DC Machines

Revised October 6, 2008

DC Machines:

DC Motors are rapidly losing popularity.

Until recent advances in power electronics DC motors excelled in terms of speed control.

Today, induction motors with solid-state control are replacing DC motors.

DC motors are still popular in cars.

DC Generators are essentially non existent.

DC motors are the simplest motor to understand.

DC Machines :

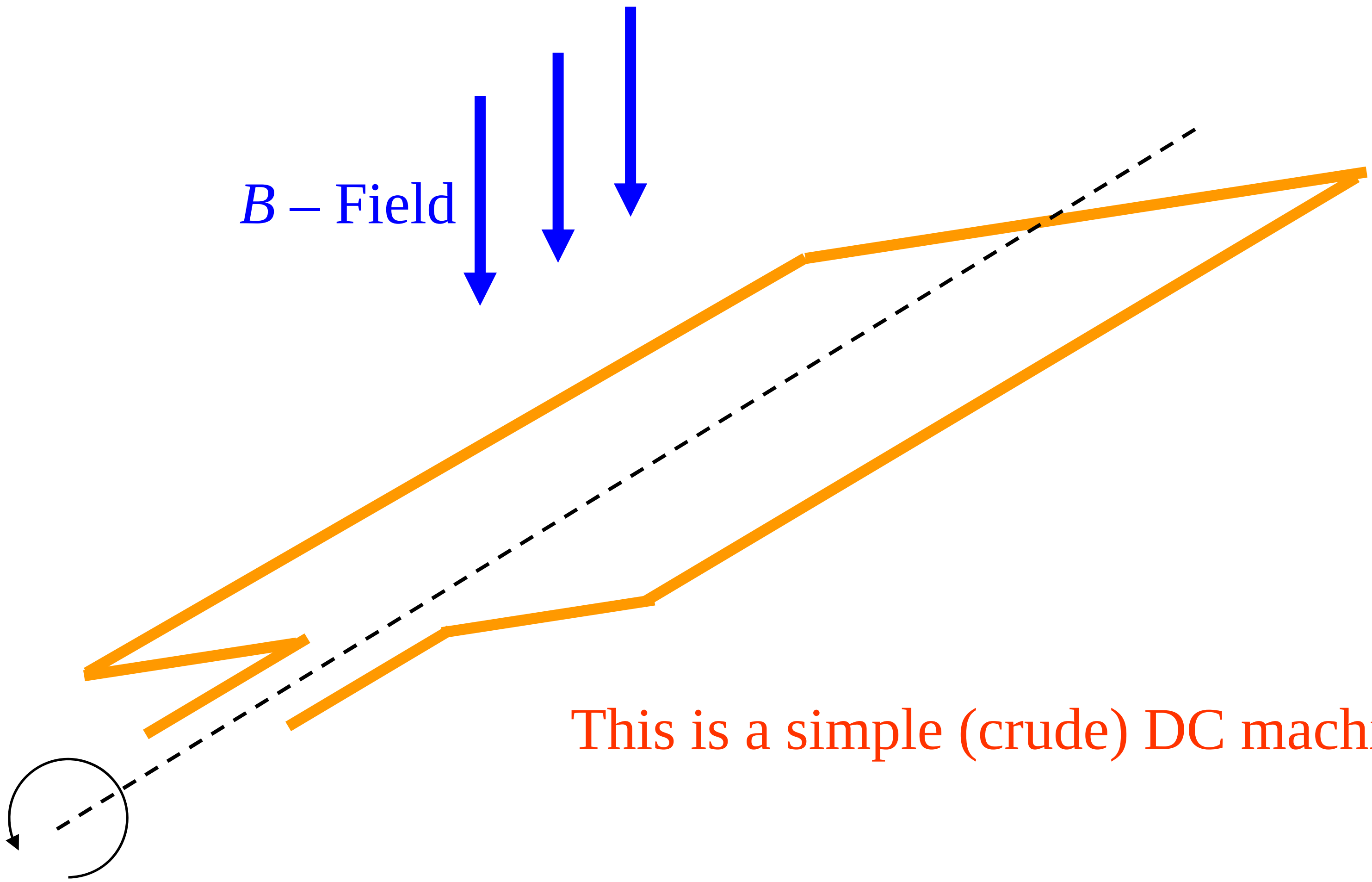
DC machines are like AC machines in that they have AC voltages and currents within them.

DC machines have a DC output only because they have a means of converting the AC into DC.

This mechanism is called a *commutator*, and DC machines are also known as *commutating machinery*.

DC Machines :

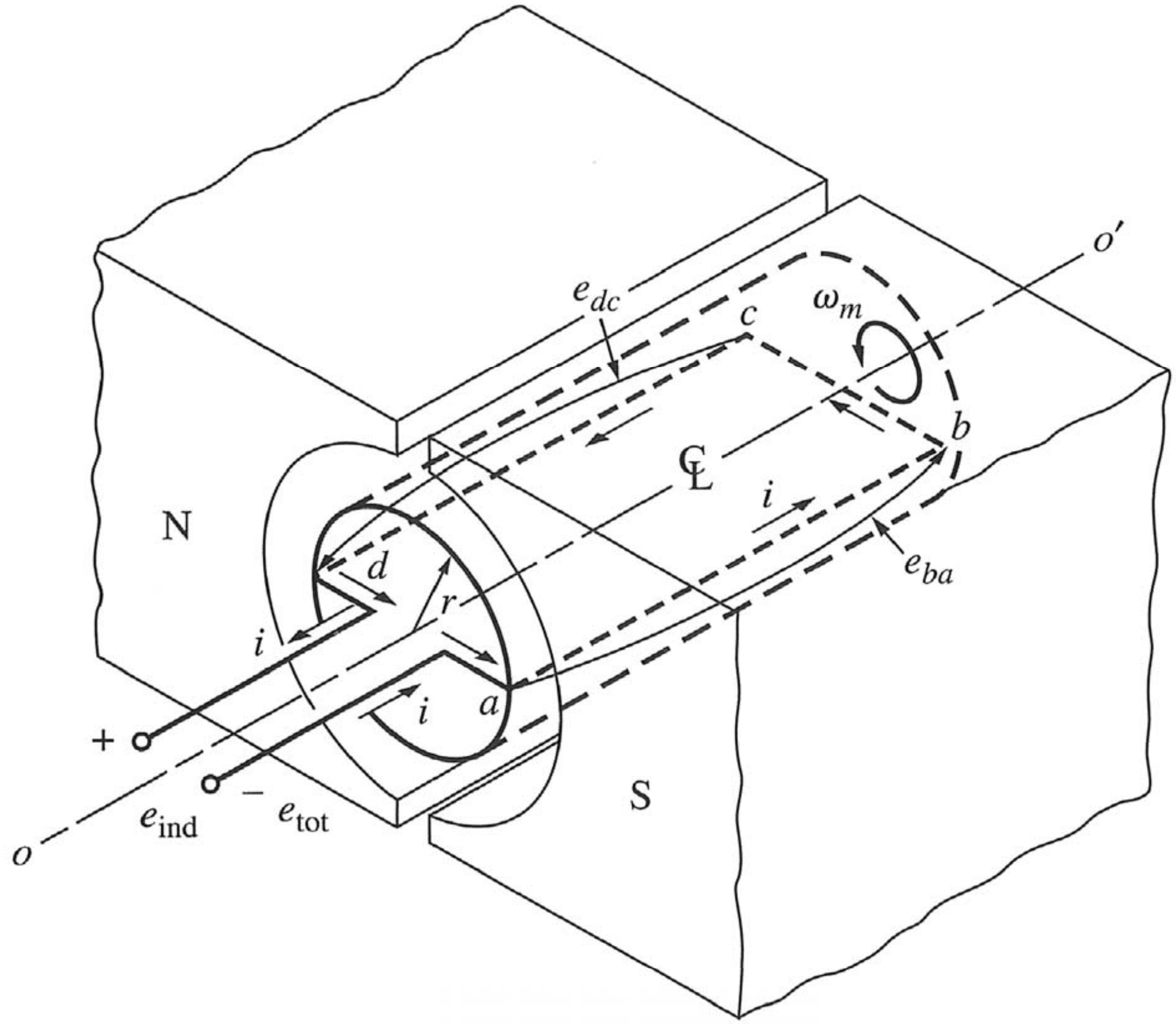
Remember this picture? (Slide 66, Note Set 1)



This is a simple (crude) DC machine.

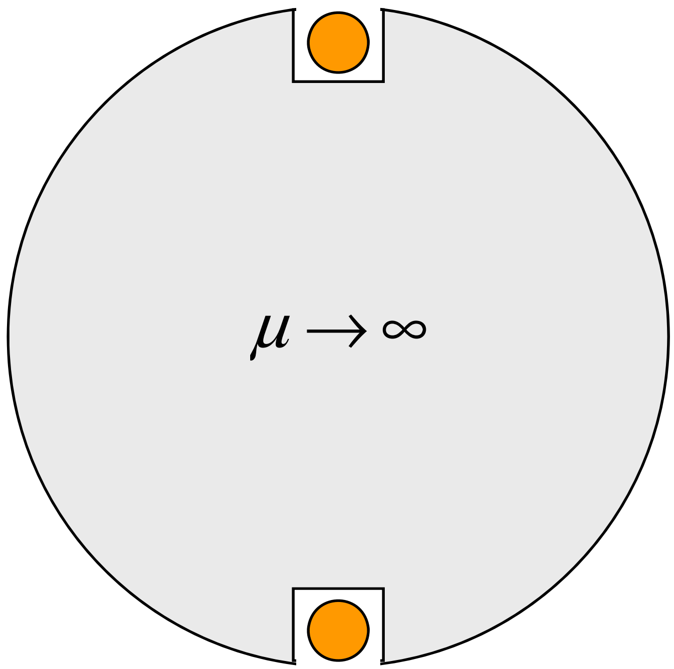
DC Machines :

A better picture:



DC Machines :

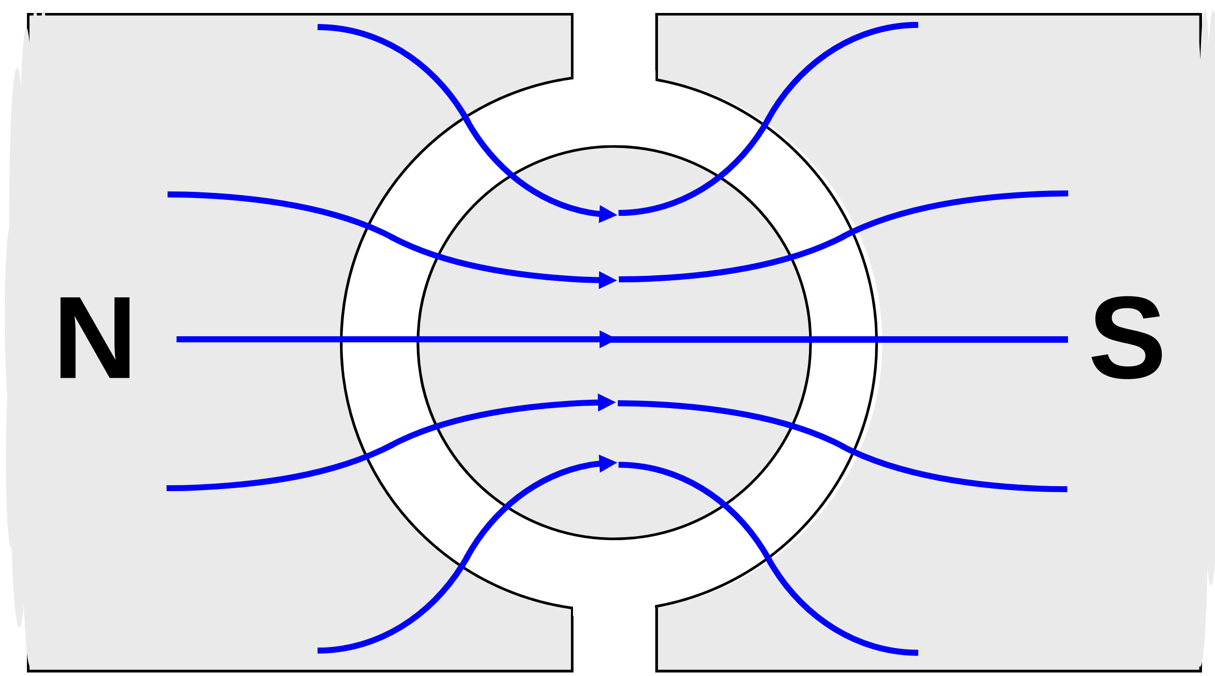
Fundamental Rotor Construction: Windings lies in carved slots in a ferromagnetic core.



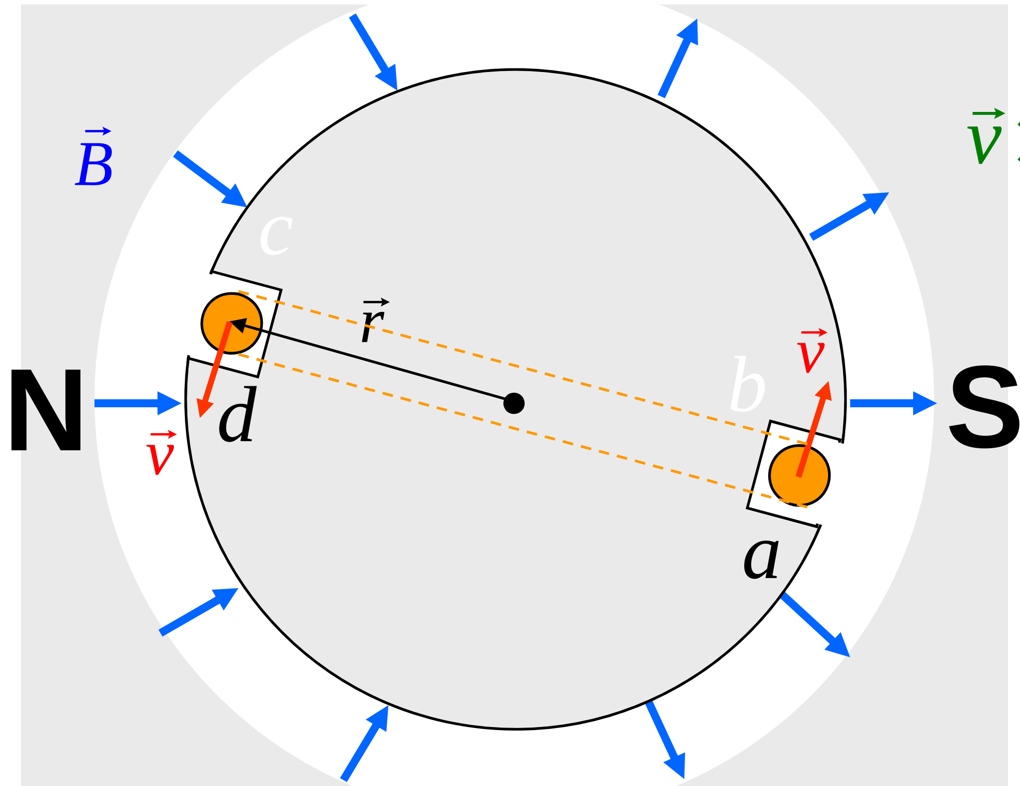
Note that for the DC generator, the rotor is the armature and the stator provides the "field" magnetic field.

