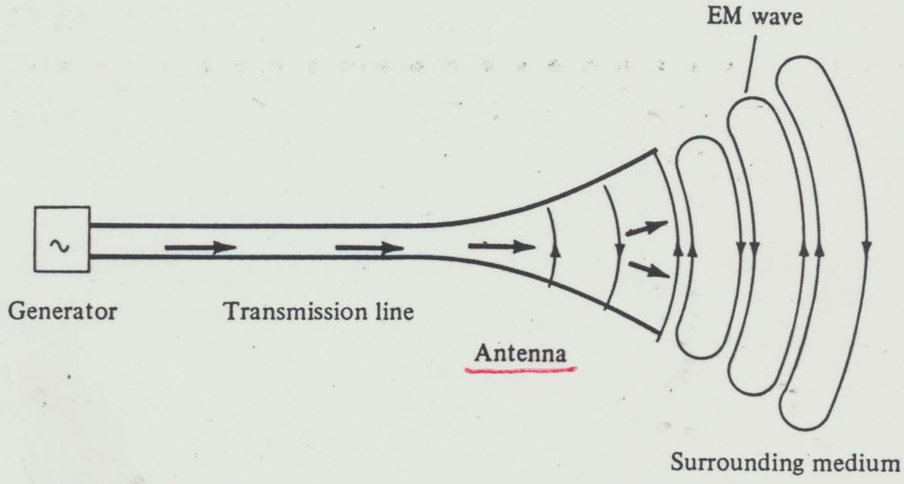
Antennas (Ch. 13)

Antennas are structures designed for radiating and receiving electromagnetic energy effectively in a prescribed manner.



Antenna as a matching device between the guiding structure and the surrounding medium.

We will not attempt a broad coverage of antenna theory; our discussion will be limited to the basic types of antennas such as the Hertzian dipole, the half-wave dipole, the quarter-wave monopole, and the small loop. For each of these types, we will determine the radiation fields by taking the following steps:

1. Select an appropriate coordinate system and determine the magnetic vector potential **A**. $\bar{A} = \frac{\mu I d\bar{l}}{4\pi r}$
2. Find **H** from $\mathbf{B} = \mu \mathbf{H} = \nabla \times \mathbf{A}$. $\bar{H} = \frac{1}{\mu} \nabla \times \bar{A}$
3. Determine **E** from $\nabla \times \mathbf{H} = \varepsilon \dfrac{\partial \mathbf{E}}{\partial t}$ or $\mathbf{E} = \eta \mathbf{H} \times \mathbf{a}_k$ assuming a lossless medium ($\sigma = 0$). $\bar{E}_s = \frac{1}{j\omega\varepsilon} \nabla \times \bar{H}_s$

Amp. Law

4. Find the far field and determine the time-average power radiated using

$$P_{rad} = \int_S \mathcal{P}_{ave} \cdot d\mathbf{S}, \ \mathrm{W} \quad \text{where} \quad \mathcal{P}_{ave} = \frac{1}{2} \mathrm{Re}\,(\mathbf{E}_s \times \mathbf{H}_s^*), \ \frac{\mathrm{W}}{\mathrm{m}^2}$$

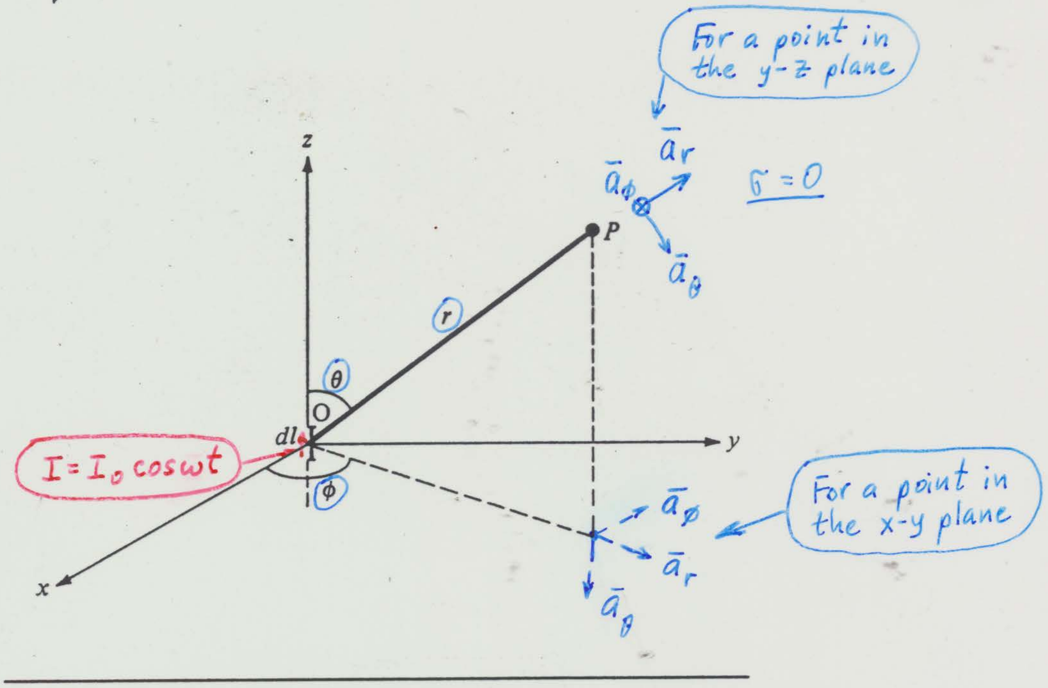
Poynting vector

Hertzian dipole

By a Hertzian dipole, we mean an infinitesimal current element Idl. It serves as a building block from which the field of a practical antenna can be calculated by integration.

1 MHz
$dl \le 6\,m$

$$dl \le \frac{\lambda}{50}$$



A Hertzian dipole carrying current $I = I_0 \cos \omega t$.

$$\begin{cases} H_{\phi s} = \dfrac{I_0\,dl}{4\pi} \sin\theta \left[\dfrac{j\beta}{r} + \dfrac{1}{r^2} \right] e^{-j\beta r} \\ E_{\theta s} = \dfrac{\eta I_0\,dl}{4\pi} \sin\theta \left[\dfrac{j\beta}{r} + \dfrac{1}{r^2} - \dfrac{j}{\beta r^3} \right] e^{-j\beta r} \\ E_{rs} = \dfrac{\eta I_0\,dl}{2\pi} \cos\theta \left[\dfrac{1}{r^2} - \dfrac{j}{\beta r^3} \right] e^{-j\beta r} \\ H_{rs} = H_{\theta s} = E_{\phi s} = 0 \end{cases}$$

1 MHz
$r \ge 500\,m$

If $\beta r \gg 1$ or $2\pi r \gg \lambda$, $(\beta = 2\pi/\lambda)$ $r \gg \lambda/2\pi$

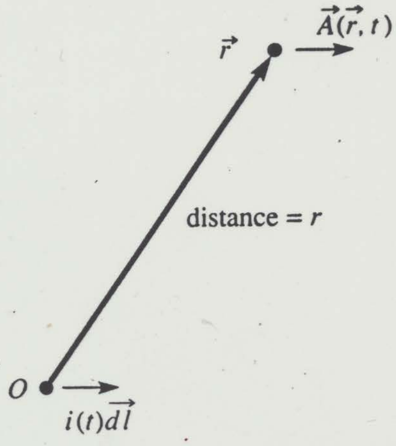
Far fields

$$\boxed{H_{\phi s} = \frac{j I_0 \beta\, dl}{4\pi r} \sin\theta\, e^{-j\beta r} \quad \Big| \quad E_{\theta s} = \eta H_{\phi s}} \qquad \left(\eta = \sqrt{\frac{\mu}{\varepsilon}}\right)$$

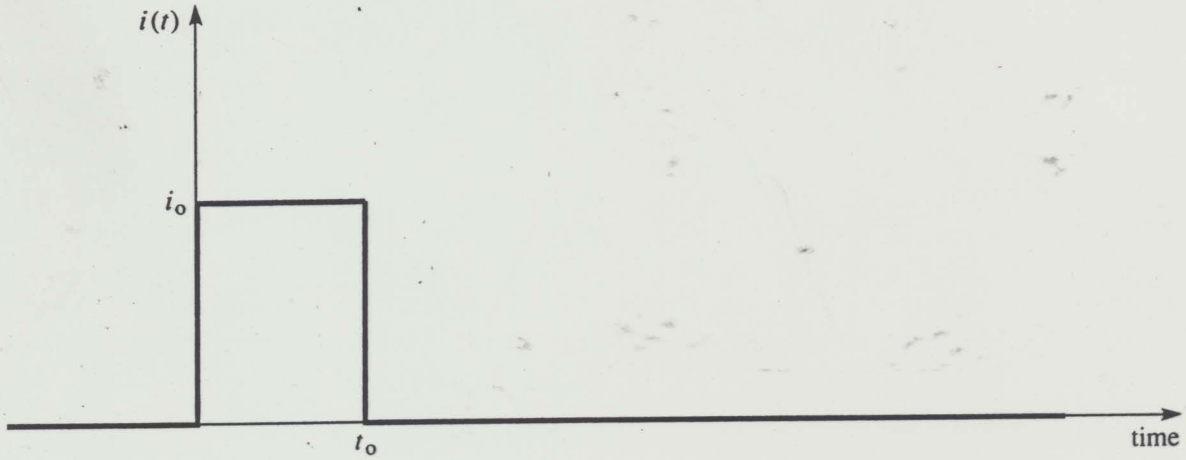
$$H_{rs} = H_{\theta s} = E_{rs} = E_{\phi s} = 0$$

$\Big(r_b = \dfrac{2d^2}{\lambda}$ $\quad r > r_b$ - far zone
$\quad r < r_b$ - near zone

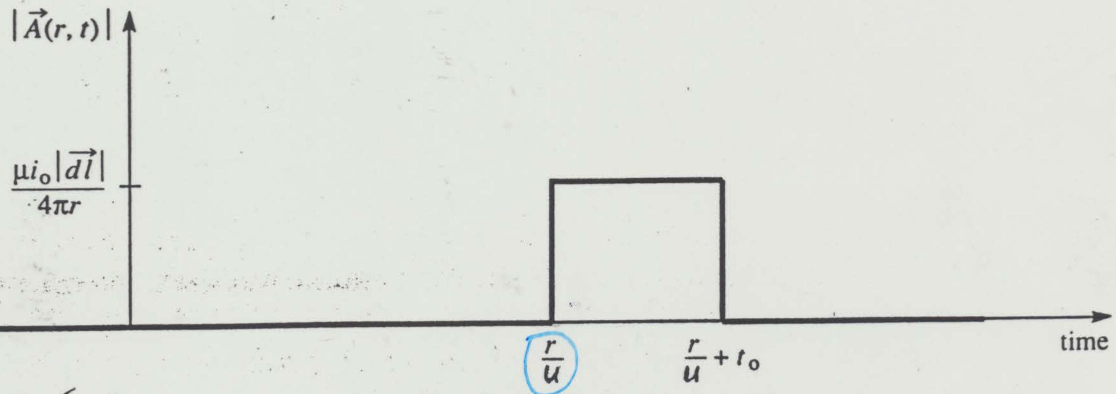
d is the largest dimension of the antenna $\Big)$



(a)



(b)



(c)

Figure 10.1 Example of retarded vector potential. If the observation point is as shown in (a), and the current at the origin is the rectangular pulse shown in (b), the retarded vector potential at the observation point is as shown in (c).

