dependent impedance will cause distortion in the derivative signal. To eliminate the distortion a resistor can be placed in series with the antenna with the resistive impedance $R$ much higher than the inductive impedance $\omega L$ of the antenna at the highest frequency of interest. The decay time constant of the overall circuit in Figures 2.3 and 2.4 will be $\tau = RC$ as long as $R$ is much smaller than the input resistance of the recorder and $C$ is much larger than the input capacitance of the recorder, both conditions being usually met. In the frequency domain the output voltage across the capacitor is

$$
V_{out} = \frac{A(j\omega B)(\frac{1}{j\omega C})}{R + j\omega L + \frac{1}{j\omega C}}
$$

Eq. 2.7

If we choose $R >> j\omega L$ and $R >> \frac{1}{j\omega C}$ for the highest and lowest frequencies of interest respectively, as discussed above, then

$$
V_{out} = \frac{AB}{RC}
$$

Eq. 2.8
8. Lightning locating systems
9. Deleterious effects of lightning and protective techniques
Effects of Lightning

1. Deleterious effects
   1.1. Death and injury to people and animals
   1.2. Damage and destruction to ground-based structures and boats
   1.3. Damage and destruction to airborne vehicles
   1.4. Damage to trees and forest fires
   1.5. Outages of power and communication lines
   1.6. Damage to sensitive solid-state electronic components

Death and injury to people

Deaths in the United States due to different meteorological causes from 1940 to 1973
(Weigel, 1976 as reported by Lopez et al., 1993)

<table>
<thead>
<tr>
<th>Deaths</th>
<th>Lightning</th>
<th>Tornado</th>
<th>Flood</th>
<th>Hurricane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>6928</td>
<td>4474</td>
<td>3075</td>
<td>1825</td>
</tr>
<tr>
<td>Annual average</td>
<td><strong>204</strong></td>
<td>132</td>
<td>90</td>
<td>54</td>
</tr>
</tbody>
</table>

The worldwide numbers of deaths and injuries due to lightning are estimated to be about 1000 and 2500 per year, respectively (Andrews et al., 1992).
NUMBER OF DEATHS BY
NATURAL HAZARDS, 1940-1981
(after Kessler, 1988)

<table>
<thead>
<tr>
<th>Hazard</th>
<th>Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIGHTNING</td>
<td>7,741</td>
</tr>
<tr>
<td>TORNADO</td>
<td>5,268</td>
</tr>
<tr>
<td>FLOOD</td>
<td>4,481</td>
</tr>
<tr>
<td>HURRICANE</td>
<td>1,923</td>
</tr>
</tbody>
</table>


[Data compiled by National Lightning Safety Institute, tel. 303-666-8817]
Fig. 3 (i) Direct strike. (ii) Contact voltage: \( U = \frac{I}{R} \times L(d+D) \), where, \( I \) is the current, \( R \) is the resistance of the tree, \( L \) is the length of the tree, and \( d \) is the distance between the tree and the point of contact. (iii) Step voltage: \( U = \frac{I}{2\pi} \times \frac{L}{d} \), where, \( I \) is the current, \( L \) is the length of the tree, and \( d \) is the distance between the tree and the point of contact. (iv) Reproduced with the permission of R. H. Golde, 1973.)
SIN & RTL FT. McCullough, AL (FISHER AND SCHNEIDER)
Figure 1. Vertical cross section of Peak in Japan Alps and positions of people at the lightning accident. (Kitagawa, 2000)
About two-thirds of the people involved in lightning accidents subsequently make a complete recovery. Most, if not all, of these survivors are probably not directly struck by lightning. The most common consequences are burns, shock, and numbness rather than death.

The most common ways in which death might be caused are believed to be circulatory arrest and respiratory arrest.

Fig. 1. Location of respiratory centre in the brain.
(Golde, 1977)

There is strong evidence that the respiratory arrest is only produced when the current passes through the respiratory center in the lower part of the brain.

The circulatory arrest is probably a result of ventricular fibrillation. An electric current passing through the heart may disturb the coordination of its individual muscle fibers so that instead of contracting simultaneously they contract individually.

Animal death due to lightning

In 1968, lightning caused the death of 464 cattle (362 killed directly, the remainder by lightning-caused fires), 13 horses (11 killed directly), 42 hogs, and 2 dogs (Uman, 1986). As opposed to humans, there are many instances of multiple deaths amongst cattle and sheep. On June 1, 1983, lightning killed 29 cows as they sheltered under a tree (Elsom, 1993).
Lightning to Wildlife. Elk Herd of 53 animals killed by lightning about August 12, 1999. Location: Mt. Evans CO. Elevation: 12,200 ft. Herd found by hunter in remote area a few days after incident occurred.
Lightning outages of electric power lines

Lightning is a major cause (40 to 50%) of electric service interruptions, resulting in $50 million per year in damage and restoration expenses (EPRI).