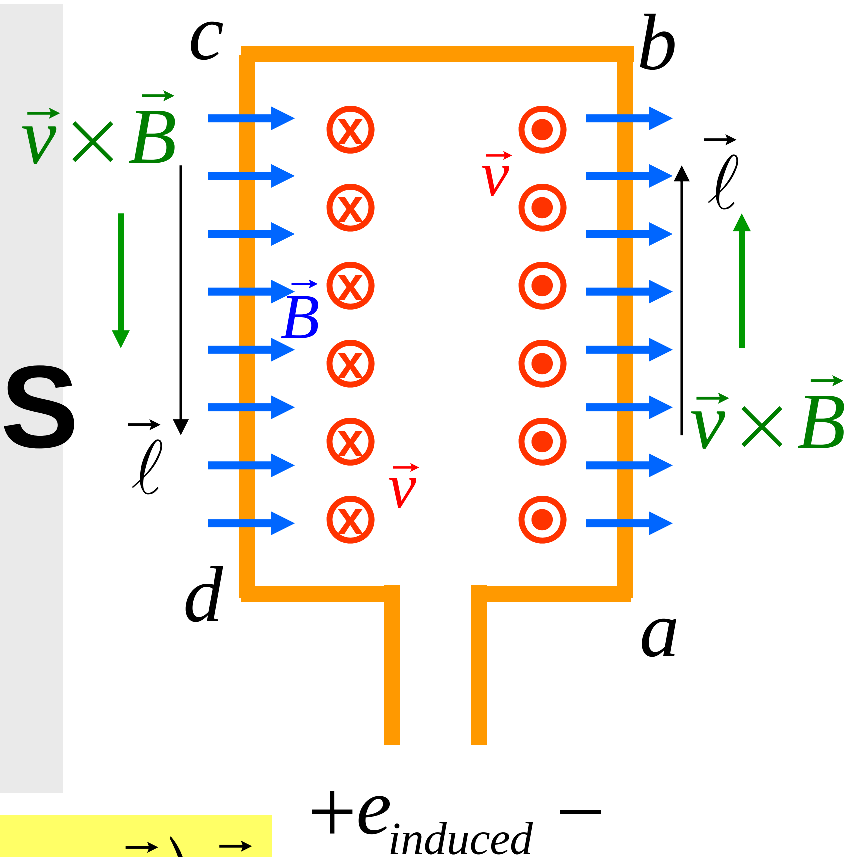
DC Motors:

Note the constant width air gap between rotor and stator. The magnetic flux takes the shortest path between rotor and stator and so is everywhere perpendicular between the two surfaces. *The magnetic flux density is constant everywhere under the pole faces.*



DC Machines : Induced Voltage in a Rotating Loop – We've done this before at the beginning of the course





$$e_{induced} = \left(\vec{v}\times\vec{B}\right)\bullet\vec{\ell}$$

DC Machines : Induced Voltage in a Rotating Loop

Segment *ab*:

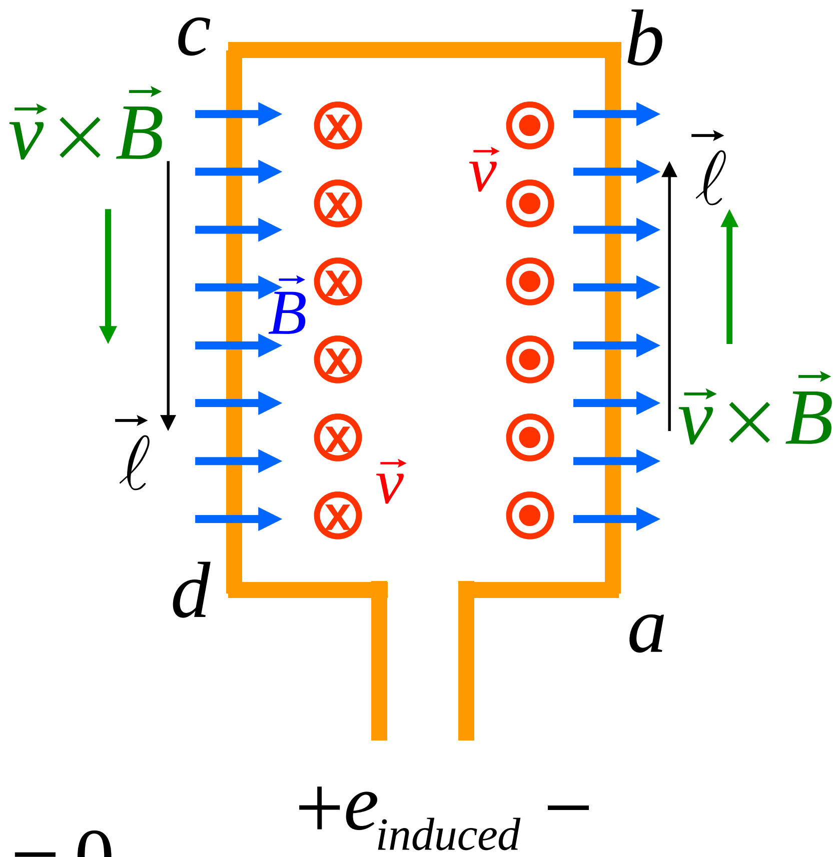
$$e_{ba} = \left(\vec{v} \times \vec{B}\right) \bullet \vec{\ell}$$

$$= vB\ell \quad \uparrow$$

Segment *cd*:

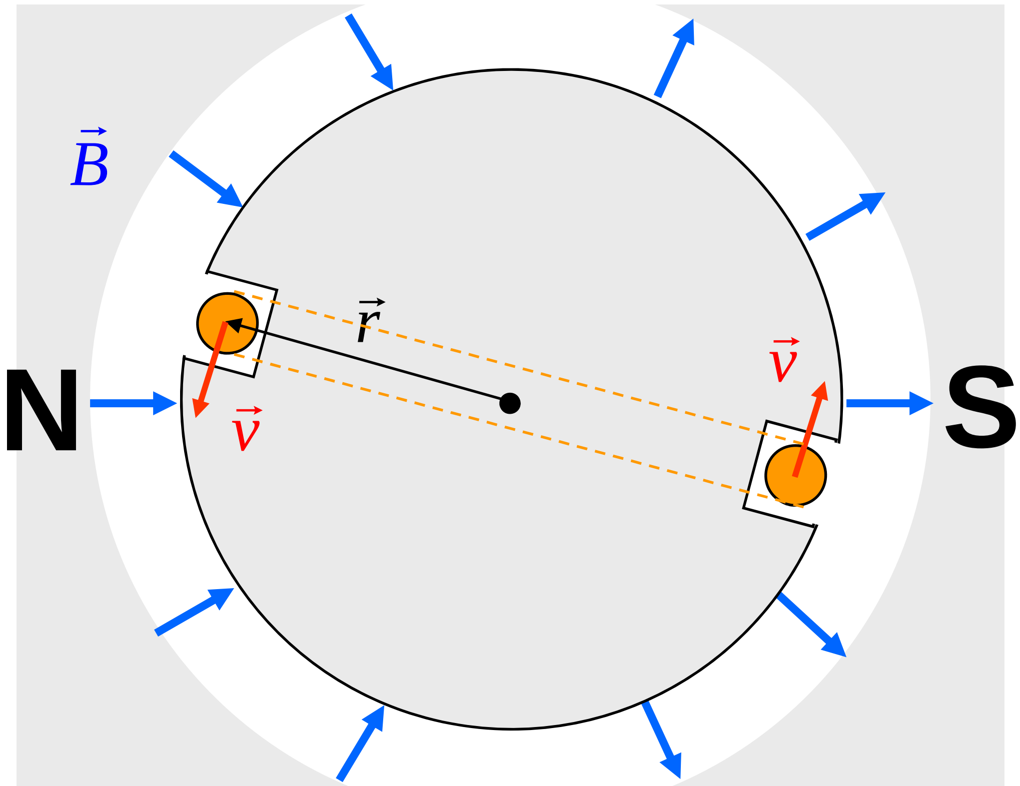
$$e_{dc} = \left(\vec{v} \times \vec{B}\right) \bullet \vec{\ell}$$

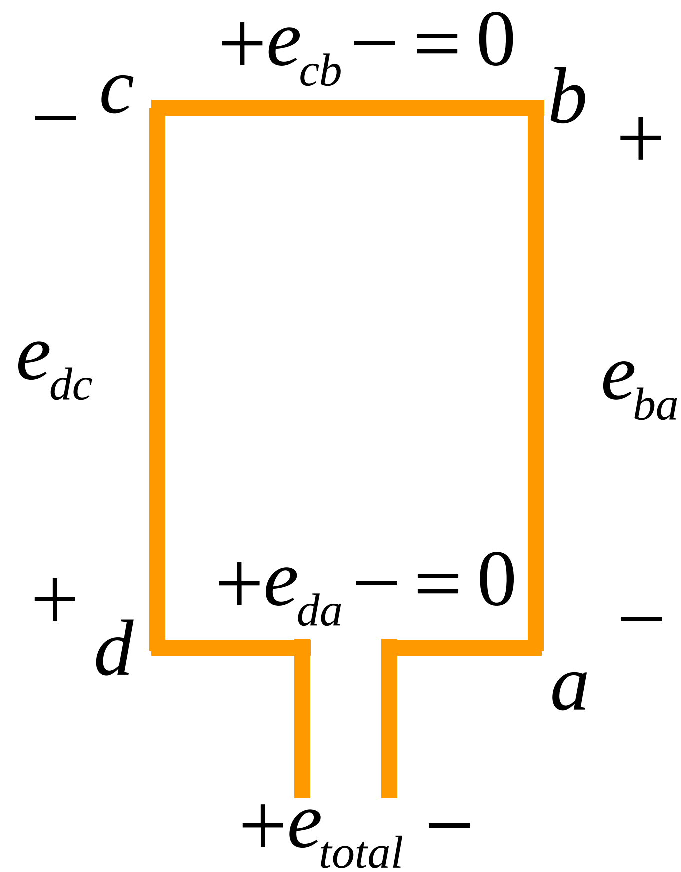
$$= vB\ell \quad \downarrow$$



$$e_{bc} = e_{da} = 0$$

DC Machines : Induced Voltage in a Rotating Loop – We've done this before at the beginning of the course