Transformation of vector components in Cartesian coordinates to spherical coordinates

$$(A_x, A_y, A_z) \rightarrow (A_r, A_\theta, A_\phi)$$

$$
\begin{aligned}
A_r &= A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z\cos\theta \\
A_\theta &= A_x \cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta \\
A_\phi &= -A_x\sin\phi + A_y\cos\phi
\end{aligned} \tag{2.26}
$$

In matrix form, the $(A_x, A_y, A_z) \rightarrow (A_r, A_\theta, A_\phi)$ vector transformation is performed according to

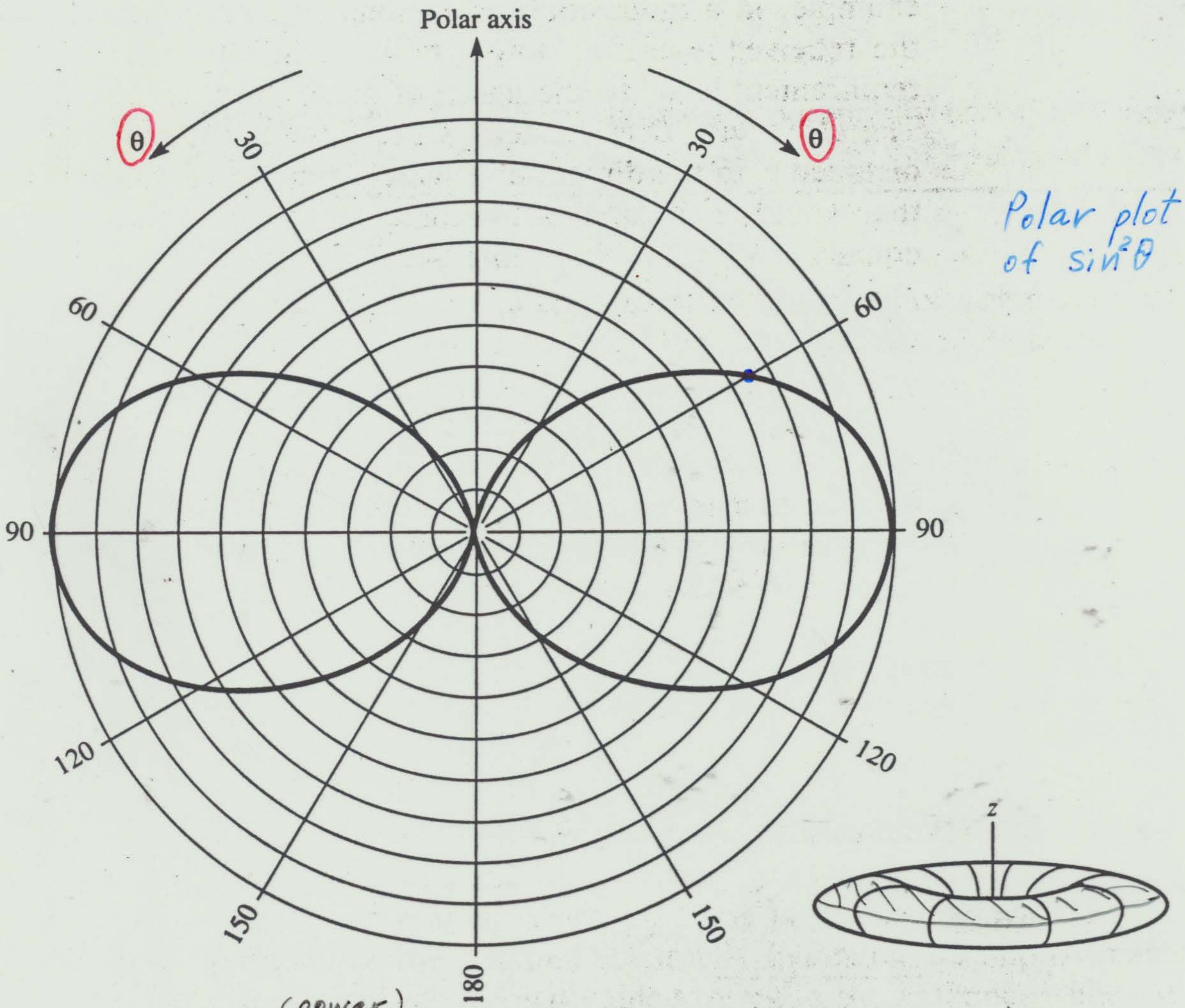
$$
\boxed{\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}} \tag{2.27}
$$

Expression of the curl of a vector field $\bar{A}$ $(\nabla \times \bar{A})$ in spherical coordinates

$$
\nabla \times \mathbf{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & (r\sin\theta)\,\mathbf{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & (r\sin\theta)\,A_\phi \end{vmatrix}
$$

$$
= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\mathbf{a}_\theta
$$

$$
+ \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\mathbf{a}_\phi
$$



Polar plot of $\sin^2\theta$

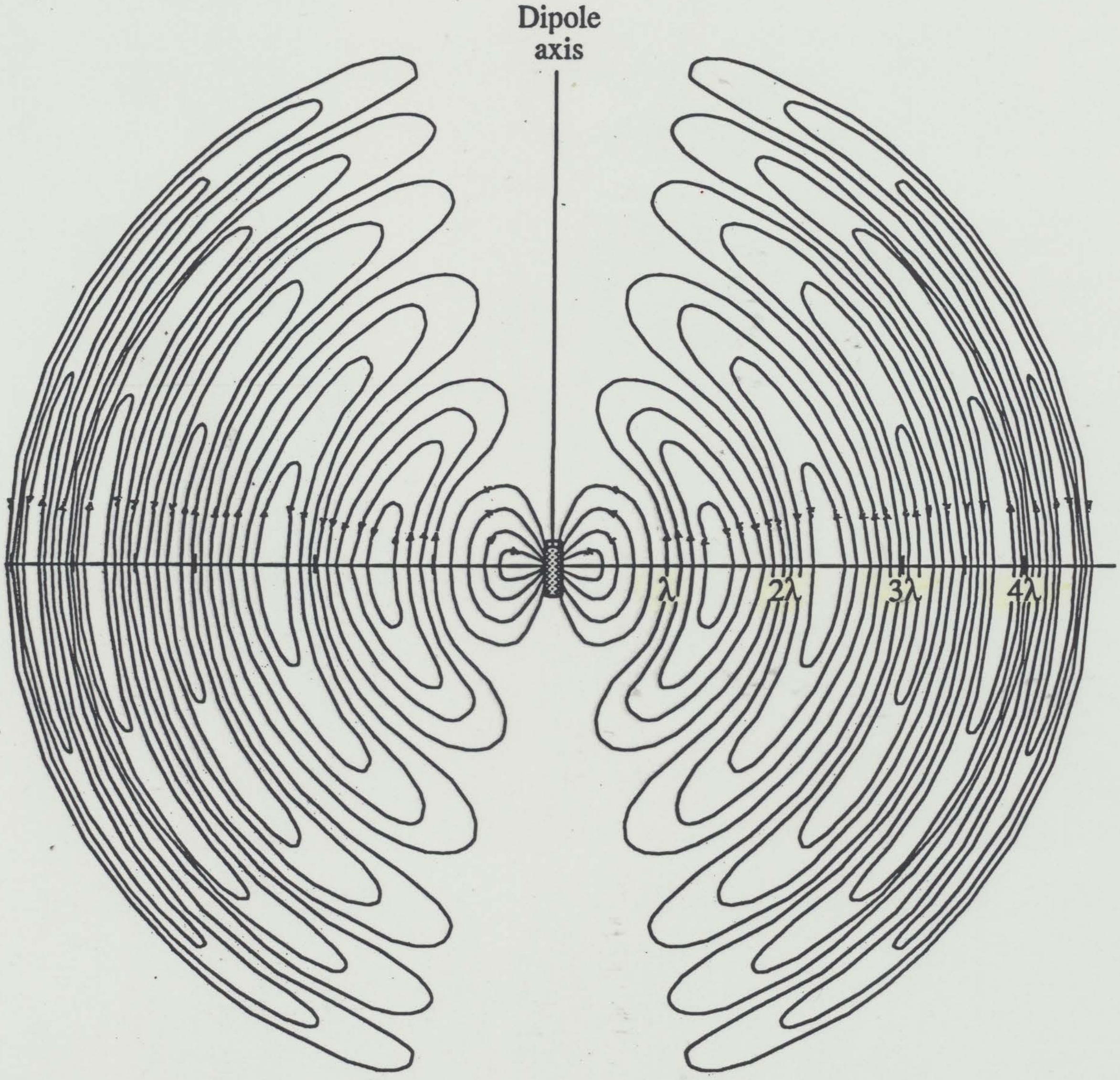
Figure 10.2 Radiation (power) pattern of the elementary dipole.

The time-average power density:

$$S_{ave} = \frac{1}{2} Re\left(\bar{E}_s \times \bar{H}_s^*\right) = \frac{1}{2} Re\left(\overbrace{E_{\theta s}}^{\eta H_{\phi s}} H_{\phi s}^* \bar{a}_r\right)$$

$$= \frac{1}{2}\eta \left|H_{\phi s}\right|^2 \bar{a}_r = \frac{1}{2}\eta\left(\frac{I_0 \beta dl}{4\pi r}\right)^2 \underline{\sin^2\theta} \quad (W/m^2)$$

$$H_{\phi s} = j\boxed{\frac{I_0\beta dl}{4\pi r}\sin\theta}\, e^{-j\beta r}$$



E- field lines for an oscillating dipole at t = const.

$r \gg \lambda/2\pi$ - far zone

Antenna radiation resistance

It is a useful measure of the amount of power radiated by an antenna.

For a Hertzian dipole,

$$S_{ave} = \frac{I_o^2 \eta \beta^2 dl^2}{32\pi^2 r^2} \sin^2\theta$$

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_{ave} \underbrace{r^2 \sin\theta \, d\theta \, d\phi}_{ds} = \frac{I_o^2 \eta \overbrace{\beta^2}^{\left(\frac{2\pi}{\lambda}\right)^2} dl^2}{32\pi^2} 2\pi \underbrace{\int_0^{\pi} \sin^3\theta \, d\theta}_{\frac{4}{3}}$$

$$P_{rad} = \frac{I_o^2 \pi \eta}{3}\left[\frac{dl}{\lambda}\right]^2 = 40\pi^2 \left[\frac{dl}{\lambda}\right]^2 I_o^2 \quad (\text{for } \eta = 120\pi)$$

This power is equivalent to the power dissipated in a fictitious resistance R_{rad} by current I_o:

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad} \Rightarrow \boxed{R_{rad} = \frac{2P_{rad}}{I_o^2}}$$

$$\boxed{R_{rad} = 80\pi^2 \left[\frac{dl}{\lambda}\right]^2} \qquad \frac{dl}{\lambda} \lessapprox \frac{1}{50} \qquad r \gg \frac{\lambda}{2\pi}$$

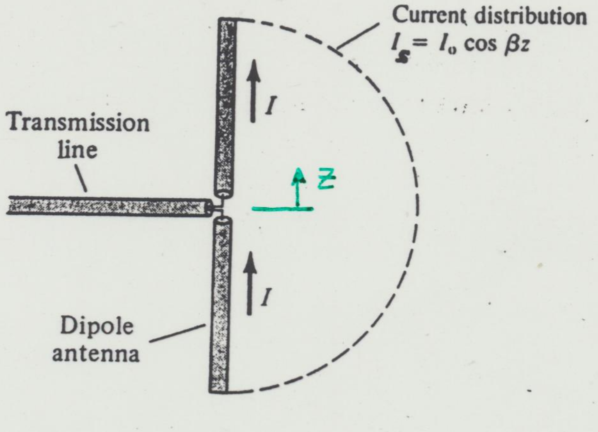
A high radiation resistance is a desirable property for an antenna.

R_{rad} is defined as the value of a hypothetical resistance that would dissipate an amount of power equal to the radiated power P_{rad} when the current in the resistance is equal to the ~~(maximum)~~ current along the antenna.