Average annual number of direct lightning strikes to power lines of different height (per 100 km, normalized to 26 thunderstorm days)

1 - Stekolnikov and Lamond (1938)
2 - Lewis and Foust (1937)
3 - Waldorf (1941)
4 - Hansson and Waldorf (1944)
5 - Schloemann et al. (1958)

Induced overvoltages

Table 8.1 (Lewis, 1965)

<table>
<thead>
<tr>
<th>Reference Class, kv</th>
<th>Basic Impulse Level, kv†</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>30</td>
</tr>
<tr>
<td>2.5</td>
<td>45</td>
</tr>
<tr>
<td>5.0</td>
<td>60</td>
</tr>
<tr>
<td>8.7</td>
<td>75</td>
</tr>
<tr>
<td>15</td>
<td>110</td>
</tr>
<tr>
<td>23</td>
<td>150</td>
</tr>
<tr>
<td>34.5</td>
<td>200</td>
</tr>
<tr>
<td>46</td>
<td>250</td>
</tr>
<tr>
<td>69</td>
<td>350</td>
</tr>
<tr>
<td>92</td>
<td>450</td>
</tr>
<tr>
<td>115</td>
<td>550</td>
</tr>
<tr>
<td>138</td>
<td>650</td>
</tr>
<tr>
<td>161</td>
<td>750</td>
</tr>
<tr>
<td>196</td>
<td>900</td>
</tr>
<tr>
<td>230</td>
<td>1050</td>
</tr>
<tr>
<td>287</td>
<td>1300</td>
</tr>
<tr>
<td>345</td>
<td>1550</td>
</tr>
<tr>
<td>360</td>
<td>1610</td>
</tr>
<tr>
<td>(500</td>
<td>1800)</td>
</tr>
</tbody>
</table>

10-km wood-pole test line, approximately 8 m high, about 400-500 kv impulse insulation level.

Figure 3 Relationship between induced voltage amplitudes and recorded flash distance.

(Eriksson and Meal, 1984)
Fig. 4. Geometry and tripout rates for five EHV lines in a common lightning environment. (Golde, 1977)

### Table 1. Lightning Faults on Distribution Lines (Florida Power Corporation)

<table>
<thead>
<tr>
<th>Substation</th>
<th>Ground Flash Density</th>
<th>Circuit Number</th>
<th>Circuit Length (Kilometer)</th>
<th>Lightning Faults</th>
<th>Predicted/Actual</th>
<th>W(35)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diston</td>
<td>16.95</td>
<td>X60</td>
<td>34.6</td>
<td>14</td>
<td>1.89/0.40</td>
<td>0.79</td>
</tr>
<tr>
<td>(end 1985-Dec. 1986)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fortieth St.</td>
<td>10.06</td>
<td>X81</td>
<td>17.5</td>
<td>9</td>
<td>1.18/0.51</td>
<td>0.58</td>
</tr>
<tr>
<td>(end 1985-Dec. 1986)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northeast</td>
<td>13.59</td>
<td>X283</td>
<td>11.3</td>
<td>7</td>
<td>1.51/0.62</td>
<td>0.59</td>
</tr>
<tr>
<td>(end 1985-Dec. 1986)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pilsbury</td>
<td>29.89</td>
<td>X253</td>
<td>34.2</td>
<td>36</td>
<td>3.33/1.05</td>
<td>0.68</td>
</tr>
<tr>
<td>(July 1985-Dec. 1987)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**BIL = 60-300 kV; average conductor height = 10 m; arresters every 400 m**
Lightning damage to electronics

A multitude of functions (highway traffic control, industrial process control, etc.) that were once performed mechanically, or electromechanically are now being accomplished by or with the aid of solid-state electronics systems which are extremely vulnerable to damage from lightning-induced voltages and currents. Voltage spikes of only 40 V can cause failure of some data and control-line interfacing components.

Modern aircraft and space vehicles rely upon microelectronics to perform light-essential functions such as flight and engine controls and navigation, often without the aid of mechanical backup. Some of these airborne systems, which operate at very low power levels and thus are especially sensitive to lightning-induced transients, are installed in airframes fabricated of fiber-reinforced plastics that do not provide the same degree of electromagnetic shielding as the conventional aluminum airframes of the past [13]. Interestingly, the deleterious signals induced in the Atlas-Centaur 67 guidance system are thought to have propagated into the vehicle through an insulating fiberglass-honeycomb nose fairing that covered the front 20 feet or so of the vehicle. (Uman, 1988)

Most failures of solid-state electronics components/systems fit into one of the following categories: hard, upset, and latent. Hard failures are permanently damaged parts. Replacements must be installed to get the system restored to normal. In some parts the damaged area on the microchip may be smaller than one tenth the cross section of a human hair, invisible to the exterior, while at the other end of the spectrum, only the mounting leads may remain on a charred circuit board.

Upset failure is a temporary malfunction that has the potential for causing serious damage. Transients have been reported as causing shutdown and permanently degraded subsystem outputs without inducing damage to onboard subsystem components [13]. Results can be disastrous, as demonstrated by the March 1987 destruction of an Atlas-Centaur rocket, which was struck by lightning shortly after launching.

Latent failures are the "walking wounded," devices which have been overstressed and slightly degraded but continue to function. At some future date these become hard failures, perhaps months or years later. One latent failure in a satellite system did not surface for 5 years [14]! It is quite possible that we often credit poor quality with what may be caused by latent failures. (Clark and Gavender, 1990)

![Graph]

Fig. 1. Frequency distribution for proportion of boats with electronics systems that had none, some, or all of their electronics systems damaged as a result of a direct lightning strike. (Thomson, 1991)

96% of 67 boats struck by lightning sustained at least some damage to electronics.

For a typical maximum distance from the mast r = 2 m and dI/dt = 100 kA/μs, the induced emf is about 10 kV/m² of equivalent loop area. Printed circuit boards may have dimensions of the order of 100 cm² and are subjected to induced voltages of about 100 V.
NOTES -

A = 20 feet (6 m) maximum spacing for 10 inch (254 mm) air terminal height or 25 feet (7.6 m) maximum spacing for 24 inch (610 mm) air terminal height.

B = 2 feet (610 mm) maximum spacing from corner, roof edge or ridge end.

*Figure 6.2 revised July 8, 1998*