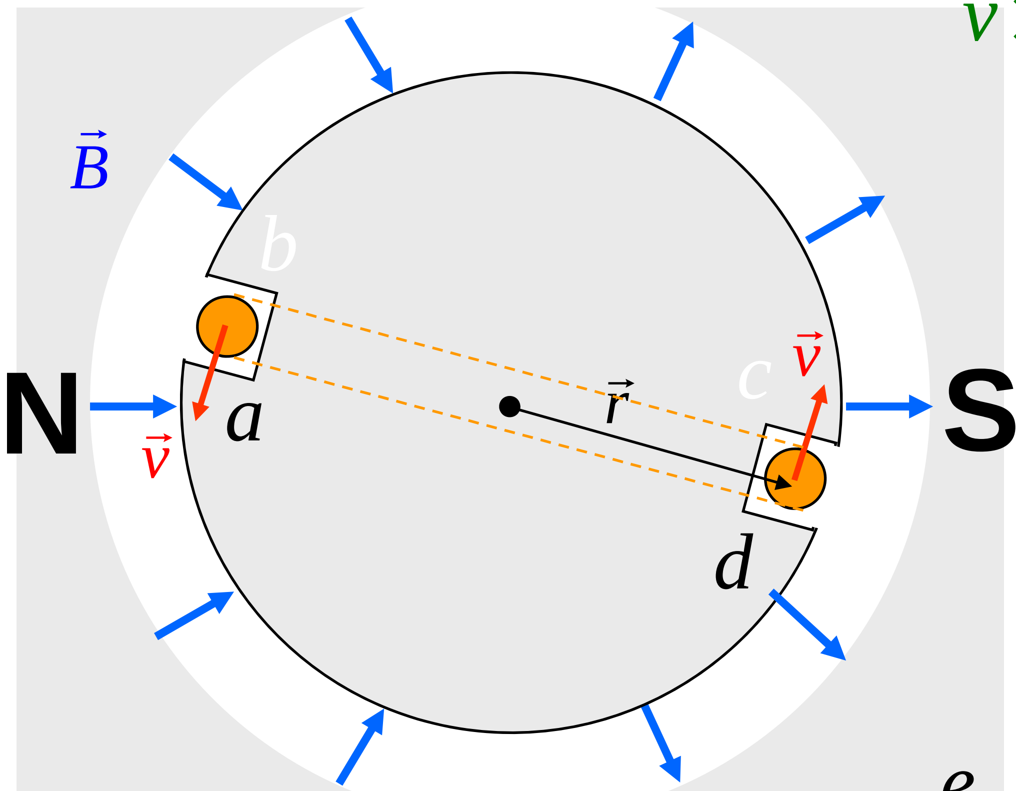


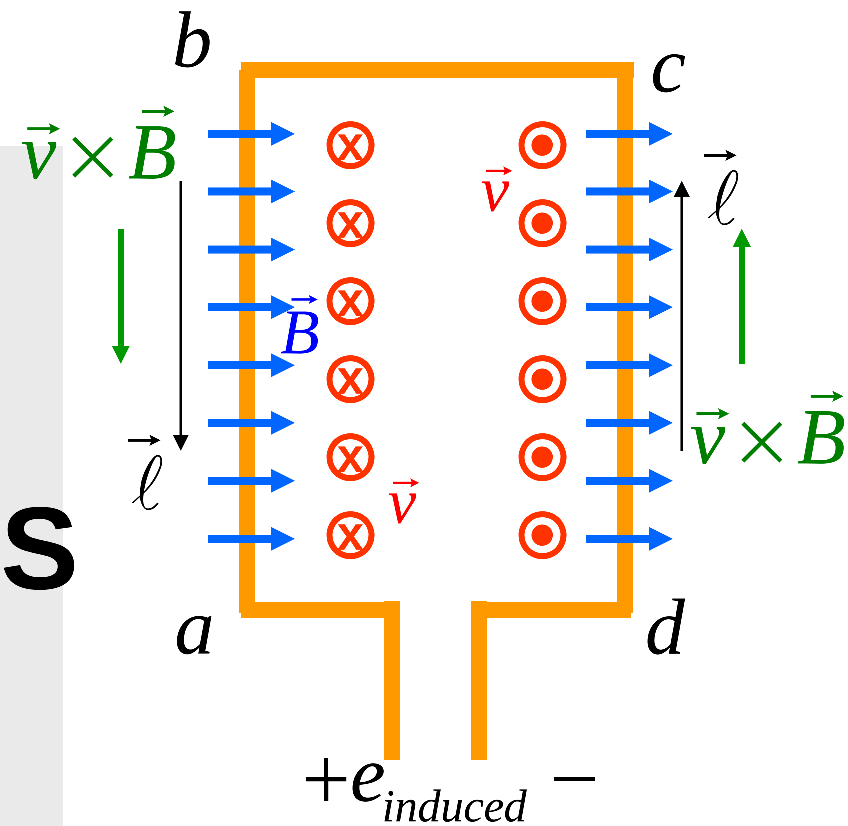
$$= e_{ba} + e_{cb} + e_{cd} + e_{da}$$

$$= 2vB\ell$$

DC Machines : Induced Voltage in a Rotating Loop –

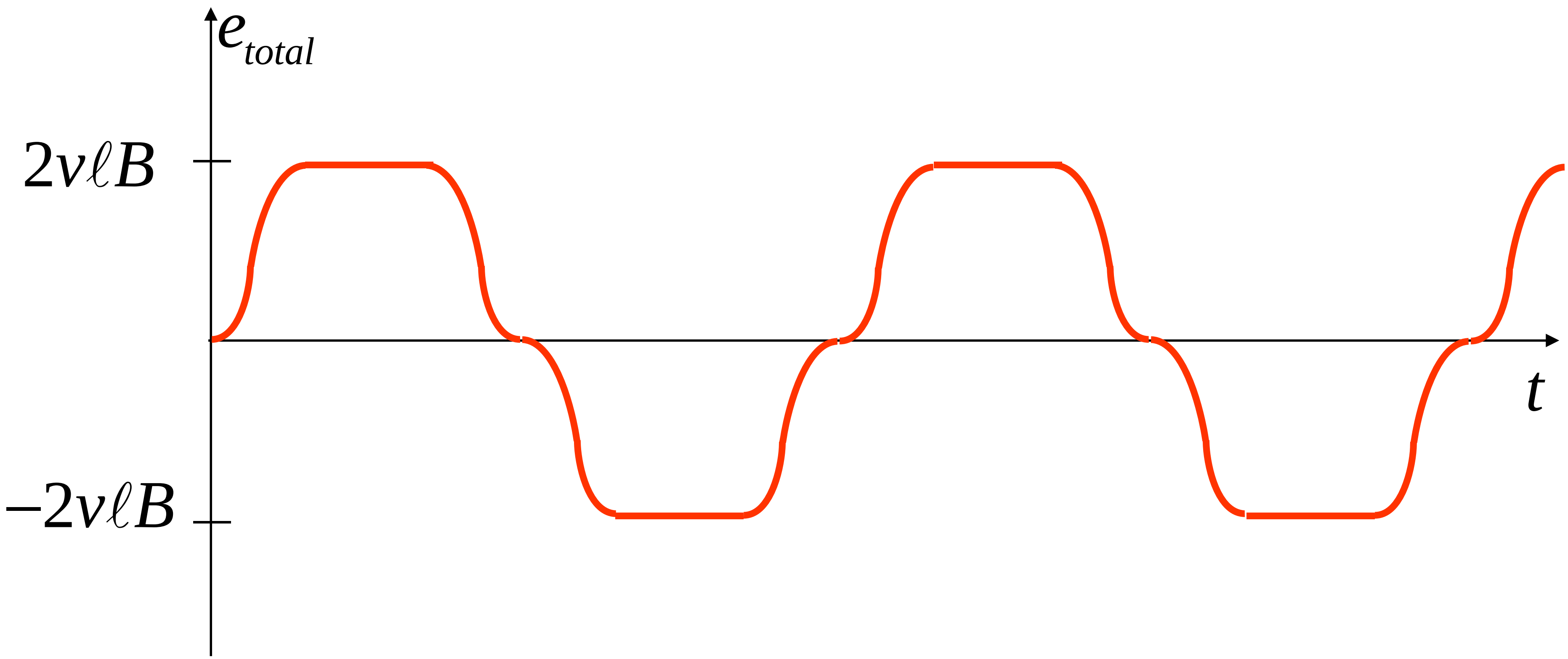
When the loop rotates by 180°:





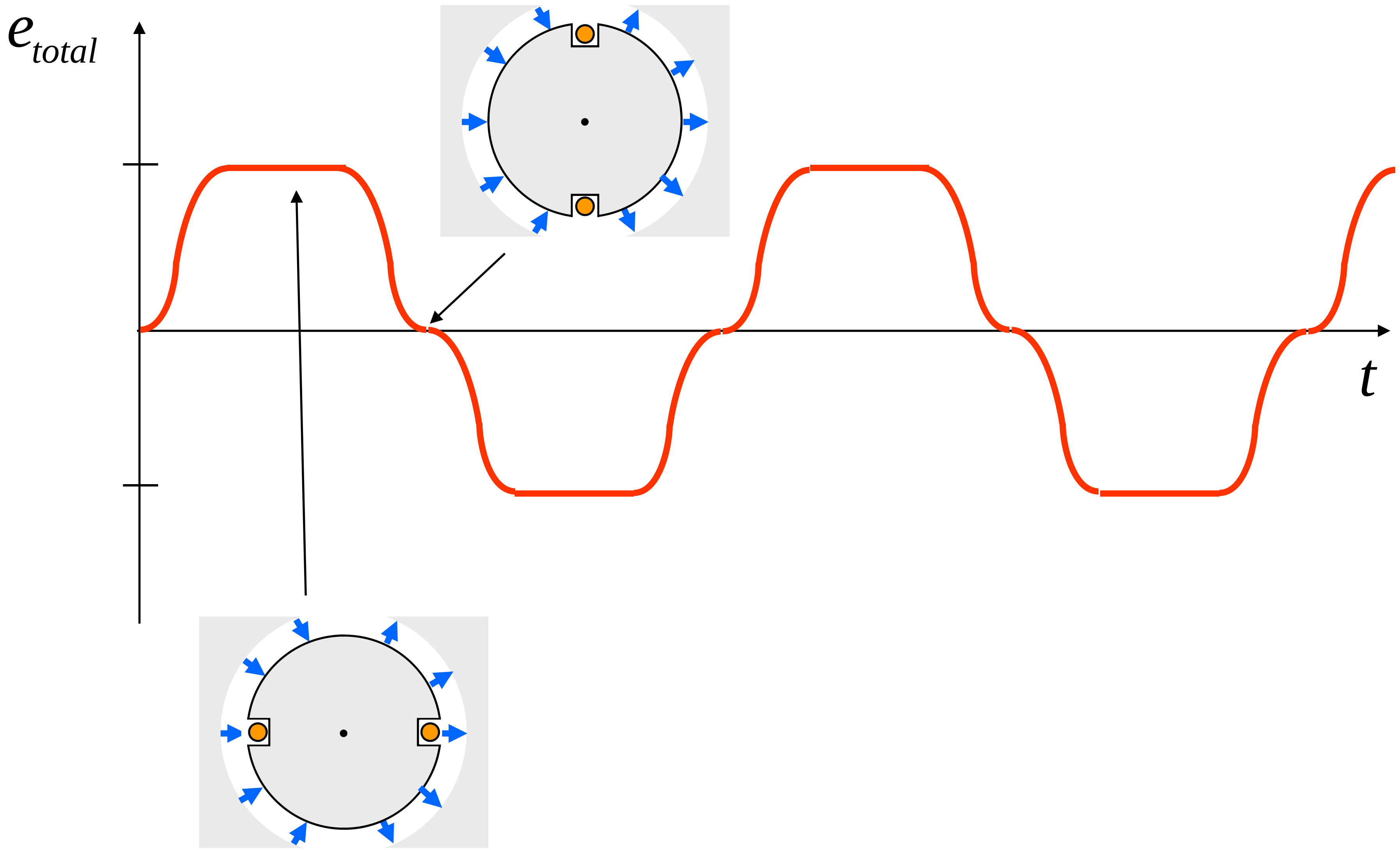
$$\left.\begin{aligned} e_{ab} > 0 \Rightarrow e_{ab} = -e_{ba} \\ e_{cd} > 0 \Rightarrow e_{cd} = -e_{dc} \end{aligned}\right\} \begin{aligned} &signs \\ &flip \end{aligned}$$

DC Machines : Induced Voltage in a Rotating Loop



Clearly an AC voltage.

DC Machines : Induced Voltage in a Rotating Loop



DC Motors: Induced Voltage in a Rotating Loop

Alternate expressions:

Rotor Surface Area

$$e_{total} = 2vB\ell = 2(r\omega)B\ell$$

$$= (2r\ell)\omega B = (2\pi r\ell)\frac{\omega B}{\pi}$$

$$= \frac{2}{\pi}A_P\omega B$$

Rotor Surface Area-per Pole

$$A_P = \pi r\ell$$

DC Machines : Induced Voltage in a Rotating Loop

Alternate expressions:

$$e_{total} = \frac{2}{\pi}\omega A_P B$$

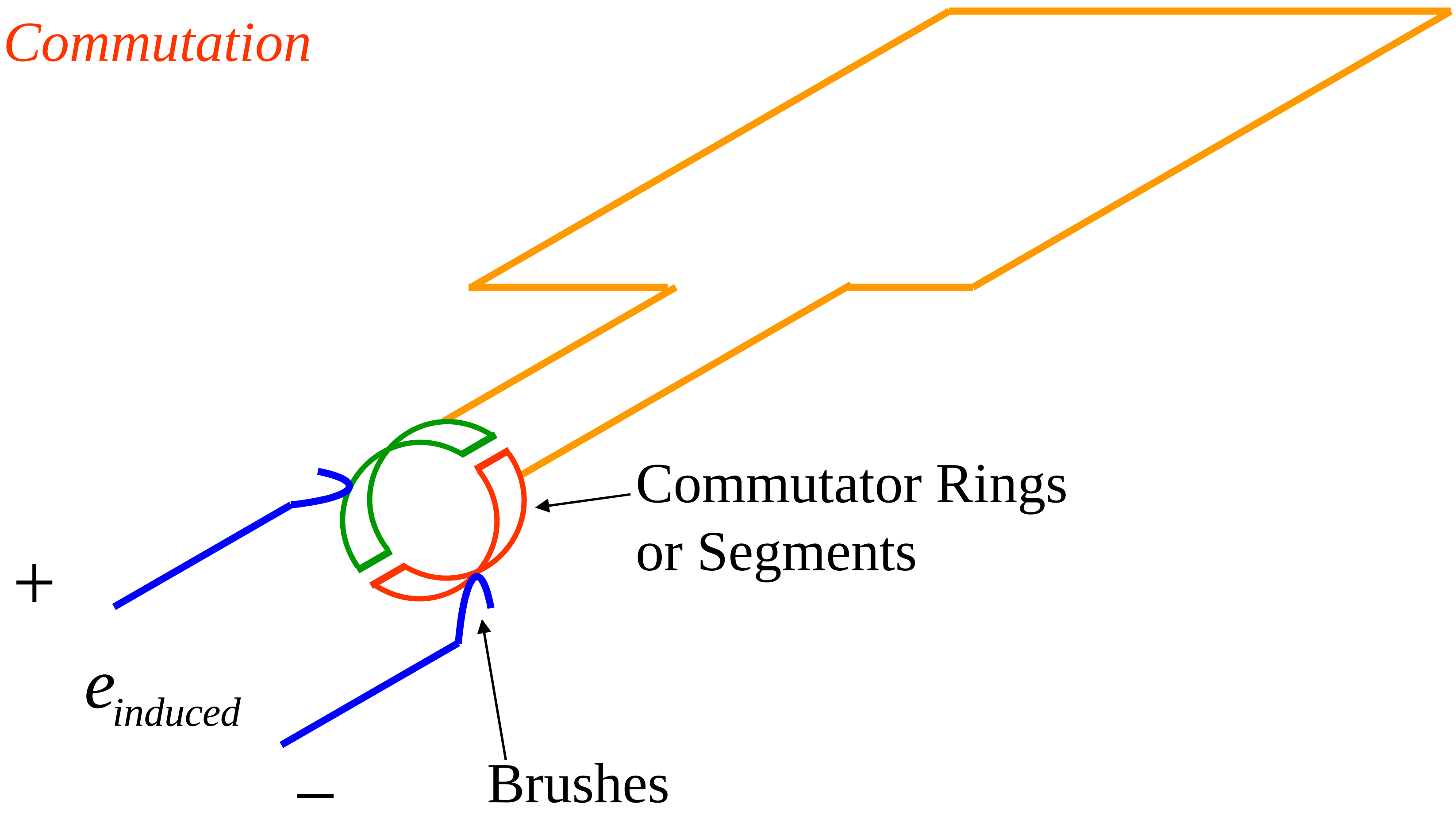
$$= \frac{2}{\pi}\omega\phi$$

The voltage generated in the machine is equal to the product of the flux in the gap time the speed of rotation of the machine.

How do we get a DC voltage out of the loop?

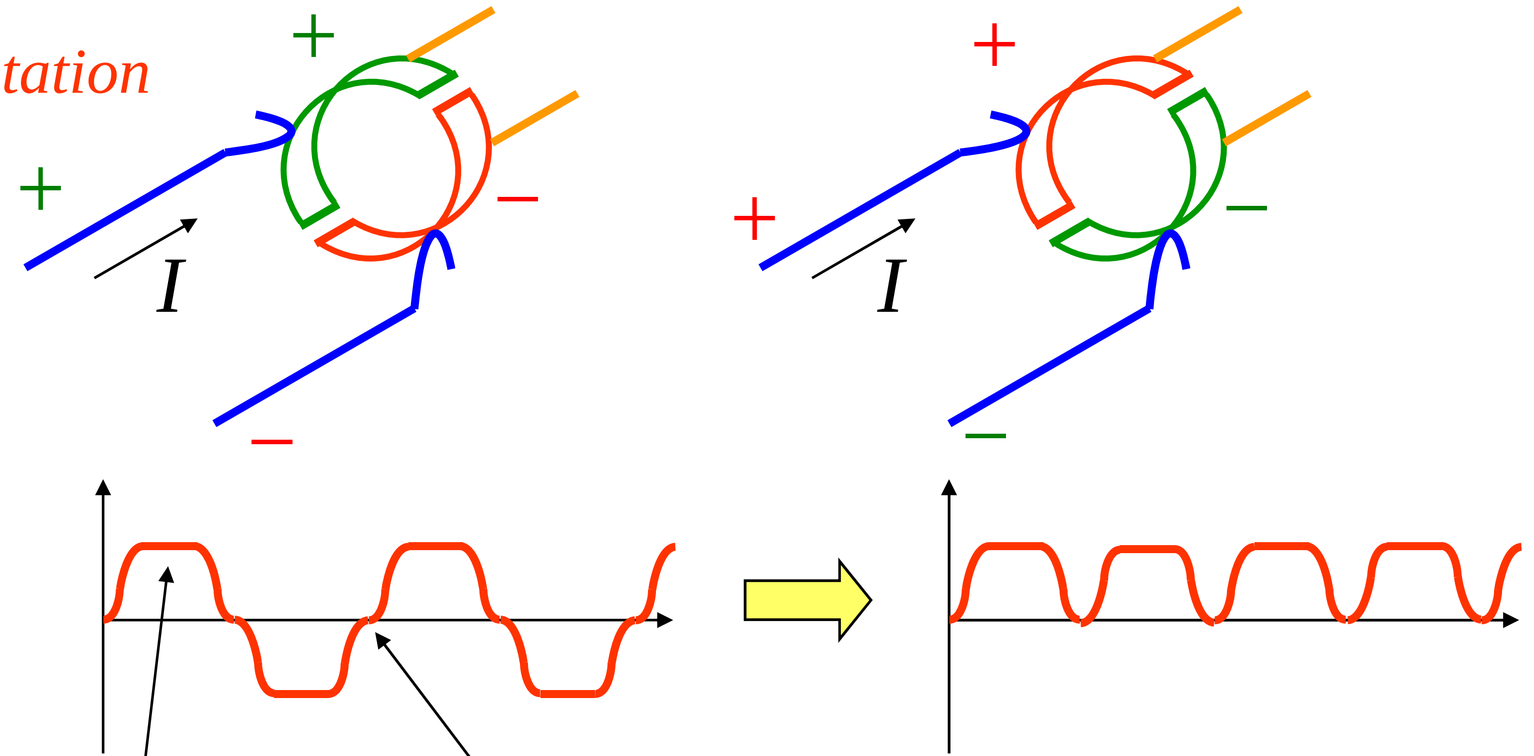
DC Machines : Getting DC voltage out of the loop –