The real part of antenna input impedance:

$$R_{in} = R_{rad} + R_{loss}$$

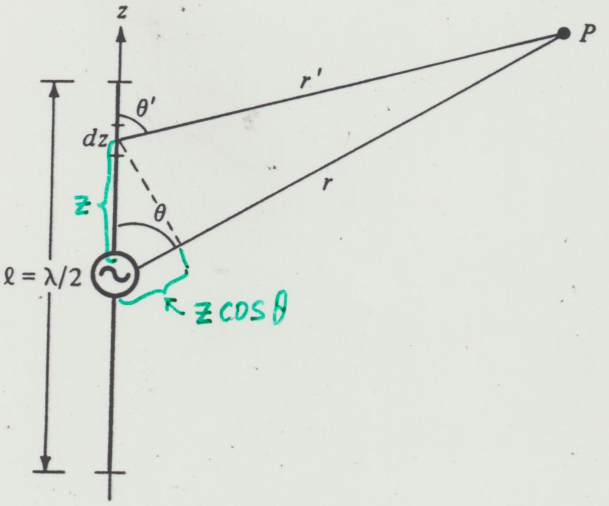
Half-wave dipole antenna



(a)

$$i(t) = I_o \cos\beta z \cos\omega t$$
$$= Re[\underbrace{I_o \cos\beta z}_{I_s}\, e^{j\omega t}]$$

$$-\frac{\lambda}{4} \le z \le \frac{\lambda}{4} \qquad \beta = 2\pi/\lambda$$



(b)

$$dA_{zs} = \frac{\mu I_o \cos\beta z \, dz}{4\pi r'} e^{-j\beta r'}$$

If $r \gg l$, we may set

$\frac{1}{r'} \approx \frac{1}{r}$ and $\theta' \approx \theta$

However, in the exponent we use

$r' = r - z\cos\theta$, not $r' \approx r$

$$e^{-j\beta r'} = e^{-j\beta r} e^{j\beta z\cos\theta}$$

$$A_{zs} = \int_{-\lambda/4}^{\lambda/4} dA_{zs} = \frac{\mu I_o}{4\pi r} e^{-j\beta r} \int_{-\lambda/4}^{\lambda/4} e^{j\beta z\cos\theta} \cos\beta z \, dz = \frac{\mu I_o e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \beta \sin^2\theta}$$

$$(0, 0, A_{zs}) \rightarrow (A_{rs}, A_{\theta s}, 0) \Rightarrow \bar{H}_s = \frac{1}{\mu} \nabla \times \bar{A}_s \Rightarrow \bar{E}_s = \frac{1}{j\omega\varepsilon} \nabla \times \bar{H}_s$$

At far zone (discarding the $1/r^3$ and $1/r^2$ terms) we get

$H_{\phi s} = \dfrac{jI_o e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$	$E_{\theta s} = \eta H_{\phi s}$

$$\bar{S}_{ave} = \frac{1}{2}\eta |H_{\phi s}|^2 \bar{a}_r = \frac{\eta I_o^2 \cos^2\left(\frac{\pi}{2}\cos\theta\right)}{8\pi^2 r^2 \sin^2\theta} \bar{a}_r$$

$$R_{rad} = \frac{2P_{rad}}{I_o^2}$$

$$P_{rad} = \int \bar{S}_{ave} \cdot d\bar{s} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S_{ave}\, r^2 \sin\theta \, d\theta \, d\phi \approx 36.56\, I_o^2$$

$$R_{rad} = 73\,\Omega$$

center-fed antennas, it is found that X_A vanishes at antenna lengths near (but not exactly) integer multiples of $\lambda/2$. At such lengths the antenna is said to be "resonant." Resonant antennas are often used to facilitate impedance matching and avoid unnecessary ohmic loss. (See Problem 2.7.) Interestingly, it is found that the thickness of the antenna wire affects X_A and the resonant length. However, R_A is nearly independent of wire thickness.

Long-wire antennas, such as the half-wave dipole, are used primarily at radio frequencies (say 0.1–30 MHz), in applications where high directivity is not required. They are simple, easy to construct, and have good efficiency and a convenient driving-point impedance. A typical installation of a half-wave dipole is shown in Fig. 10.6. The antenna (which at AM radio broadcast frequencies—about 1 MHz—would be about 150 meters long) is suspended between two tall supports. The transmitter, which is below at ground level, is connected by means of a transmission line. A standing-wave ratio near unity can be obtained by using a transmission line with a characteristic impedance of 73 ohms. (Coaxial line of this impedance is commercially available.) The transmitter is usually designed to drive a pure resistance. Residual reactance is removed by a lumped-circuit matching network such as the "pi network" shown in the figure. This network can also transform the real part of the impedance to the value for which the transmitter is designed, if this is something other than 73 ohms.

The reader may inquire as to whether an elementary dipole could be used in place of a half-wave dipole, with great saving of space. Sometimes they are; for instance, automobile AM radio antennas, which are much

For $f = 1$ MHz
$l = \lambda/2 = 150$ m

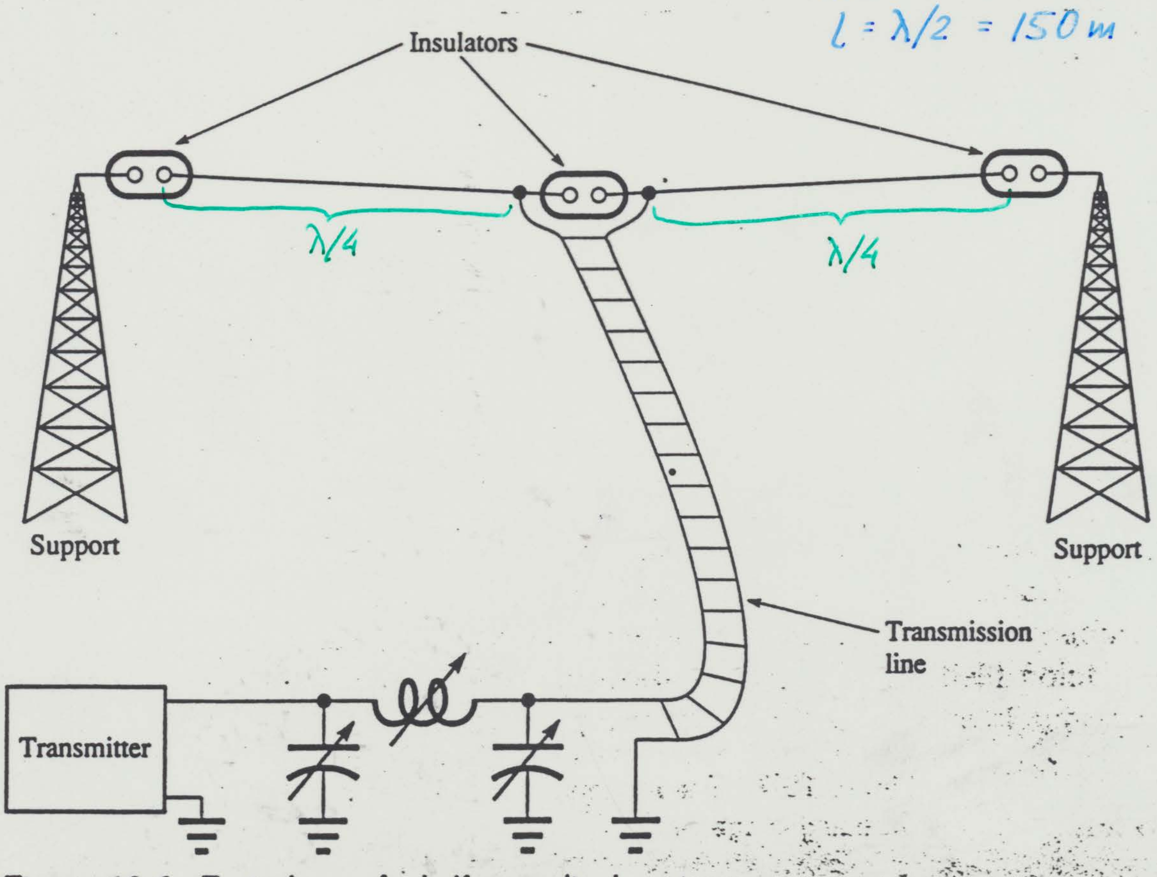
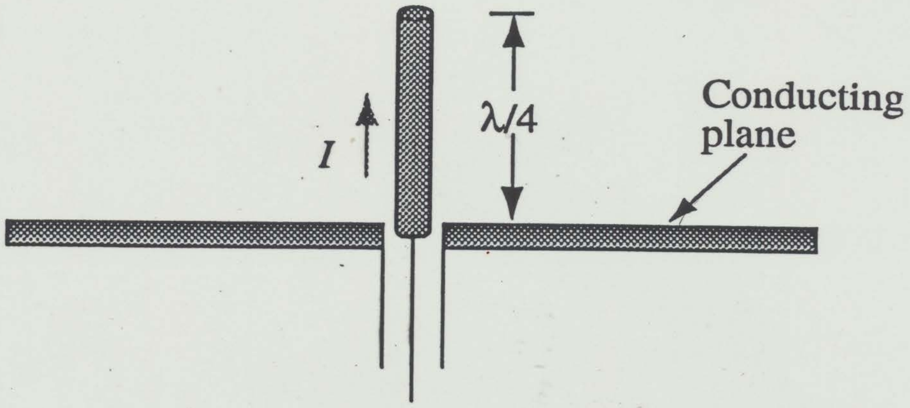


Figure 10.6 Typical use of a half-wave dipole antenna.



(a)

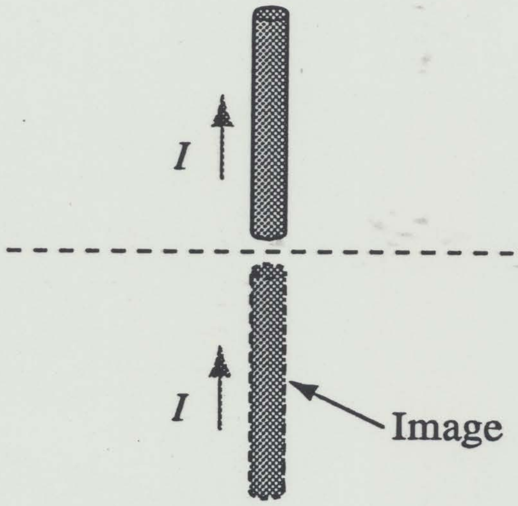
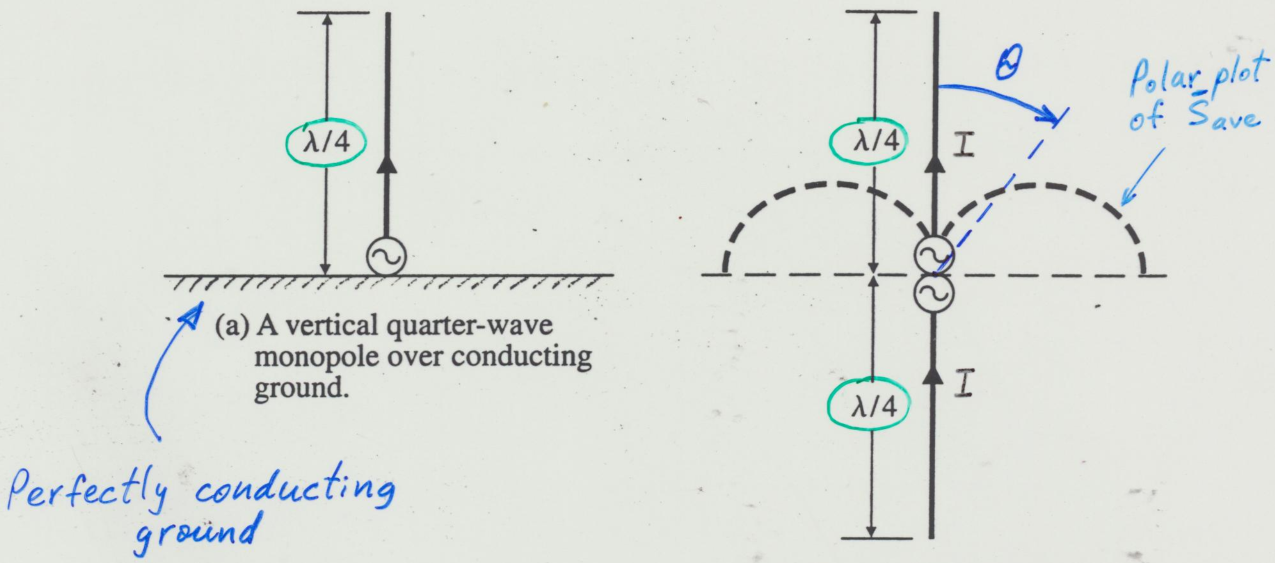


Figure 9-15

(b)

Quarter-wave monopole antenna

The quarter-wave monopole antenna consists of one half of a half-wave dipole antenna located on a conducting ground plane.