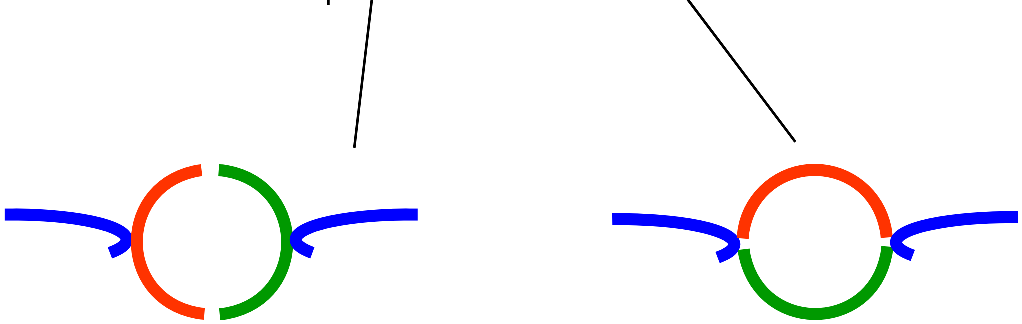
Commutation



DC Machines : Getting DC voltage out of the loop –

Commutation

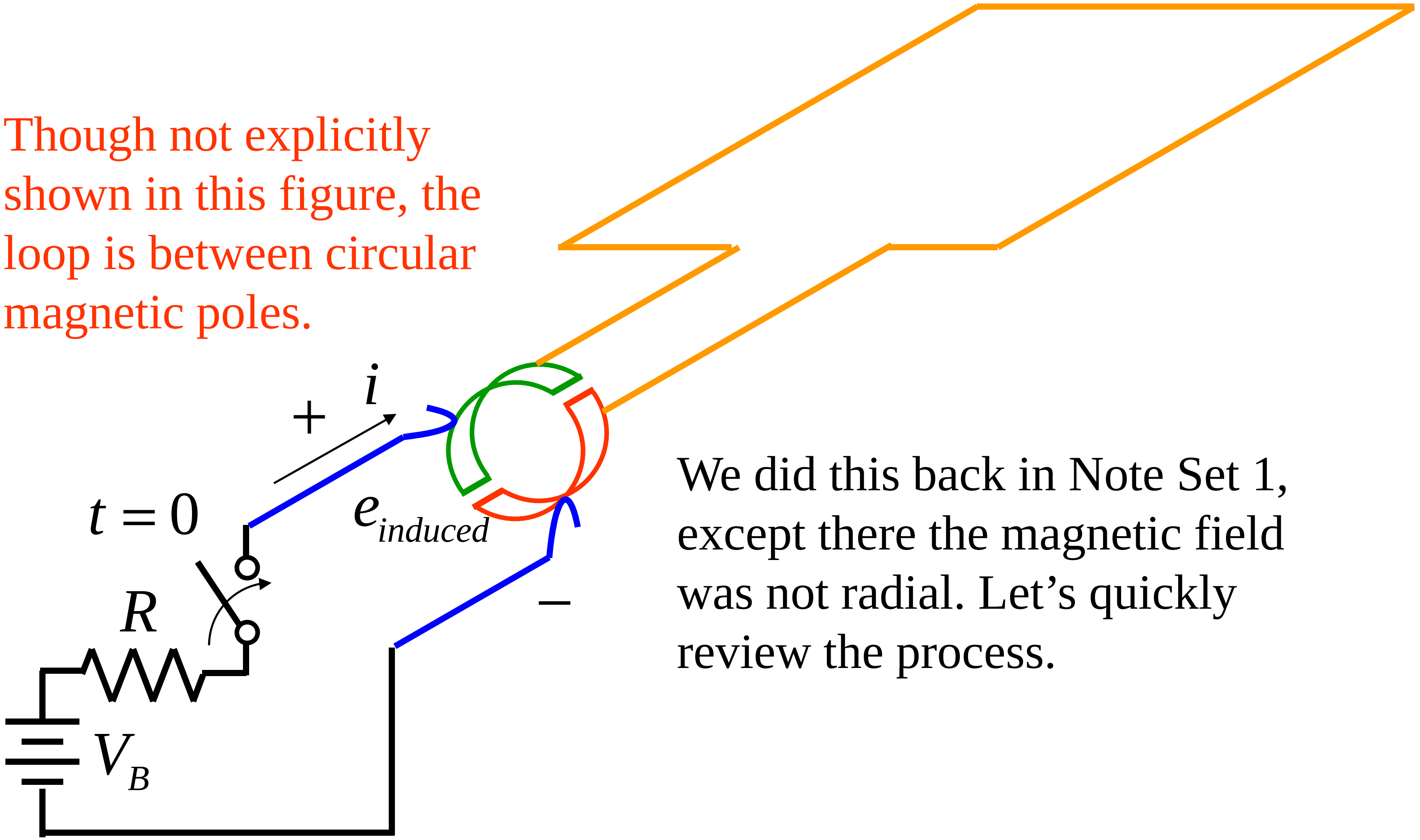




The current in the loop is AC and reverses direction every half cycle, but so do the contacts, so the current I in the brushes is DC.

DC Machines : Induced Torque in a Rotating Loop

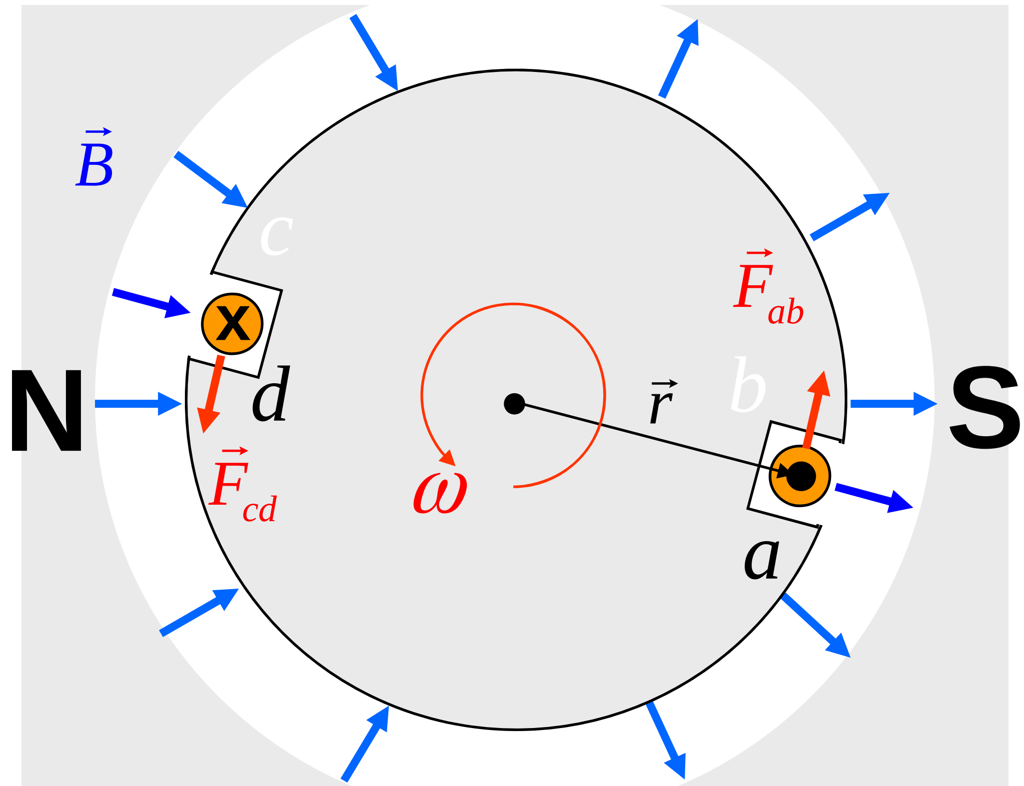
Though not explicitly shown in this figure, the loop is between circular magnetic poles.



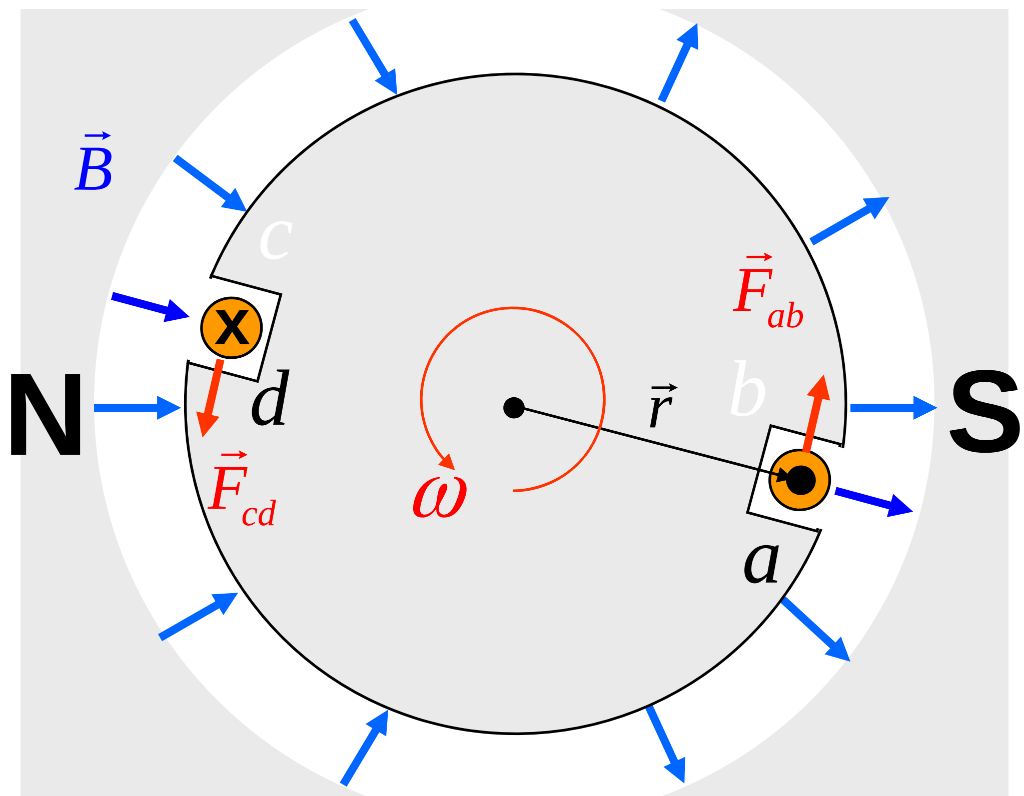
We did this back in Note Set 1, except there the magnetic field was not radial. Let's quickly review the process.

DC Machines : Induced Torque in a Rotating Loop

$$\vec{F} = i\left(\vec{\ell} \times \vec{B}\right)$$



DC Machines : Induced Torque in a Rotating Loop



Segments bc & da

$$\vec{\ell} \,||\, \vec{B} \Rightarrow \vec{F}_{bc} = \vec{F}_{da} = 0$$

$$\vec{F}_{ab} = i\left(\vec{\ell} \times \vec{B}\right) = i\ell B$$

$$\vec{\tau}_{ab} = \vec{r} \times \vec{F}_{ab} = rF_{ab} \sin 90^\circ$$
$$= ri\ell B$$

Counterclockwise

$$\vec{F}_{cd} = i\left(\vec{\ell} \times \vec{B}\right) = i\ell B$$

$$\vec{\tau}_{cd} = \vec{r} \times \vec{F}_{cd} = rF_{cd} \sin 90^\circ$$
$$= ri\ell B$$

Counterclockwise

DC Motors: Induced Torque in a Rotating Loop

$$\vec{\tau}_{induced} = \vec{\tau}_{ab} + \vec{\tau}_{cd} = 2ri\ell B$$
$$= \frac{2}{\pi}\left(r\ell\pi\right)iB = \frac{2}{\pi}iA_P B$$
$$= \frac{2}{\pi}i\phi$$

$$\boxed{\vec{\tau}_{induced} = \frac{2}{\pi}i\phi}$$

Commutation in a 4-Pole DC Machine

Commutation is the process of converting the AC voltages and currents produced by a machine into DC voltages and currents at the terminal (and vice versa).

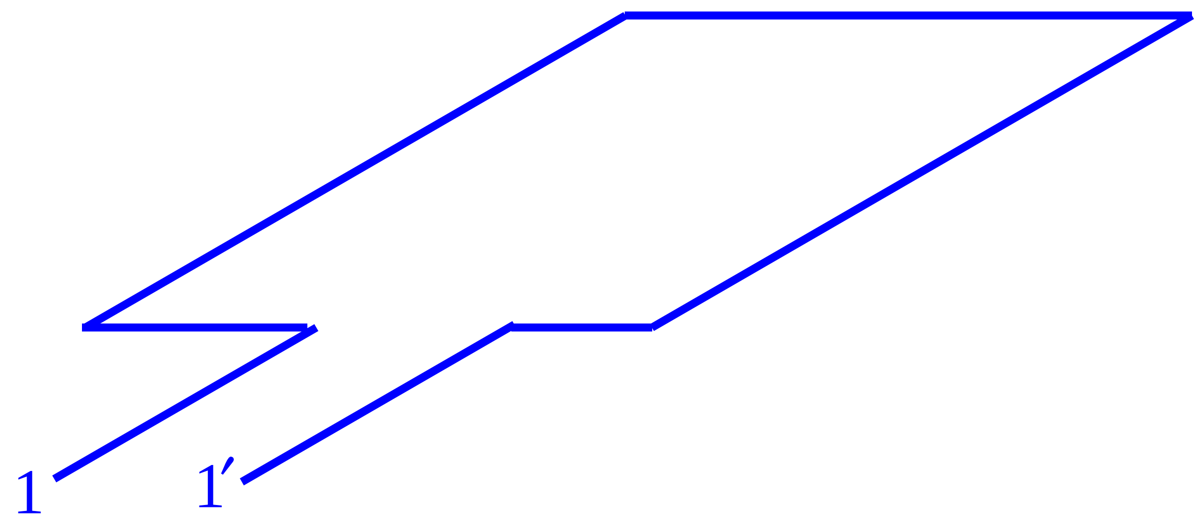
Commutation is the most critical part in the design and operation of a DC machine.

Practical commutation methods have problem that we will need to understand and deal with.

First let us look at a somewhat more sophisticated (and more realistic) commutation scheme than what we have considered thus far.

Commutation in a 4-Pole DC Machine

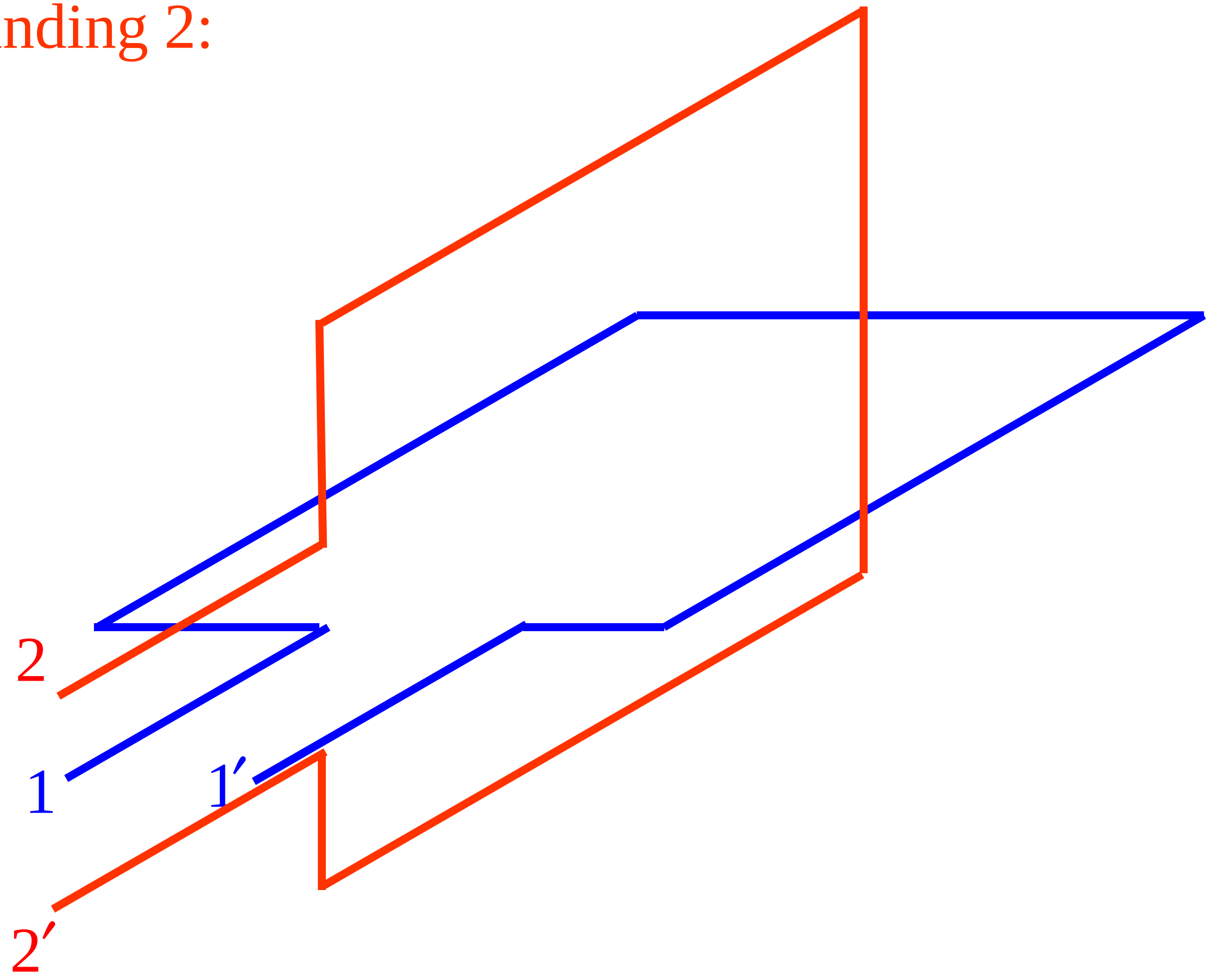
Rotor Winding 1:



Commutation in a 4-Pole DC Machine

Rotor Winding 1:

Rotor Winding 2:

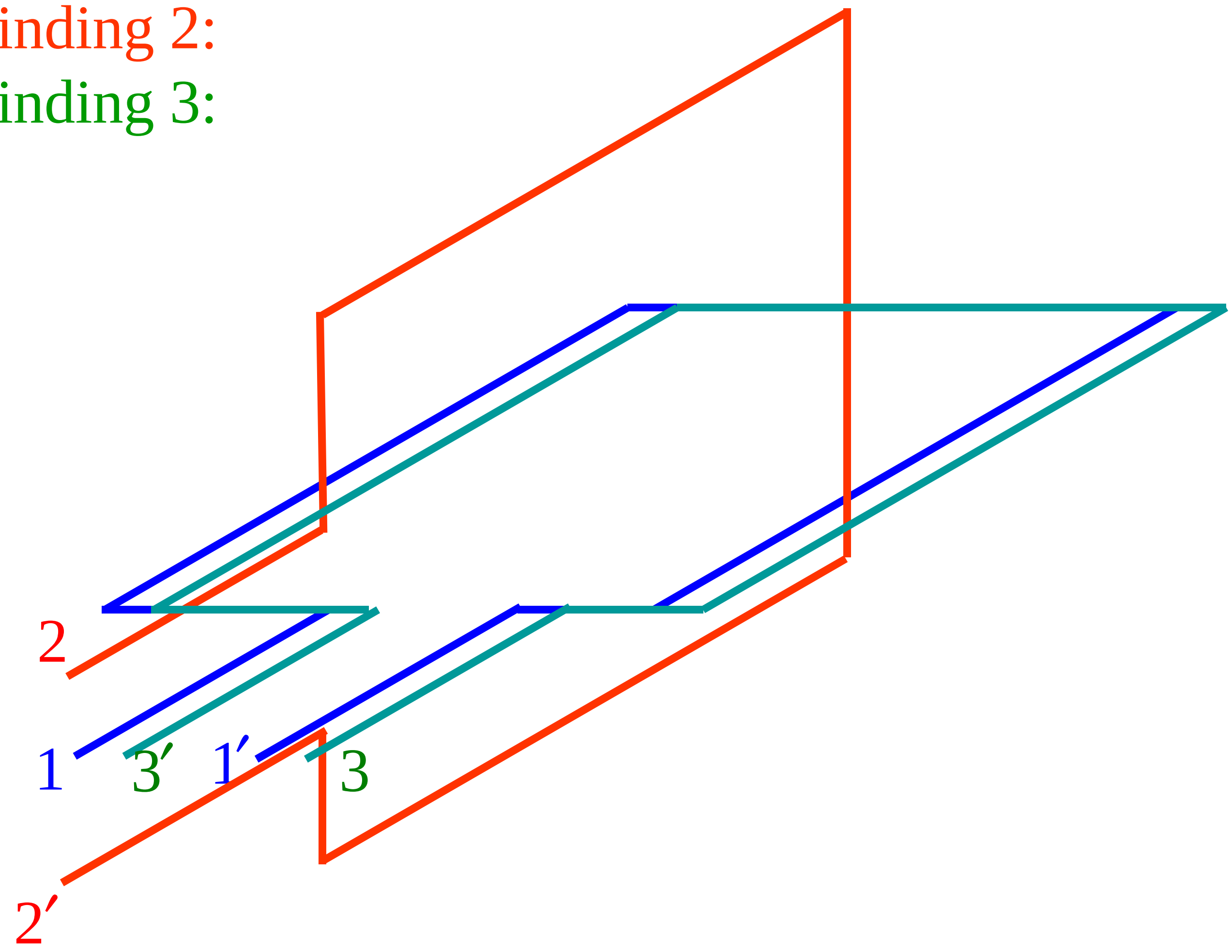


Commutation in a 4-Pole DC Machine

Rotor Winding 1:

Rotor Winding 2:

Rotor Winding 3:



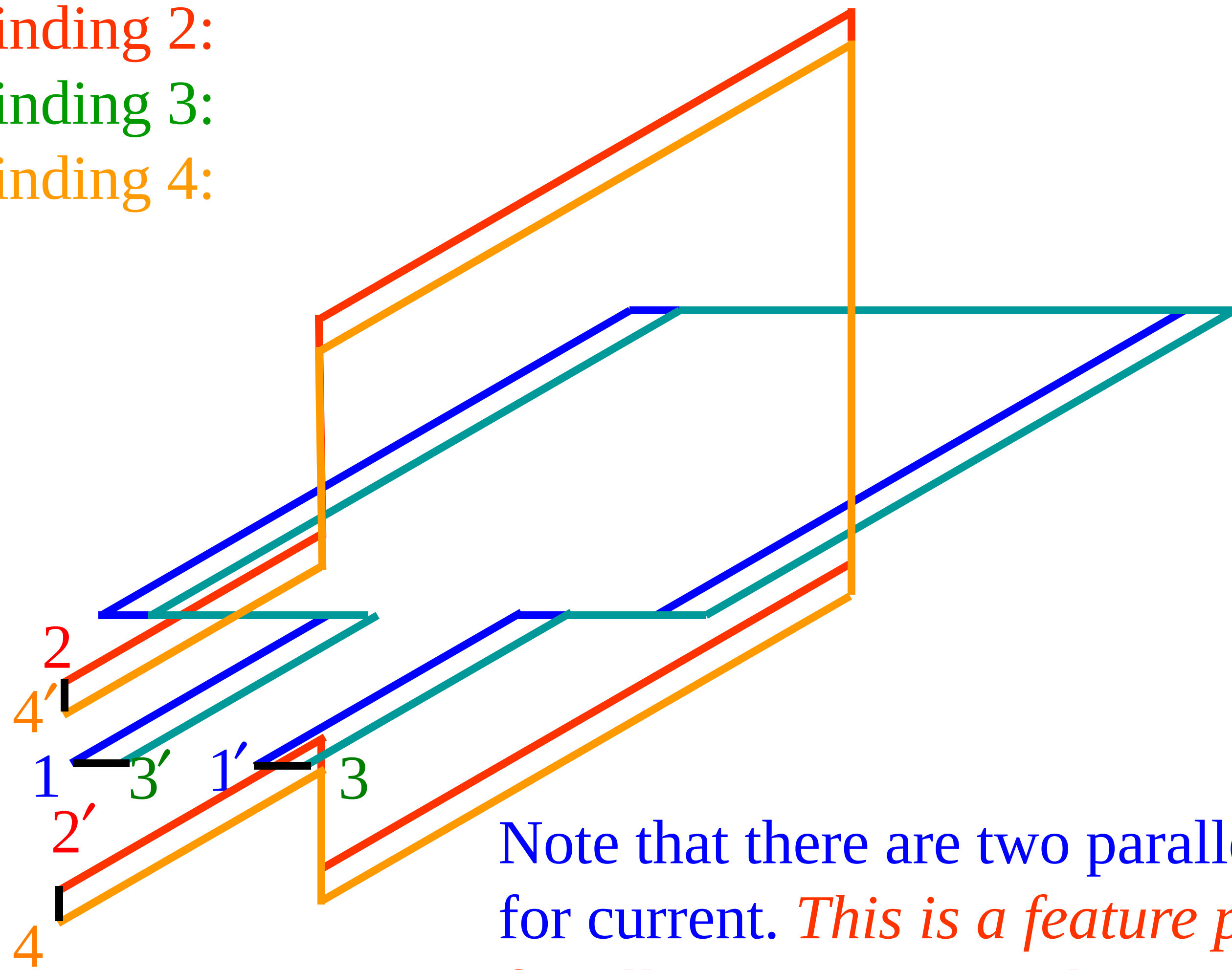
Commutation in a 4-Pole DC Machine

Rotor Winding 1:

Rotor Winding 2:

Rotor Winding 3:

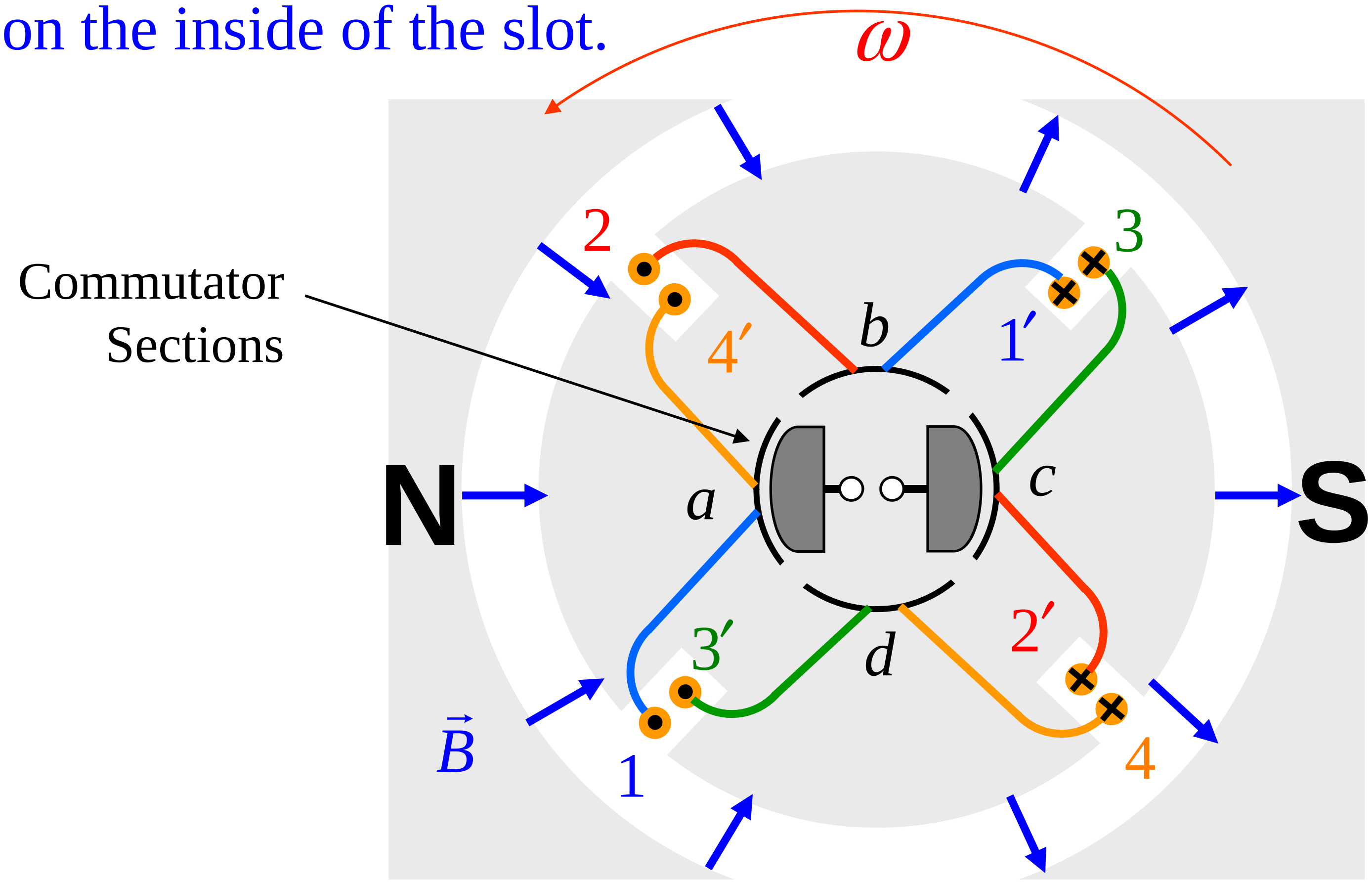
Rotor Winding 4:



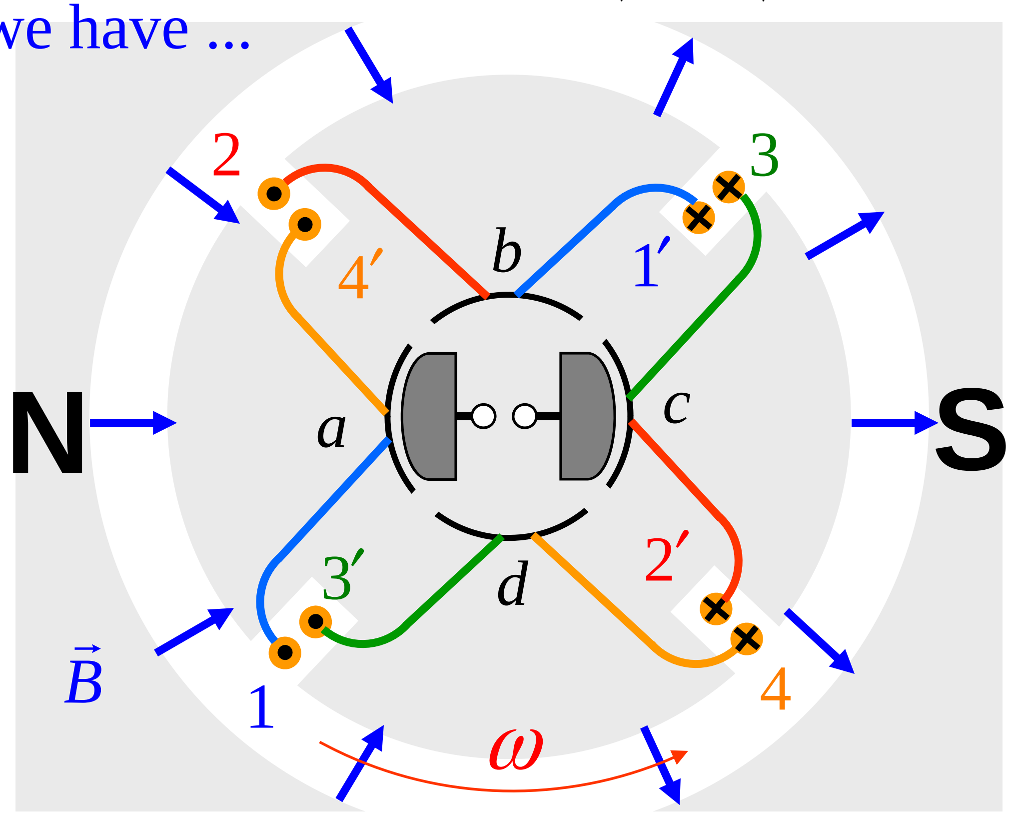
Note that there are two parallel paths for current. *This is a feature present for all commutation schemes.*

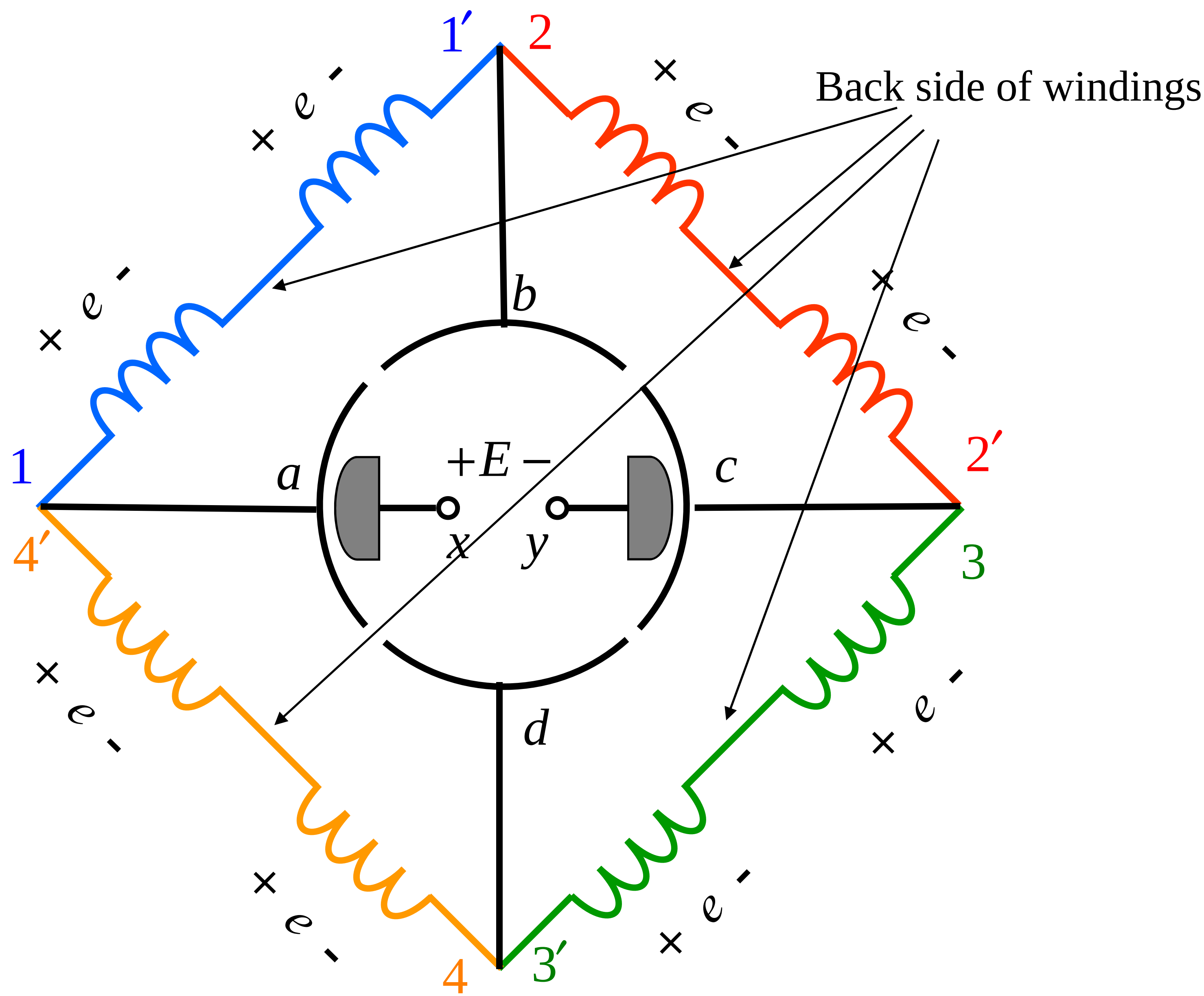
Commutation in a 4-Pole DC Machine The winding loops are buried in slots in the rotor steel core.

Note the unprimed ends are on the outside, while the primed ends are on the inside of the slot.



Commutation in a 4-Pole DC Machine At the instant shown (call this $\omega t = 0$), the ends 1, 2, 3', and 4' of the loop are under the north pole face while ends 1', 2', 3, and 4 are under the south pole face. The voltages at the north end will be positive with respect to those at the south end as determined from $e = \left(\vec{v} \times \vec{B}\right) \bullet \vec{\ell}$ as usual. Equivalently, we have ...



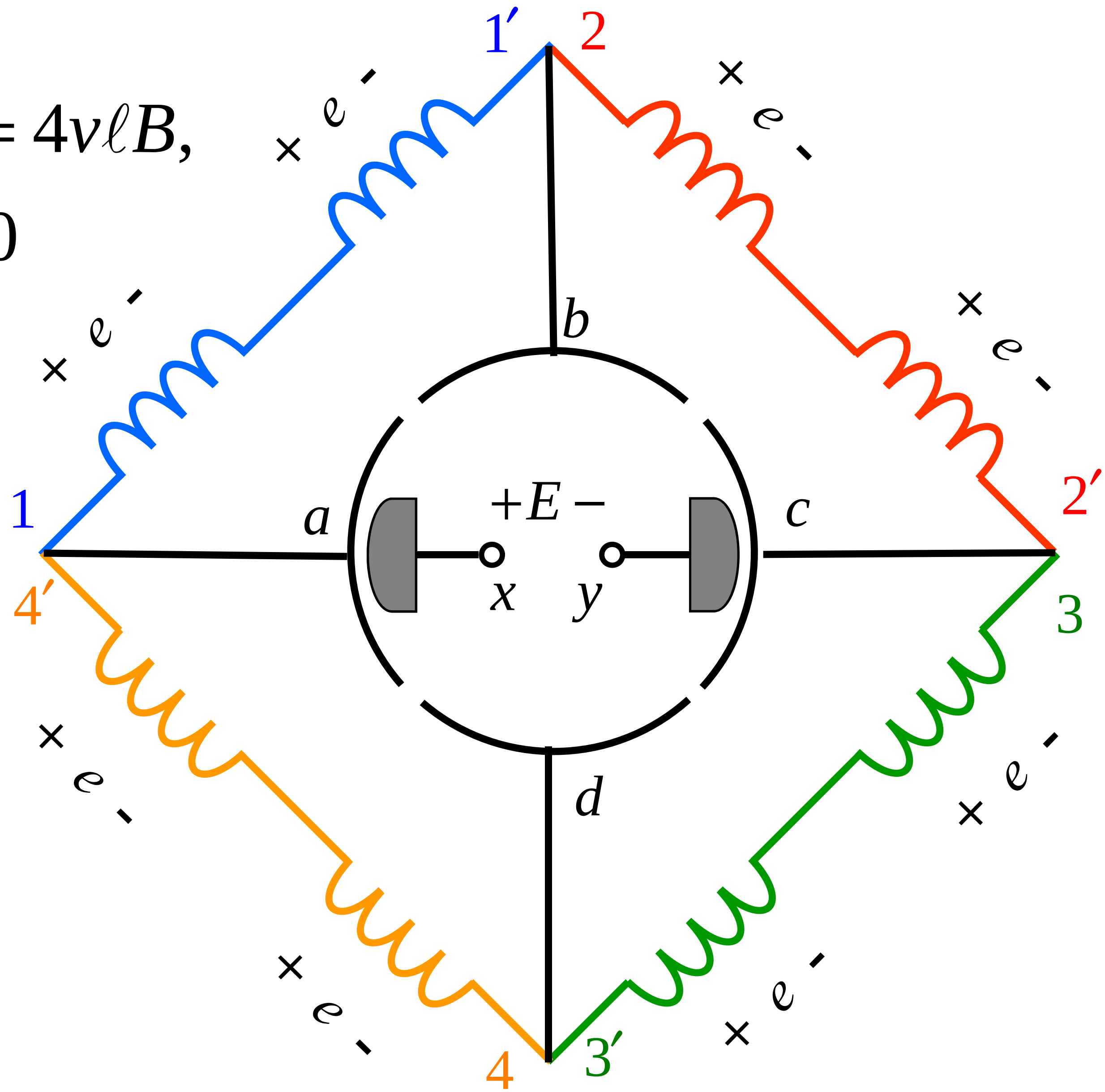


Back side of windings
1′
2
2′
3
3′
4
4′
1
a
b
c
d
+E –
x
y
× e -
× e -
× e -
× e -
× e -
× e -
× e -
× e -

Clearly,

$$E = 4e = 4v\ell B,$$

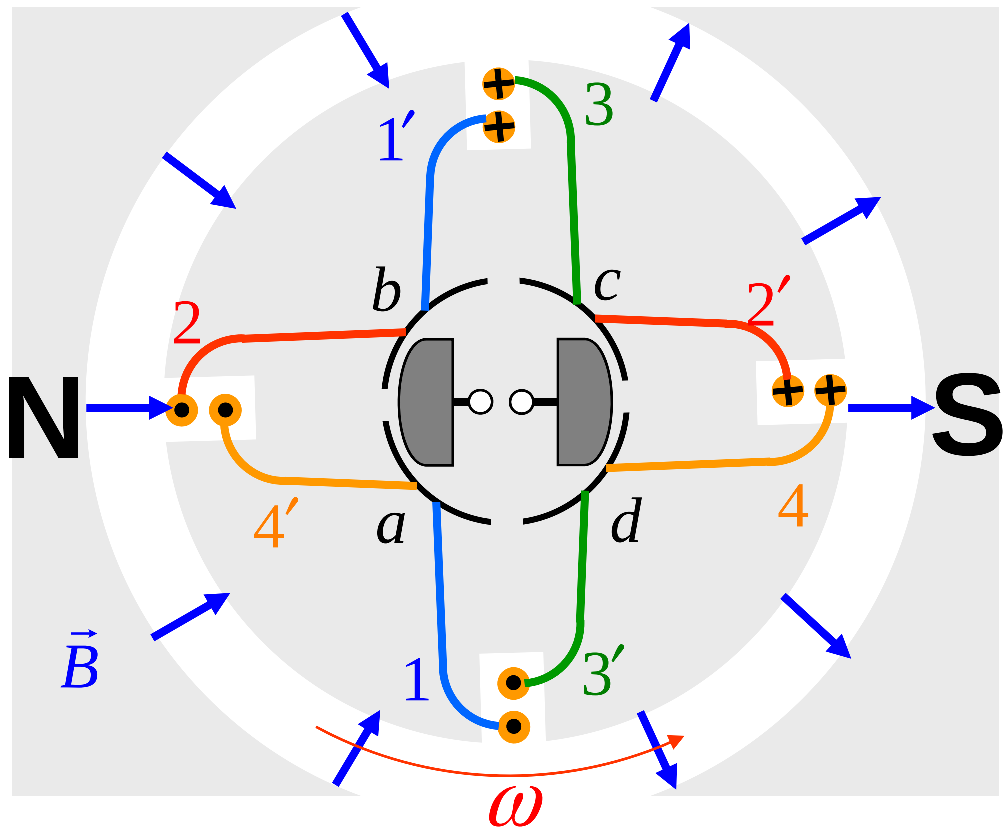
$$\omega t = 0$$



As the machine continues to rotate, consider now the case for

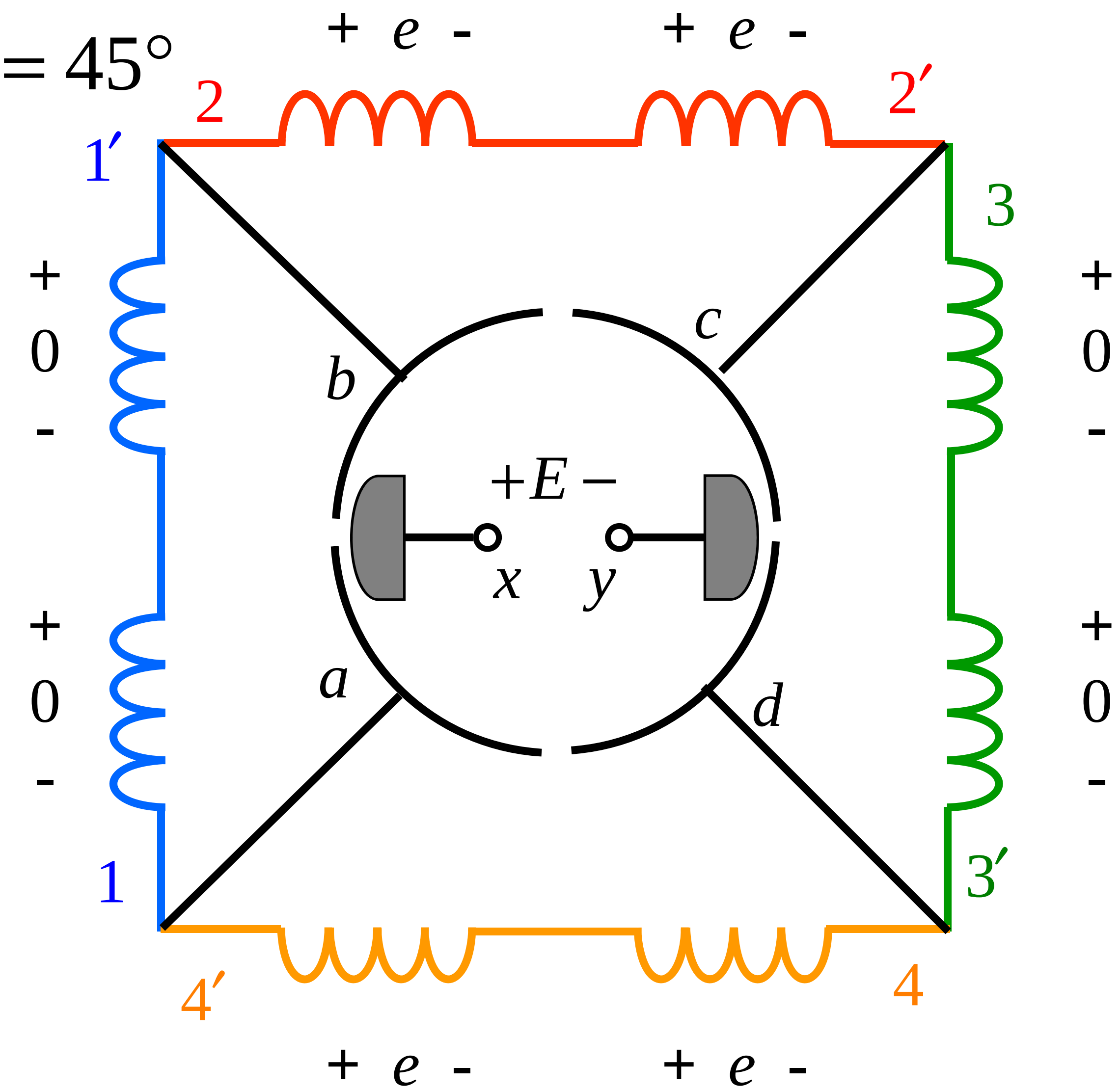
$$\omega t = 45°$$

Commutation in a 4-Pole DC Machine For ωt = 45º, clearly no voltage is induced in loops 1 and 3. At this point, the commutator brushed are shorting out commutator segments *ab* and *cd*. Our equivalent model is...