(a) A vertical quarter-wave monopole over conducting ground.

(b) Equivalent half-wave dipole radiating into upper half-space.

FIGURE 10-5 Quarter-wave monopole over a conducting ground and its equivalent half-wave dipole.

$$H_{\phi s} = \frac{j I_o e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta} \qquad E_{\theta s} = \eta H_{\phi s}$$

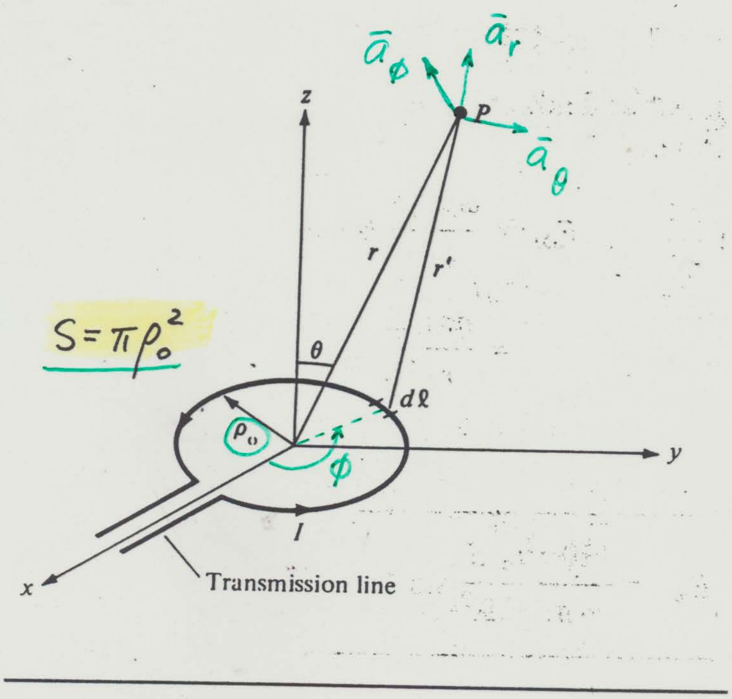
$$\bar{S}_{ave} = \frac{1}{2}\eta |H_{\phi s}|^2 \bar{a}_r$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \bar{S}_{ave} \cdot d\bar{s} \approx \underline{18.28\, I_o^2}$$

$$R_{rad} = \frac{2 P_{rad}}{I_o^2} = \underline{36.5\ \Omega}$$ (one-half of the radiation resistance of a half-wave antenna in free space)

Small loop antenna ($\rho_o \ll \lambda$)

It is used in direction finding and as a TV antenna for ultra-high frequencies (UHF). ← 300-3000 MHz



The small loop antenna.

$S = \pi \rho_o^2$

$$\bar{A} = \oint_L \frac{\mu [I] d\bar{l}}{4\pi r'}$$

$$[I_s] = I_o e^{-j\beta r'}$$

$$\bar{A}_s = \frac{\mu I_o}{4\pi} \oint_L \frac{e^{-j\beta r'}}{r'} d\bar{l}$$

It can be shown that $A_{rs} = A_{\theta s} = 0$ and

$$A_{\phi s} = \frac{\mu I_o S}{4\pi r^2} (1 + j\beta r) e^{-j\beta r} \sin\theta$$

$$\bar{H}_s = \frac{1}{\mu} \nabla \times \bar{A}_s \qquad \bar{E}_s = \frac{1}{j\omega\varepsilon} \nabla \times \bar{H}_s$$

$$E_{\phi s} = \frac{-j\omega\mu I_o S}{4\pi} \sin\theta \left[\frac{j\beta}{r} + \frac{1}{r^2} \right] e^{-j\beta r}$$

$$H_{rs} = \frac{j\omega\mu I_o S}{2\pi\eta} \cos\theta \left[\frac{1}{r^2} - \frac{j}{\beta r^3} \right] e^{-j\beta r}$$

$$H_{\theta s} = \frac{j\omega\mu I_o S}{4\pi\eta} \sin\theta \left[\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3} \right] e^{-j\beta r}$$

$$E_{rs} = E_{\theta s} = H_{\phi s} = 0$$

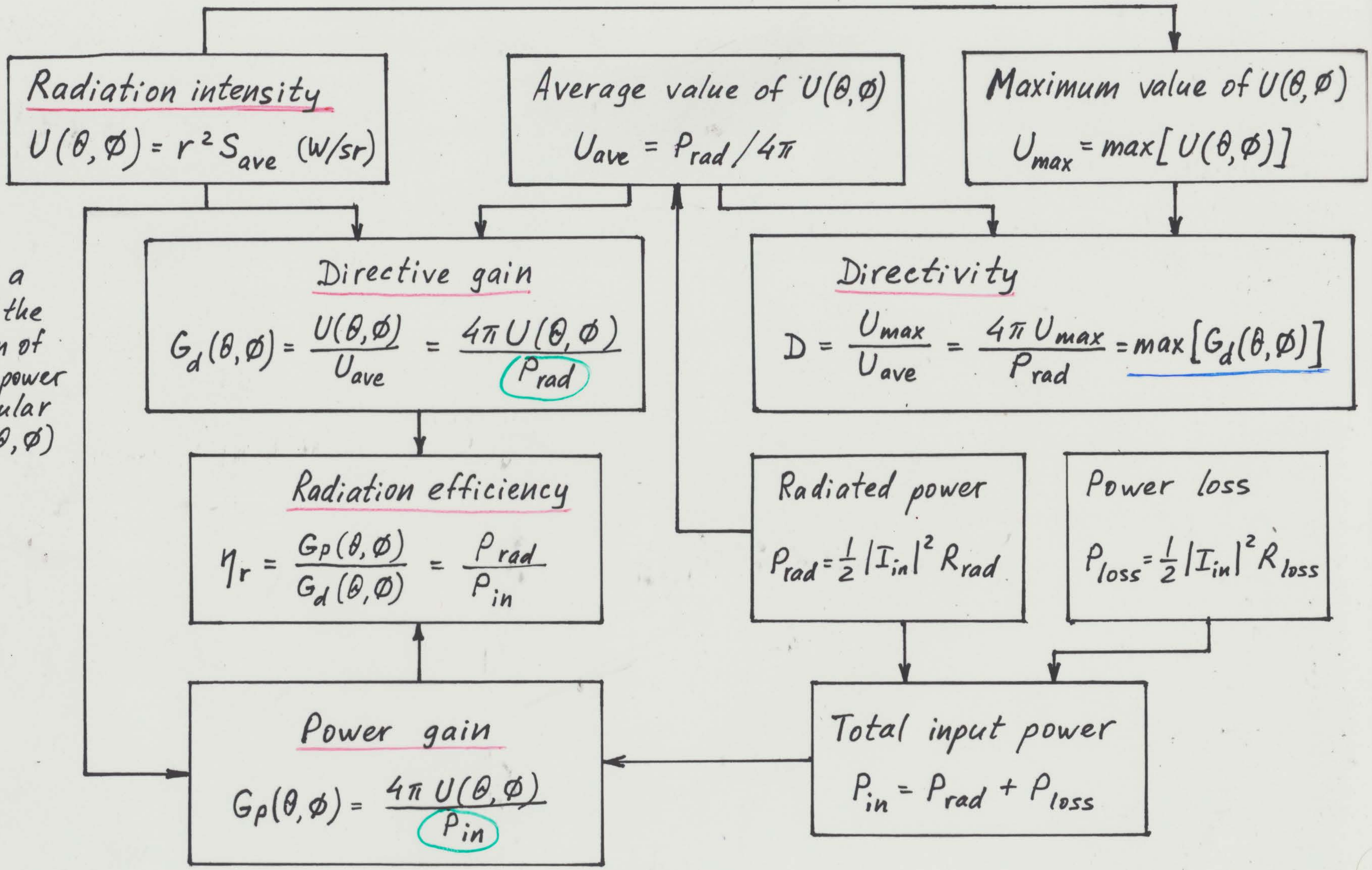
Far fields

$$\boxed{E_{\phi s} = \frac{120\pi^2 I_o}{r} \frac{S}{\lambda^2} \sin\theta \, e^{-j\beta r} \quad \Big| \quad H_{\theta s} = -\frac{E_{\phi s}}{\eta}} \qquad (\eta = 120\pi)$$

$$E_{rs} = E_{\theta s} = H_{rs} = H_{\phi s} = 0$$

$$R_{rad} = \frac{2 P_{rad}}{I_o^2} \qquad \boxed{R_{rad} = \frac{320\pi^4 S^2}{\lambda^4}} \qquad \rho_o \ll \lambda \qquad r \gg \frac{\lambda}{2\pi}$$

Antenna Characteristics



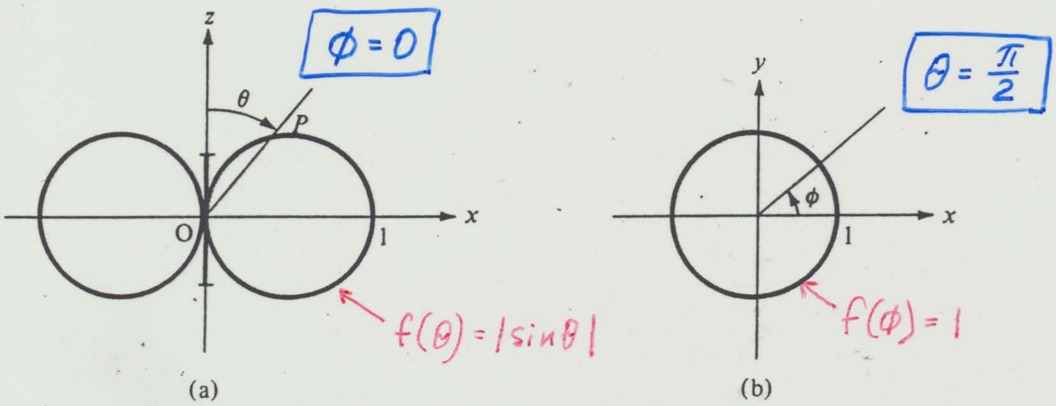
$G_d(\theta,\phi)$ is a measure of the concentration of the radiated power in a particular direction (θ,ϕ)

$$S_{ave} = \frac{G_d}{4\pi r^2} P_{rad}$$

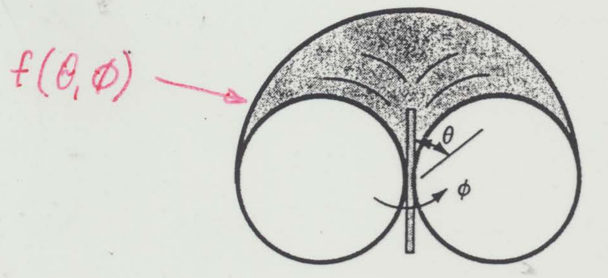
$$(P_{rad} \equiv P_t)$$

Antenna Patterns

No physical antennas radiate uniformly in all directions in space. The graph that describes the relative far-zone field strength versus direction at a fixed distance from an antenna is called the radiation pattern (field pattern) of the antenna.



(a) (b)



(c)

$$f(\theta) = \frac{|E_s|}{\max|E_s|} = |\sin\theta|$$

$$|E_s| = \frac{\eta I_o \beta dl}{4\pi r}\sin\theta$$

$$\max|E_s| = \frac{\eta I_o \beta dl}{4\pi r}$$

$$r = \text{const}$$

Field patterns of the Hertzian dipole: **(a)** normalized *E*-plane or vertical pattern (elevation) (ϕ = constant = 0), **(b)** normalized *H*-plane or horizontal pattern (azimuth) ($\theta = \pi/2$), **(c)** three-dimensional pattern.

When the square of $|E_s|$ is plotted it is called the power pattern. For the Hertzian dipole,

$$f^2(\theta) = \sin^2\theta$$

$$\left(S_{ave} = \frac{1}{2}\frac{|E_{\theta s}|^2}{\eta} \right)$$

$S_{ave}(\theta) \qquad \phi = 0 \qquad r = const$

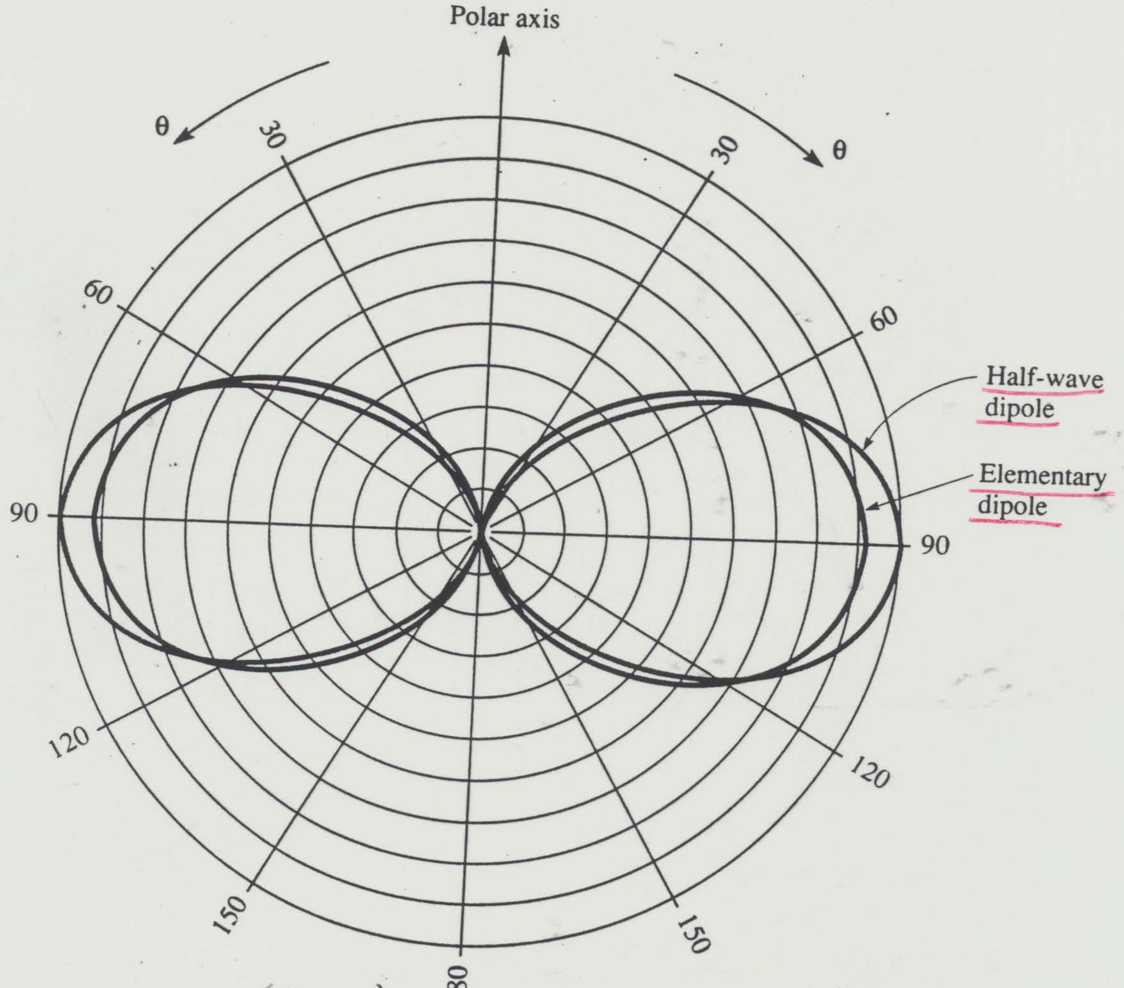


Figure 10.5 Radiation (power) pattern of the half-wave dipole. The pattern of the elementary dipole is also shown for comparison. The amplitudes of the two patterns are chosen to make the total radiated powers equal.

Hertzian dipole: $$S_{ave} = \frac{\eta I_0^2 \beta^2 dl^2}{32\pi^2 r^2} \sin^2\theta$$

Half-wave dipole: $$S_{ave} = \frac{\eta I_0^2 \cos^2\left(\frac{\pi}{2}\cos\theta\right)}{8\pi^2 r^2 \sin^2\theta}$$

Radiation intensity

The radiation intensity of an antenna is defined as

$$U(\theta, \phi) = r^2 S_{ave} \quad (\text{W/sr})$$

Then, the total average power radiated can be expressed as

$$P_{rad} = \oint_S \bar{S}_{ave} \cdot d\bar{S} = \oint_S \underbrace{S_{ave} r^2}_{U(\theta,\phi)} \overbrace{\underbrace{\sin\theta \, d\theta \, d\phi}_{d\Omega}}^{dS} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta,\phi) \, d\Omega \quad (\text{W})$$

where $d\Omega$ is the differential solid angle defined as the subtended area dS divided by r^2. A solid angle is measured in steradians (sr). The solid angle of a sphere is $\Omega = (4\pi r^2)/r^2 = 4\pi$ sr. The solid angle of a hemisphere is 2π sr.

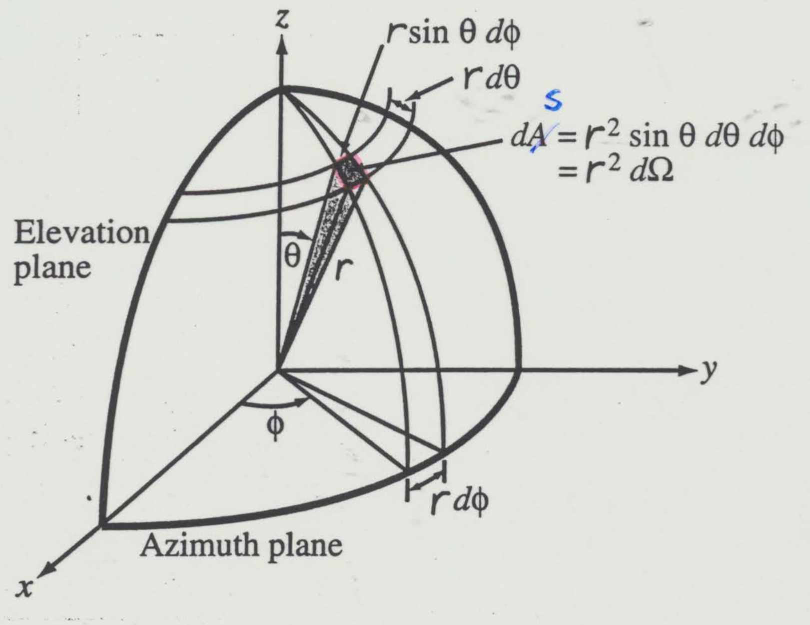


Figure 9-8: Definition of solid angle $d\Omega = \sin\theta \, d\theta \, d\phi$.

Radiation intensity is the time-average power per unit solid angle. It depends on θ and ϕ. The average value of $U(\theta, \phi)$ is the total radiated power divided by 4π sr; that is,

$$U_{ave} = \frac{P_{rad}}{4\pi}$$

Antenna Arrays

To obtain better directivity, several antennas can be combined to form an array. The individual antennas (elements of the array) are positioned and excited in such a way that their radiated fields in the desired direction add, while radiation in undesired directions tends to cancel.

$I_{1s} = I_o e^{j\alpha}$

$I_{2s} = I_o$

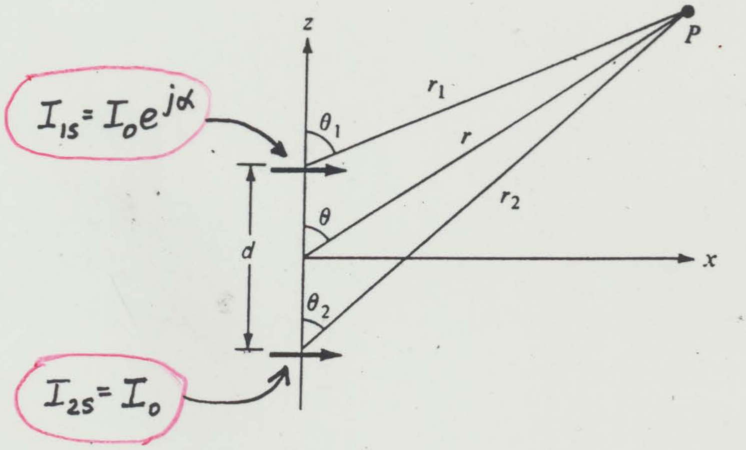


Figure 13.10 A two-element array. (Hertzian dipoles)

Far fields:

$$\bar{E}_{1s} = \frac{j\eta\beta I_o e^{j\alpha} dl}{4\pi r_1} \cos\theta_1 \, e^{-j\beta r_1} \bar{a}_{\theta_1}$$

$$\bar{E}_{2s} = j\frac{\eta\beta I_o \, dl}{4\pi r_2} \cos\theta_2 \, e^{-j\beta r_2} \bar{a}_{\theta_2}$$

$$\bar{E}_s = \bar{E}_{1s} + \bar{E}_{2s}$$

Since P is very far from the array,

$$\theta_1 \approx \theta_2 \approx \theta$$

$$\bar{a}_{\theta_1} \approx \bar{a}_{\theta_2} \approx \bar{a}_\theta$$

In the amplitude, we can set $r_1 \approx r_2 \approx r$, but in the phase, we use

$$\boxed{r_1 \approx r - \frac{d}{2}\cos\theta} \qquad \boxed{r_2 \approx r + \frac{d}{2}\cos\theta} \qquad \Big| \times (-j\beta)$$

$$\bar{E}_s = \frac{j\eta\beta I_o \, dl}{4\pi r} \cos\theta \, e^{-j\beta r} e^{j\alpha/2} \left[e^{j(\beta d\cos\theta)/2} e^{j\alpha/2} + e^{-j(\beta d\cos\theta)/2} e^{-j\alpha/2} \right] \bar{a}_\theta$$

$$= \underbrace{\frac{j\eta\beta I_o \, dl}{4\pi r} \cos\theta \, e^{-j\beta r}}_{\text{Field due to single element at origin}} \; \underbrace{e^{j\alpha/2} \, 2\cos\Big[\tfrac{1}{2}\overbrace{(\beta d\cos\theta + \alpha)}^{\psi}\Big]}_{\text{Normalized array factor (AF)}} \bar{a}_\theta$$

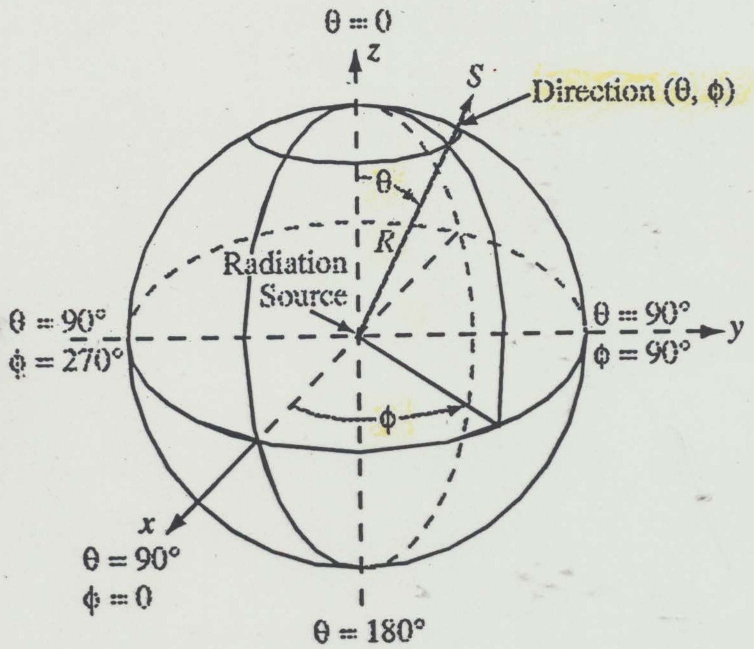
The field pattern of the array is

$$f(\theta, \phi) = \frac{|\bar{E}_s|}{\max|\bar{E}_s|} = \overbrace{|\cos\theta|}^{} \times \underbrace{\left|\cos\frac{\psi}{2}\right|}_{}$$

Field pattern of a single element (unit pattern)

Group pattern

This property is called the principle of pattern multiplication (applies only to arrays with identical elements).