Now,

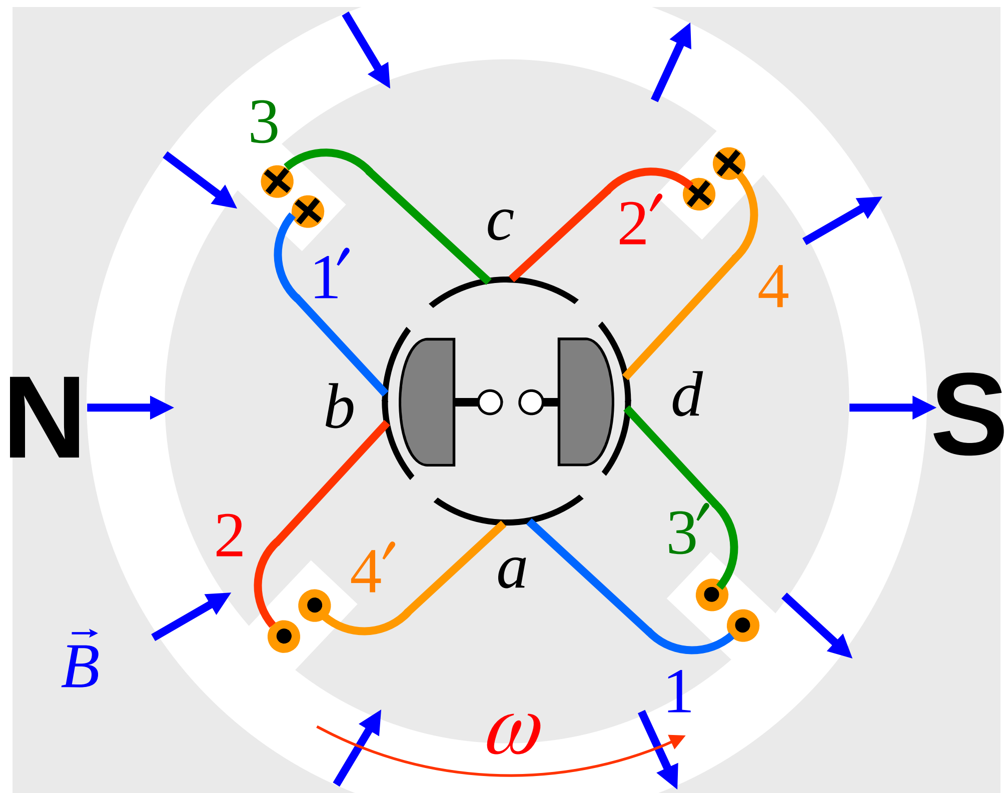
$$E = 2e, \quad \omega t = 45°$$



As the machine continues to rotate, consider now the case for

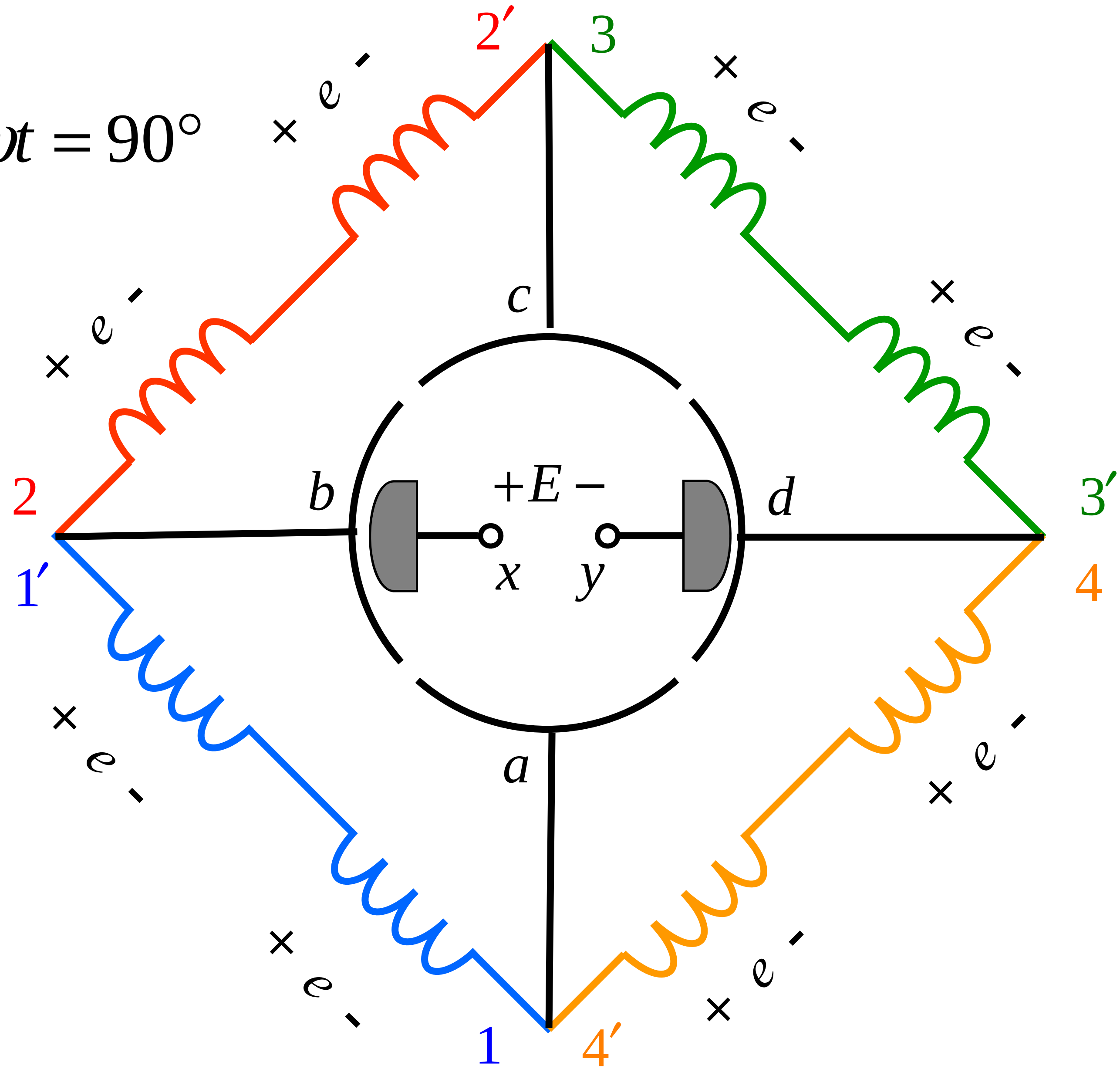
$$\omega t = 90^\circ$$

Commutation in a 4-Pole DC Machine For $\omega t = 90^{o}$, now loop ends 1', 2, 3, and 4' are positive with respect to 1, 2', 3', and 4, respectively

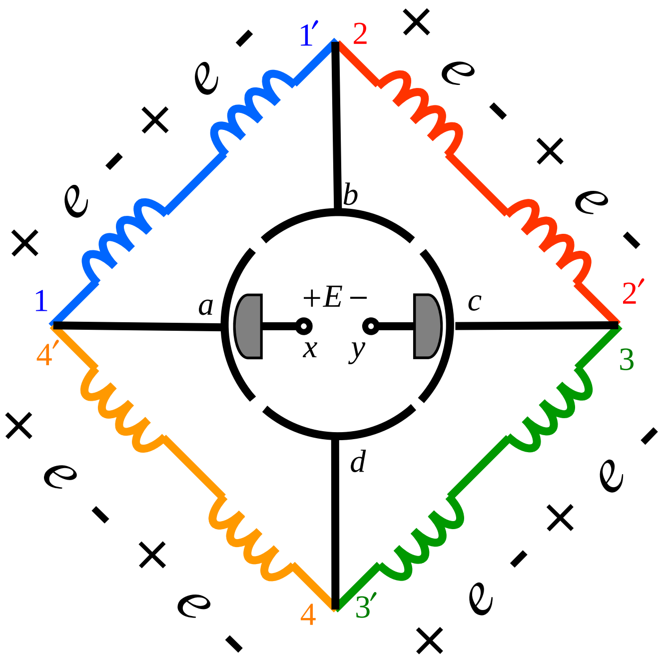


Now,

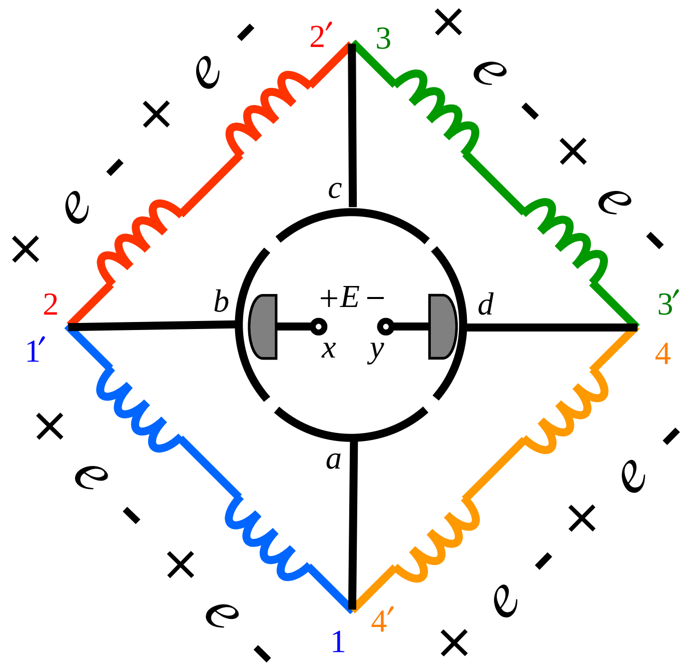
$$E = 4e \quad \omega t = 90^\circ$$



Comparing $\omega t = 0$ with $\omega t = 90^{\circ}$, we see that the polarity across loops 1 and 3 has reversed, but so have their connections, so the terminal polarity remains the same. *This is the essence of commutation.*

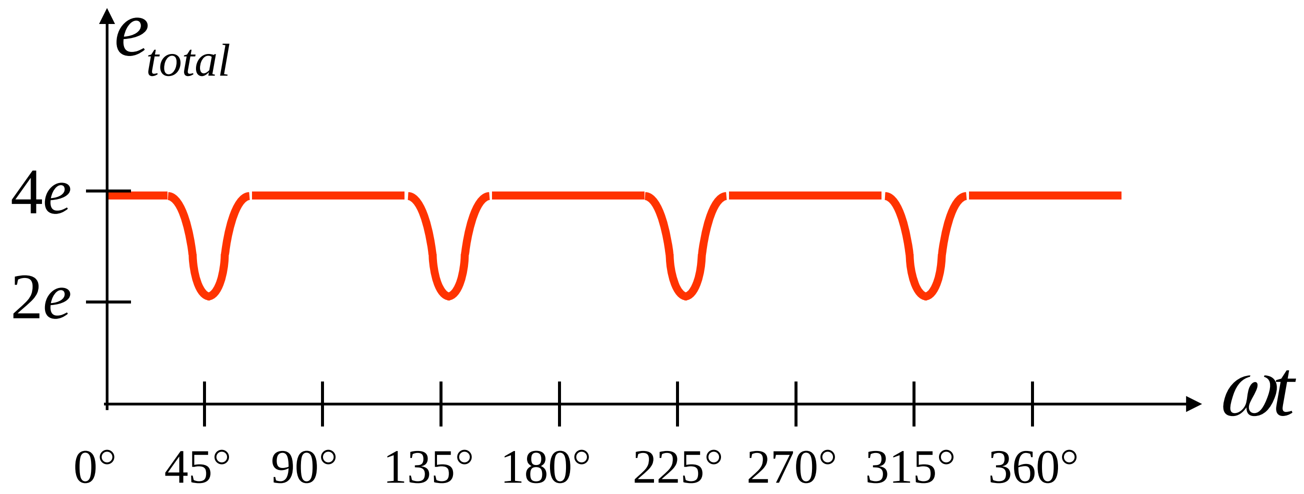


$$\omega t = 0^{\circ}$$



$$\omega t = 90^{\circ}$$

Output Voltage: Note that this is a better approximation to DC than is a single loop. As the number of loops on the rotor increases, the approximation to a DC voltage gets better and better.



Real DC machines have a similar construction to the four-pole machine considered, except they have more loops and more commutation segments.

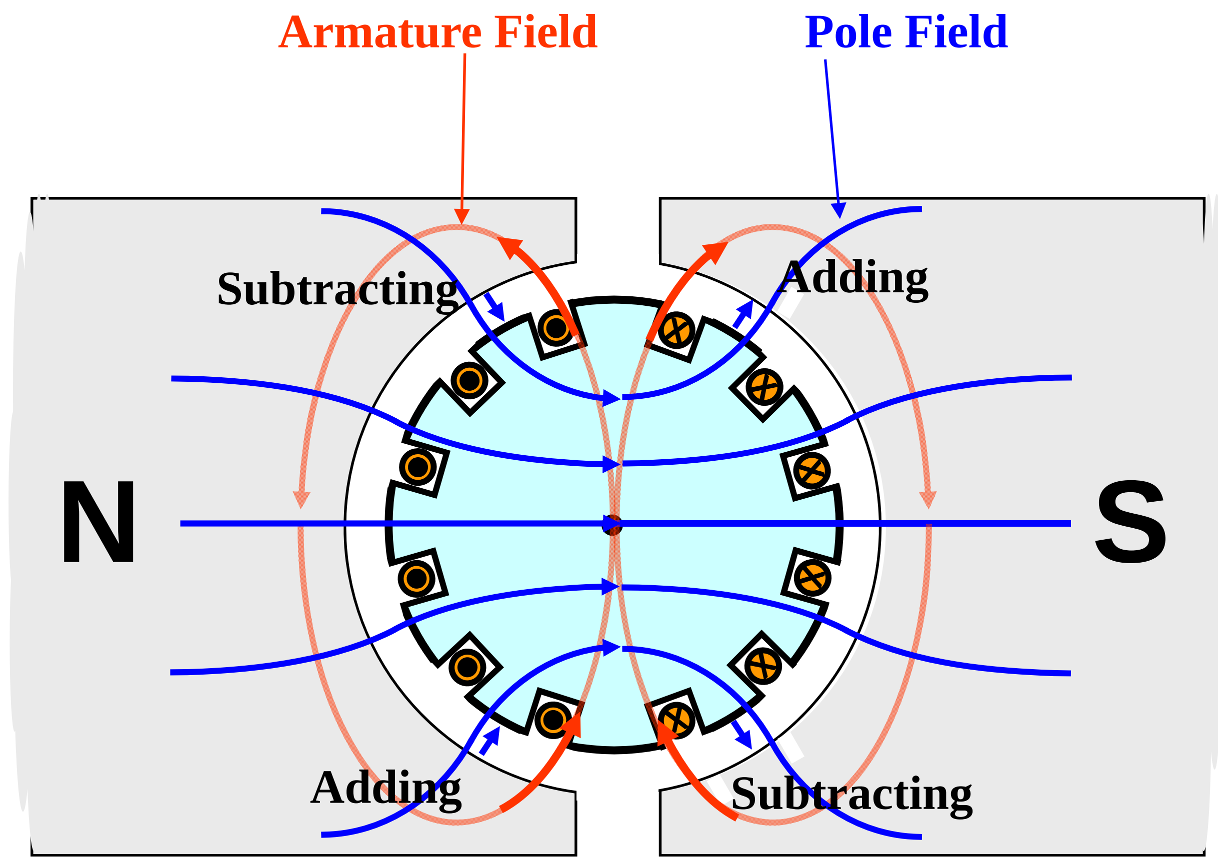
Practical Issues Associated with Commutation

1. Armature reaction – Current flowing in the rotor armature winding produces its own magnetic field that interacts with the pole field due to the permanent magnet field causing a distortion in the uniform radial field desired.

2. *Ldi/dt* Voltages – The commutator segments shorted by the brushes cause what is known as *inductive kickback*.