θ = 0
z
S
Direction (θ, ϕ)
θ
R
Radiation Source
θ = 90°
ϕ = 270°
θ = 90°
ϕ = 90°
y
ϕ
x
θ = 90°
ϕ = 0
θ = 180°

Example. Plot the H-plane (x-y plane) field patterns of two parallel dipoles for the following two cases:

(a) $d = \lambda/2$, $\alpha = 0$

(b) $d = \lambda/4$, $\alpha = -\frac{\pi}{2}$

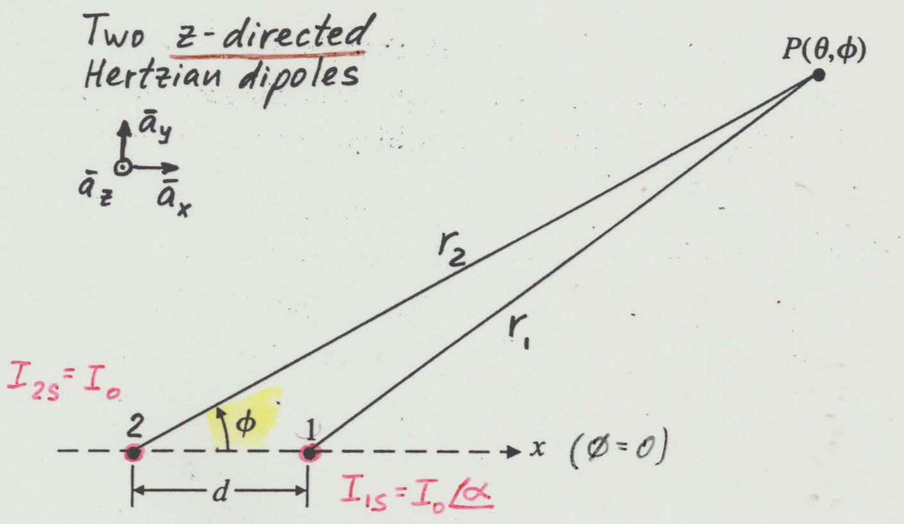


FIGURE 10-6 A two-element array.

$f(\phi) = UP \times GP$

In the H-plane ($\theta = \pi/2$) each dipole is omnidirectional, so that $UP = 1$ and $f(\phi) = GP$

$$\boxed{GP = \left|\cos\left[\tfrac{1}{2}(\beta d \cos\phi + \alpha)\right]\right|}$$

$\beta = 2\pi/\lambda$

(a) $d = \lambda/2$ $(\beta d = \pi)$, $\alpha = 0$

$$GP = \left|\cos\left(\frac{\pi}{2}\cos\phi\right)\right|$$

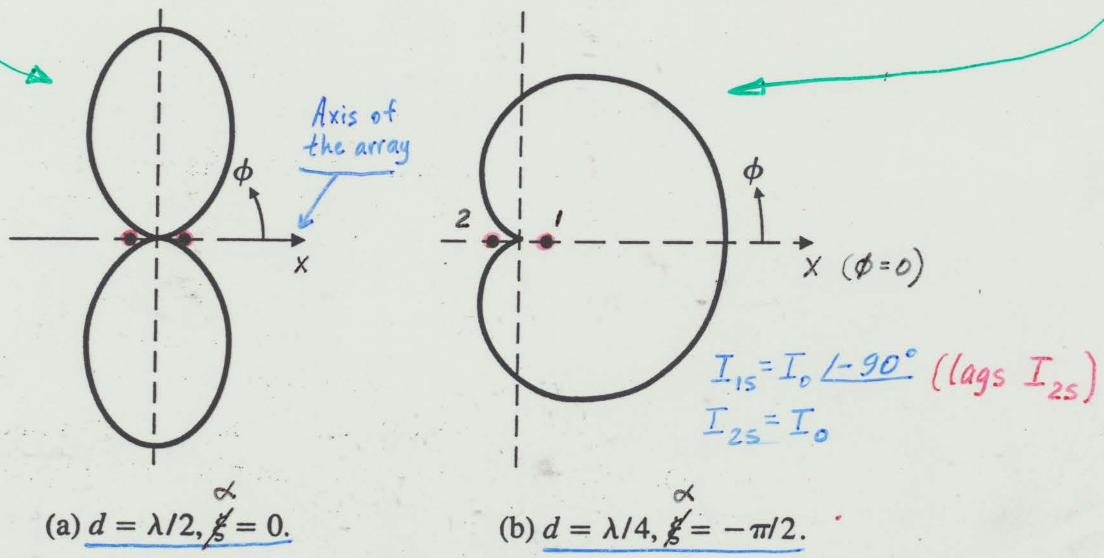
This pattern has its maximum at $\phi = \pm\pi/2$; that is, in a direction normal to the axis of the array. This is a broadside array.

(b) $d = \lambda/4$ $(\beta d = \frac{\pi}{2})$, $\alpha = -\frac{\pi}{2}$

$$GP = \left|\cos\left[\frac{\pi}{4}(\cos\phi - 1)\right]\right|$$

The pattern maximum is at $\phi = 0$; that is in a direction along the array axis. This is an endfire array.

FIGURE 10-7 *H*-plane radiation patterns of two-element parallel dipole array.



(a) $d = \lambda/2$, $\alpha = 0$. (b) $d = \lambda/4$, $\alpha = -\pi/2$.

The H-plane is perpendicular to the axes of the dipoles.

Broadside array

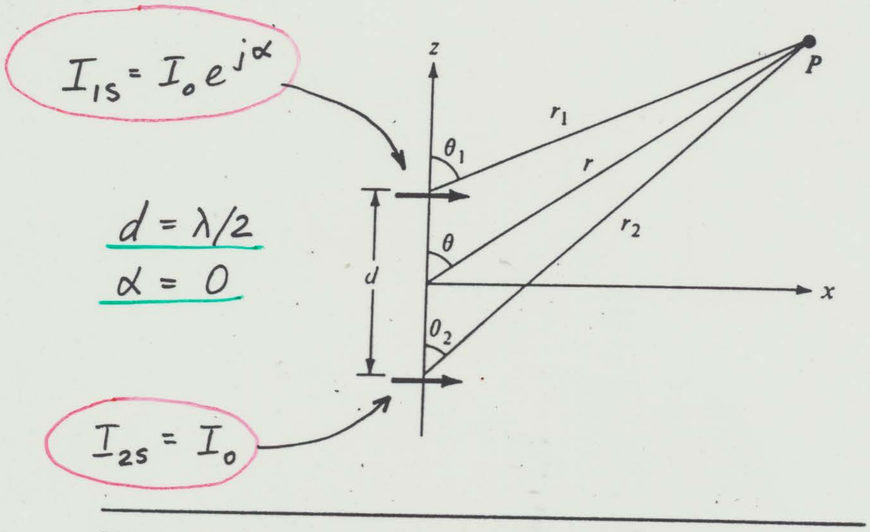
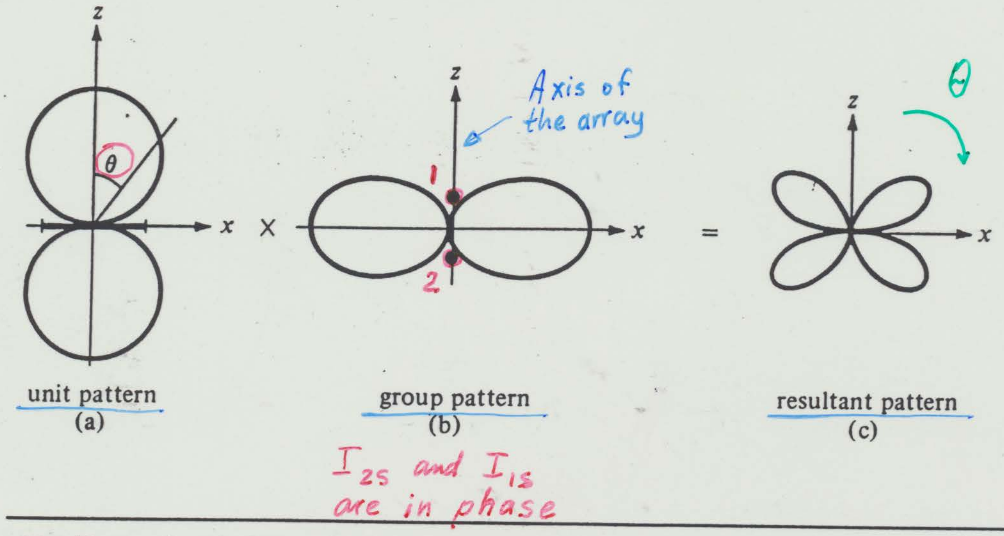


Figure 13.10 A two-element array.

The radiating elements in a broadside array are fed in phase.

E-plane field patterns



For Example 13.6(a); field patterns in the plane containing the axes of the elements. (E-plane)

Endfire array

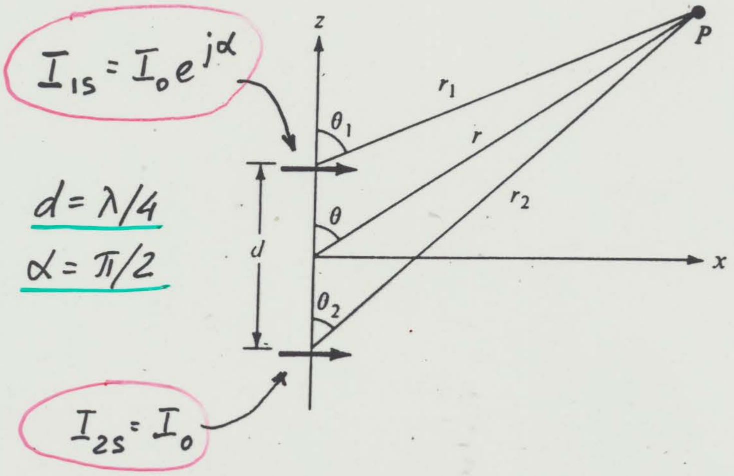
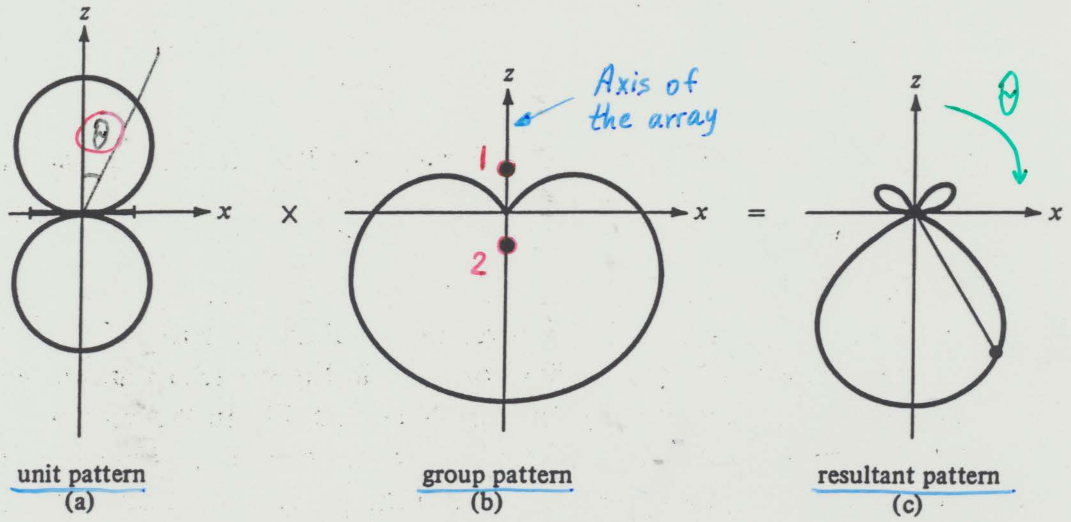


Figure 13.10 A two-element array.

The phase of a radiating element in an endfire array usually lags by an amount equal to $\beta d = (2\pi/\lambda)d$ where d is the distance that the element is displaced in the direction of maximum radiation.

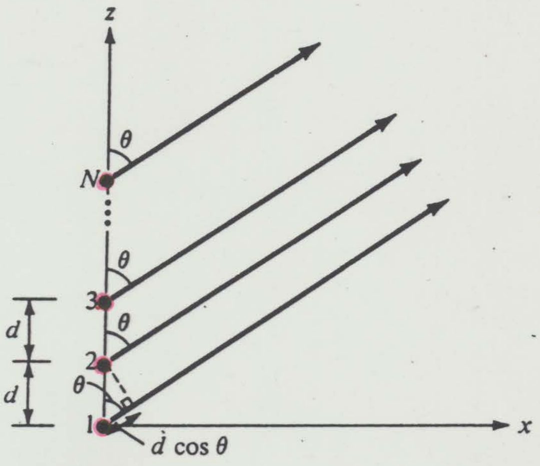
E-plane field patterns



I_{2s} lags I_{1s}

For Example 13.6(b); field patterns in the plane containing the axes of the elements. (E-plane)

N-element array



An N-element uniform linear array.

Assumptions:

1. The array is linear in that the elements are spaced equally along a straight line and lie along the z-axis.

2. The array is uniform so that each element is fed with current of the same magnitude but of progressive phase shift α,

$$I_{1s} = I_o \angle 0$$
$$I_{2s} = I_o \angle \alpha$$
$$I_{3s} = I_o \angle 2\alpha$$
$$\vdots$$
$$I_{Ns} = I_o \angle (N-1)\alpha$$

For the uniform linear array, the array factor is the sum of the contributions by all the elements, Thus,

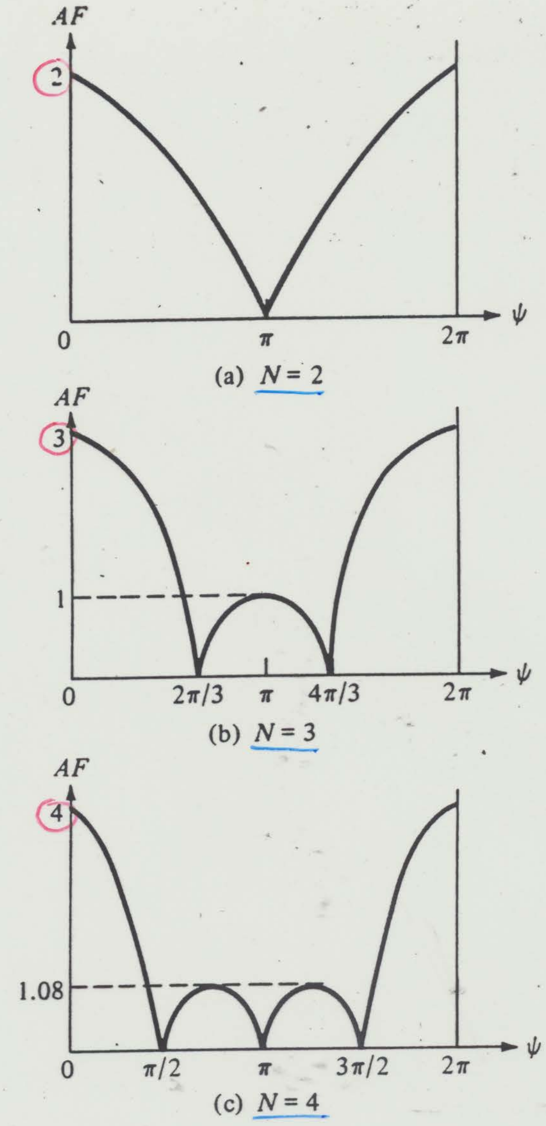
$$AF = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{j(N-1)\psi} = \frac{1 - e^{jN\psi}}{1 - e^{j\psi}}$$

where $\psi = \beta d \cos\theta + \alpha$ $\quad (\beta = 2\pi/\lambda)$

$$AF = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1} = \frac{e^{jN\psi/2}}{e^{j\psi/2}} \frac{e^{jN\psi/2} - e^{-jN\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}} = e^{j(N-1)\psi/2} \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

Dropping the phase factor $e^{j(N-1)\psi/2}$ (it would not be present if the array were centered about the origin) we get

$$\boxed{AF = \frac{\sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}}} \qquad \psi = \beta d \cos\theta + \alpha$$



$\psi = \beta d \cos\theta + \alpha$

Figure 13.12 Array factor for uniform linear array.

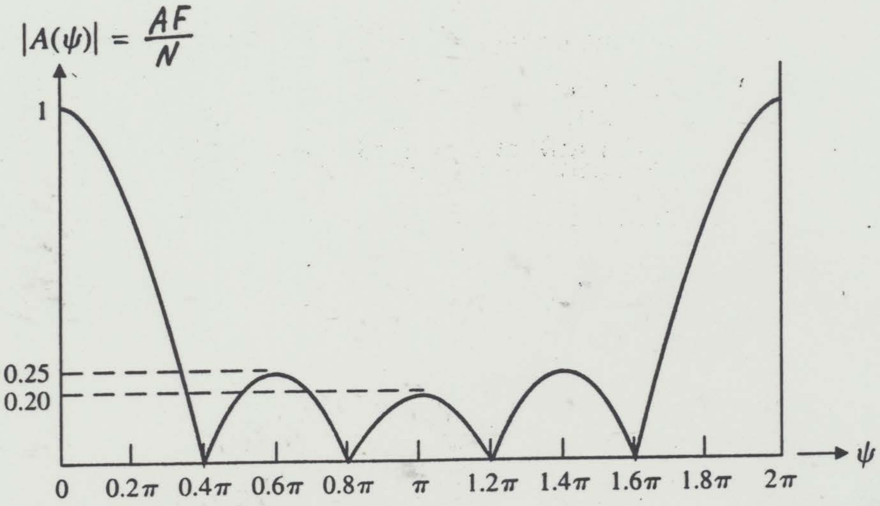
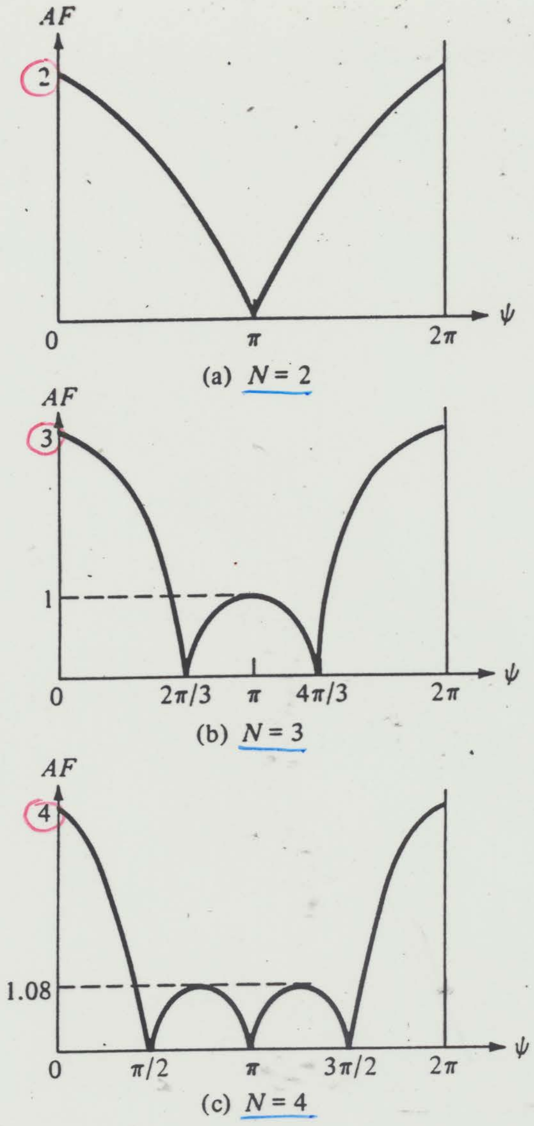


FIGURE 10-11 Normalized array factor of a five-element uniform linear array.



$\psi = \beta d \cos\theta + \alpha$

Figure 13.12 Array factor for uniform linear array.

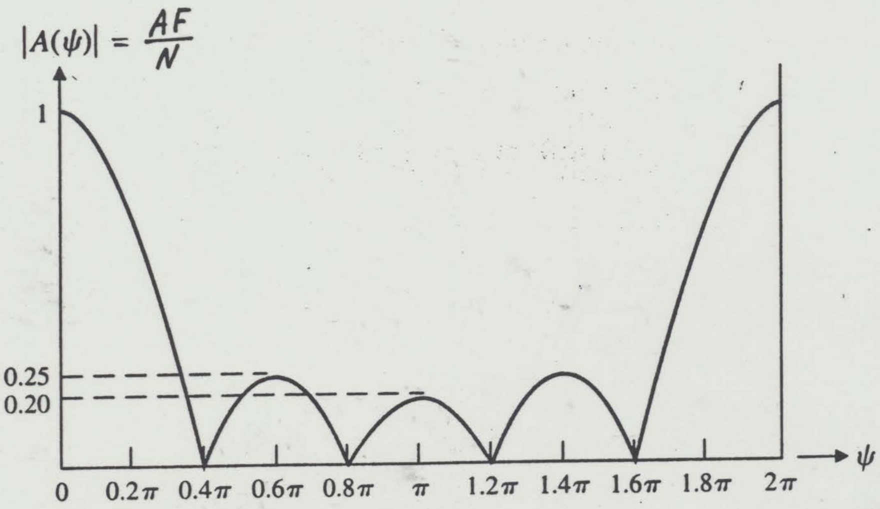


FIGURE 10-11 Normalized array factor of a five-element uniform linear array.

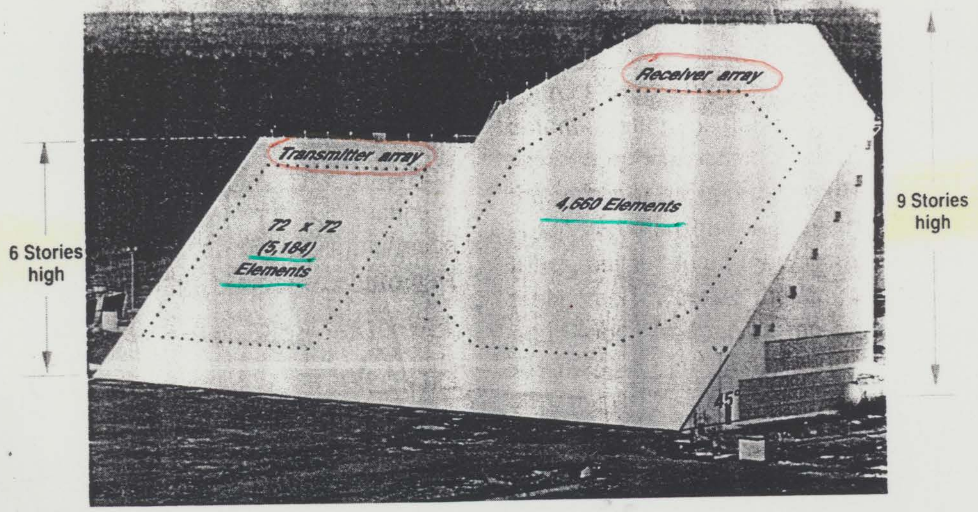


Figure 9-25: The AN/FPS-85 Phased Array Radar Facility in the Florida panhandle, near the city of Freeport. A several-mile no-fly zone surrounds the radar installation as a safety concern for electroexplosive devices, such as ejection seats and munitions, carried on military aircraft.

and to minimize it along directions of low population density or a direction corresponding to an area serviced by another station operating at the same frequency (to avoid undesirable interference effects). When two or more antennas are used together, the combination is called an ***antenna array***.

The AM broadcast antenna array is only one application example of antenna arrays; they are used extensively in numerous communication systems and radar applications as well. Antenna arrays provide the antenna designer the flexibility to obtain high directivity, narrow beams, low side lobes, steerable beams, and shaped antenna patterns. Figure 9-25 shows a photograph of a very large radar system consisting of a transmitter array composed of 5,184 individual dipole antenna elements and a receiver array composed of 4,660 elements. The radar system, part of the Space Surveillance Network operated by the U.S. Air Force, operates at 442 MHz and transmits a combined peak power of 30 MW!

Although an array need not consist of similar radiating elements, most arrays usually use identical elements, such as dipoles, slots, horn antennas, or parabolic dishes, excited by the same type of current or field distribution. The antenna elements comprising an array may be arranged in various configurations, but the most common are the linear one-dimensional configuration, wherein the elements are arranged along a straight line, and the two-dimensional lattice configuration, wherein the elements

Effective area of a receiving antenna

A receiving antenna extracts energy from an incident wave and delivers it to a load. The ability of an antenna to capture energy from an incident wave of power density S_{ave} (W/m^2) and to convert it into a received (intercepted) power P_r (W) for delivery to a matched load is characterized by the effective area (effective aperture or receiving cross section) A_e:

$$\boxed{A_e = \frac{P_r}{S_{ave}} \quad (m^2)} \qquad P_r = A_e S_{ave}$$

The Thevenin impedance of an antenna in the receiving mode is equal to the input impedance of the antenna in the transmitting mode.

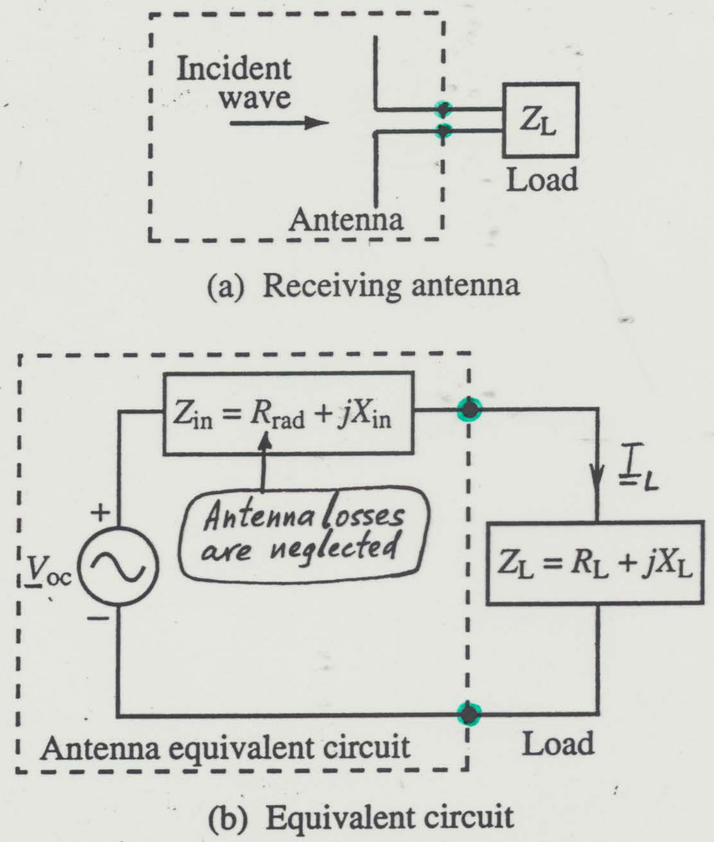


Figure 9-18: Receiving antenna represented by an equivalent circuit.

For maximum power transfer, the load impedance should be equal to the complex conjugate of the generator impedance:

$$Z_L = Z_{in}^*, \text{ then } \underline{I}_L = \underline{V}_{oc}/(2R_{rad})$$

The time-average power delivered to the matched load ($Z_L = Z_{in}^*$) is

$$P_r = \frac{1}{2}|\underline{I}_L|^2 R_{rad} = \frac{1}{2}\left[\frac{|\underline{V}_{oc}|}{2R_{rad}}\right]^2 R_{rad} = \frac{|\underline{V}_{oc}|^2}{8R_{rad}}$$

For the Hertzian dipole ($dl \ll \lambda$),

$$R_{rad} = 80\pi^2 (dl/\lambda)^2$$

and

$$V_{oc} = E\,dl$$

where E is the effective field strength parallel to the dipole axis.

$$P_r = \frac{E^2 \cancel{dl^2} \lambda^2}{8\cdot 80\pi^2 \cancel{dl^2}} = \frac{E^2\lambda^2}{640\pi^2}$$

The power density carried by the wave at the antenna (E parallel to the dipole axis) is

$$S_{ave} = \frac{E^2}{2\eta} = \frac{E^2}{240\pi}$$

Then

$$A_e = \frac{P_r}{S_{ave}} = \frac{\cancel{E^2}\lambda^2\, \cancel{240}^{\,3}\,\cancel{\pi}}{\cancel{640}_{\,8}\,\pi^{\cancel{2}}\, \cancel{E^2}} = \frac{3\lambda^2}{8\pi} = 1.5\frac{\lambda^2}{4\pi} = D\frac{\lambda^2}{4\pi}$$

where $D = 1.5$ is the directivity of the Hertzian dipole.

If the direction of the incident electromagnetic wave makes an angle θ with the dipole axis (E is not parallel to the dipole axis), the effective area will be

$$\boxed{A_e(\theta,\phi) = \frac{\lambda^2}{4\pi} G_d(\theta,\phi)}$$

($G_d(\theta,\phi) = 1.5\sin^2\theta$ for Hertzian dipole)

This latter formula is valid for any antenna.

Friis transmission formula

Antennas are in the far-field region of each other.

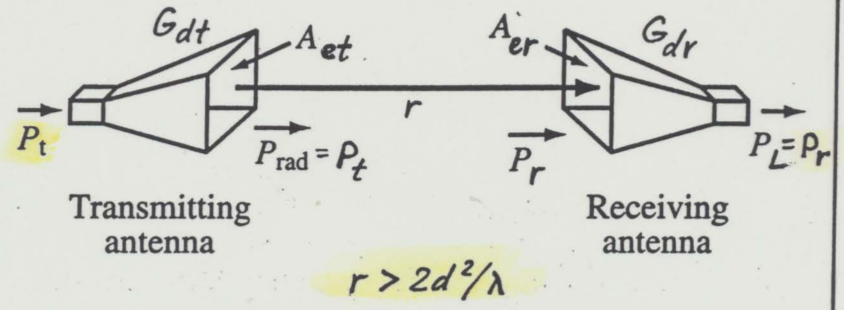


Figure 9-19: Transmitter–receiver configuration.

The power density at the receiving antenna is

$$S_{ave} = \frac{P_t}{4\pi r^2} G_{dt}$$

where G_{dt} is the directive gain of the transmitting antenna (in the direction of the receiving antenna)

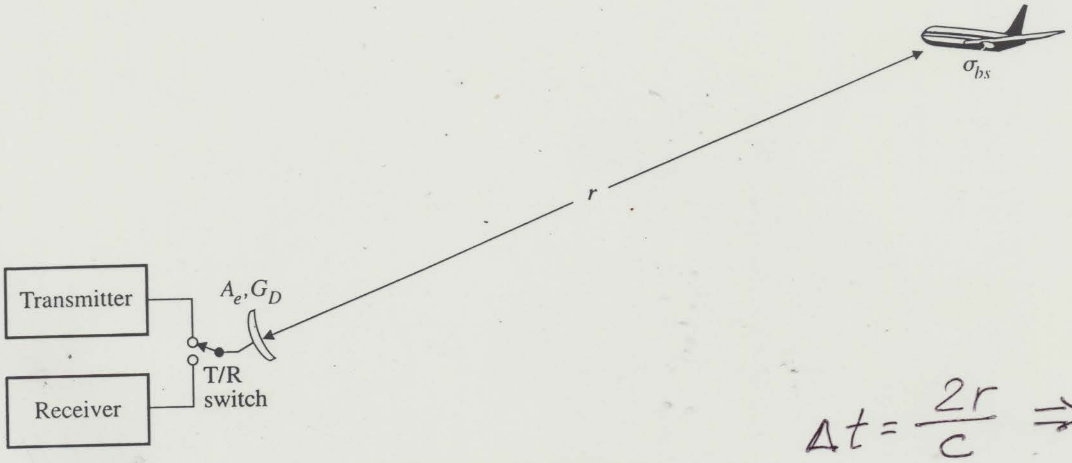
Power received by the receiving antenna is

$$P_r = A_{er} S_{ave} = \frac{\lambda^2}{4\pi} G_{dr} S_{ave}$$

Substituting equation for S_{ave} we get

$$\boxed{P_r = G_{dr} G_{dt} \left[\frac{\lambda}{4\pi r}\right]^2 P_t}$$

This is referred to as the Friis formula which characterizes the coupling between two antennas in terms of their directive gains, separation distance, and frequency of operation.



$\Delta t = \frac{2r}{c} \Rightarrow r$

FIGURE 10-13 A monostatic radar system.

Now consider a radar system that uses the same antenna for transmitting short pulses of time-harmonic radiation and for receiving the energy scattered back from a target, as depicted in Fig. 10-13.† For a transmitted power P_t the power density at a target at a distance r away is (see Eq. 10-77)

$$\mathscr{P}_{av} = \frac{P_t}{4\pi r^2} G_D(\theta, \phi), \tag{10-81}$$

where $G_D(\theta, \phi)$ is the directive gain of the antenna in the direction of the target. If σ_{bs} denotes the backscatter or radar cross section of the target, then the equivalent power that is scattered isotropically is $\sigma_{bs}\mathscr{P}_{av}$, which results in a power density at the antenna $\sigma_{bs}\mathscr{P}_{av}/4\pi r^2$. Let A_e be the effective area of the antenna. We have the following expression for the received power:

$$\begin{aligned} P_L &= A_e \sigma_{bs} \frac{\mathscr{P}_{av}}{4\pi r^2} \\ &= A_e \sigma_{bs} \frac{P_t}{(4\pi r^2)^2} G_D(\theta, \phi). \end{aligned} \tag{10-82}$$

By using Eq. (10-75), Eq. (10-82) becomes

Radar equation

$$\boxed{\frac{P_L}{P_t} = \frac{\sigma_{bs}\lambda^2}{(4\pi)^3 r^4} G_D^2(\theta, \phi),} \tag{10-83}$$

which is called the ***radar equation***. In terms of the antenna effective area A_e instead of the directive gain $G_D(\theta, \phi)$, the radar equation can be written as

† A radar system employing a common antenna for transmitting and receiving at the same site and using a T/R (xmt/rcv) switch is called a *monostatic radar*.