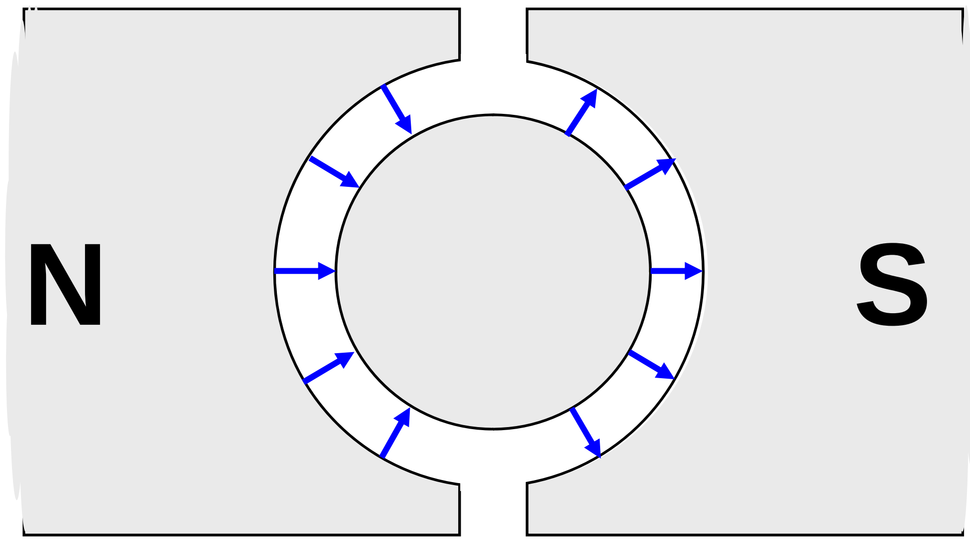
Each of these issues (problems) and ways to fix them are now discussed.

Armature Reaction

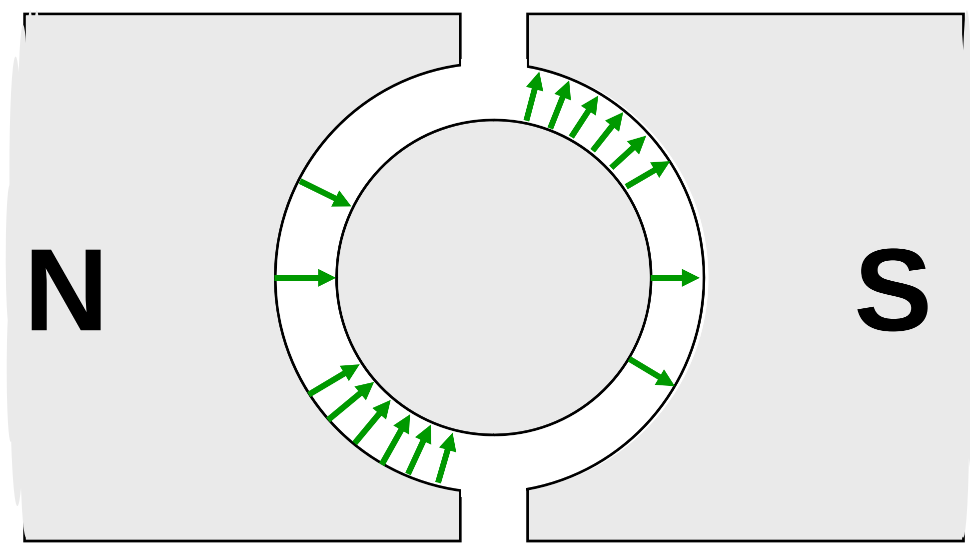


Armature Reaction

What we want:

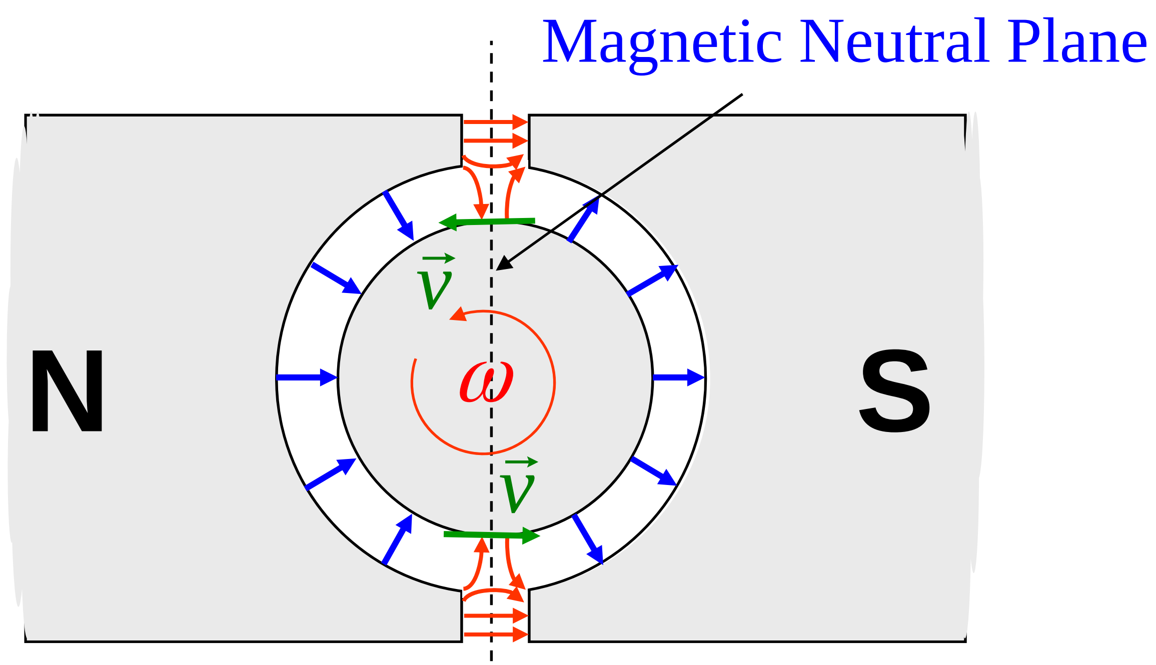


What we get:

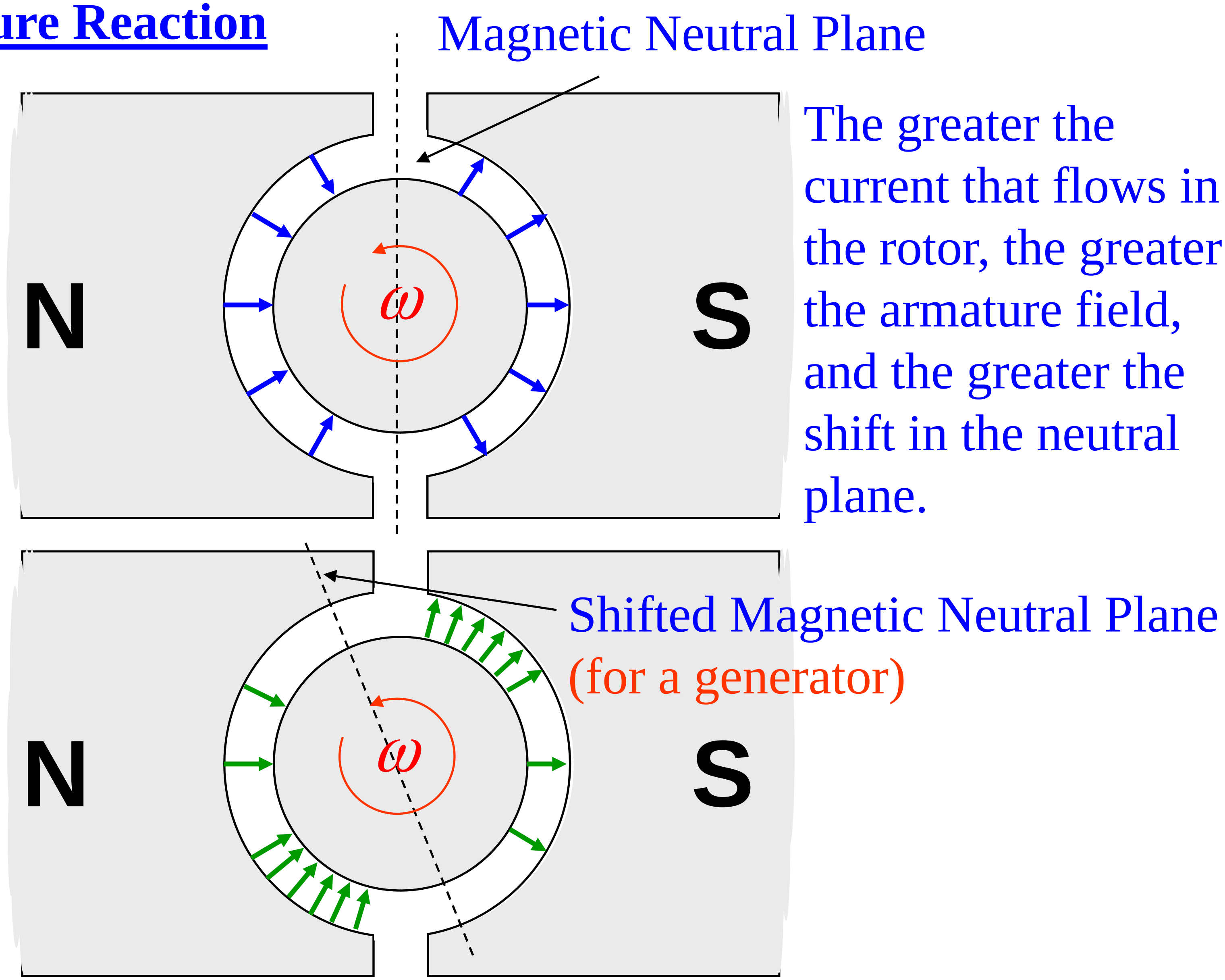


Armature Reaction

The *Magnetic Neutral Plane* is the plane in the machine where the velocity of the rotor wires is parallel to the magnetic flux lines so that the induced voltage in the plane is zero.

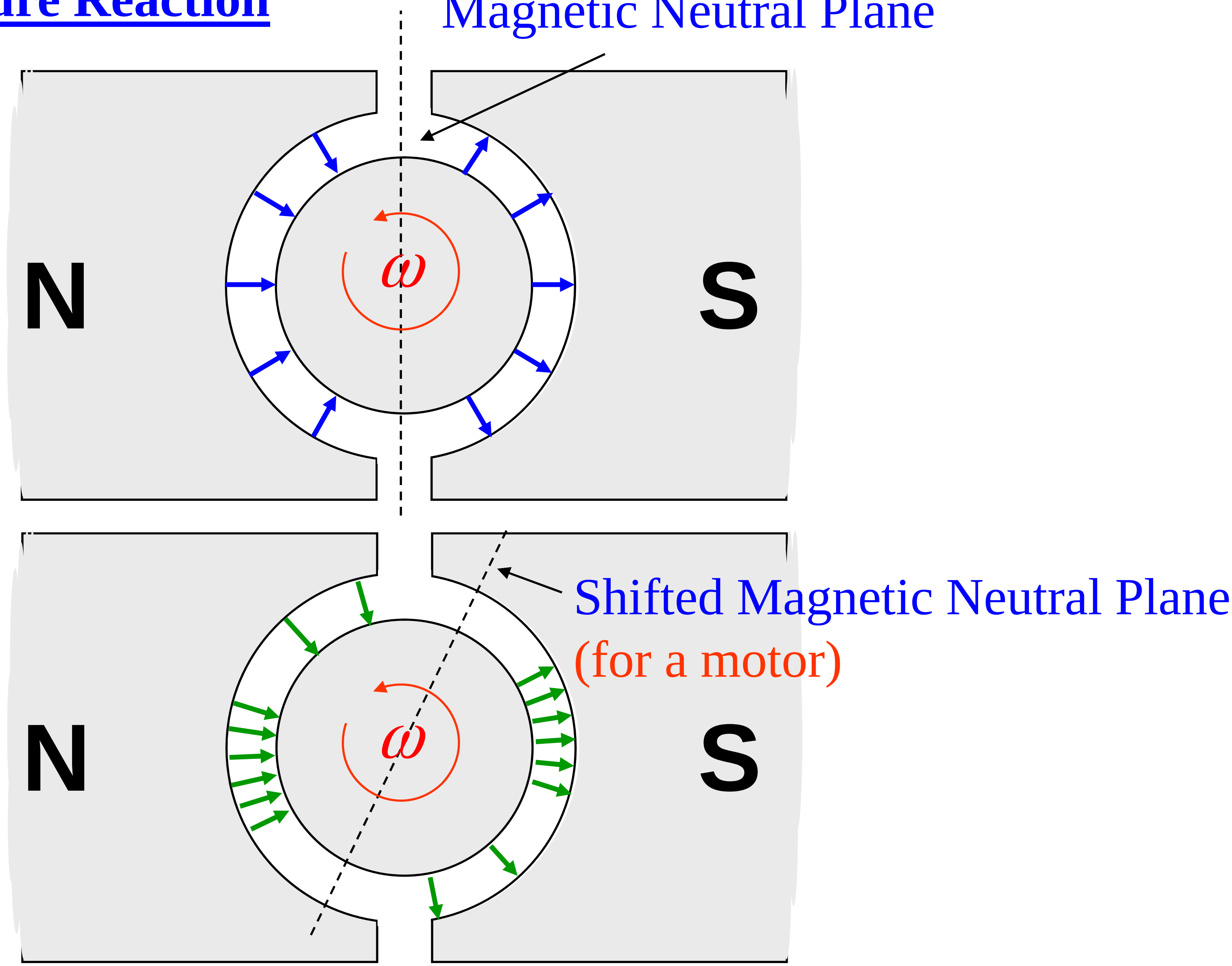


Armature Reaction



The greater the current that flows in the rotor, the greater the armature field, and the greater the shift in the neutral plane.

Armature Reaction



Armature Reaction

Why is armature reaction a problem?

The brushes must short out the commutator segments at exactly the point where the voltage across them is zero.

If the brushes were located to short out the conductors for the case of the vertical neutral plane, then when the machine is loaded the brushes will short out segments with a finite voltage across them.

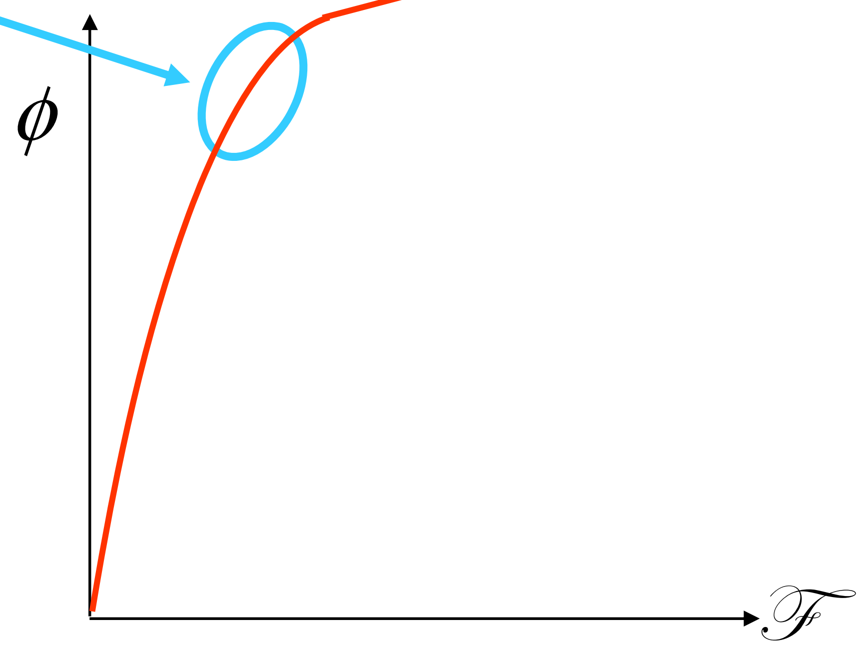
This is a serious problem.

If we placed the brushes at their full-load location, it would then spark at no-load.

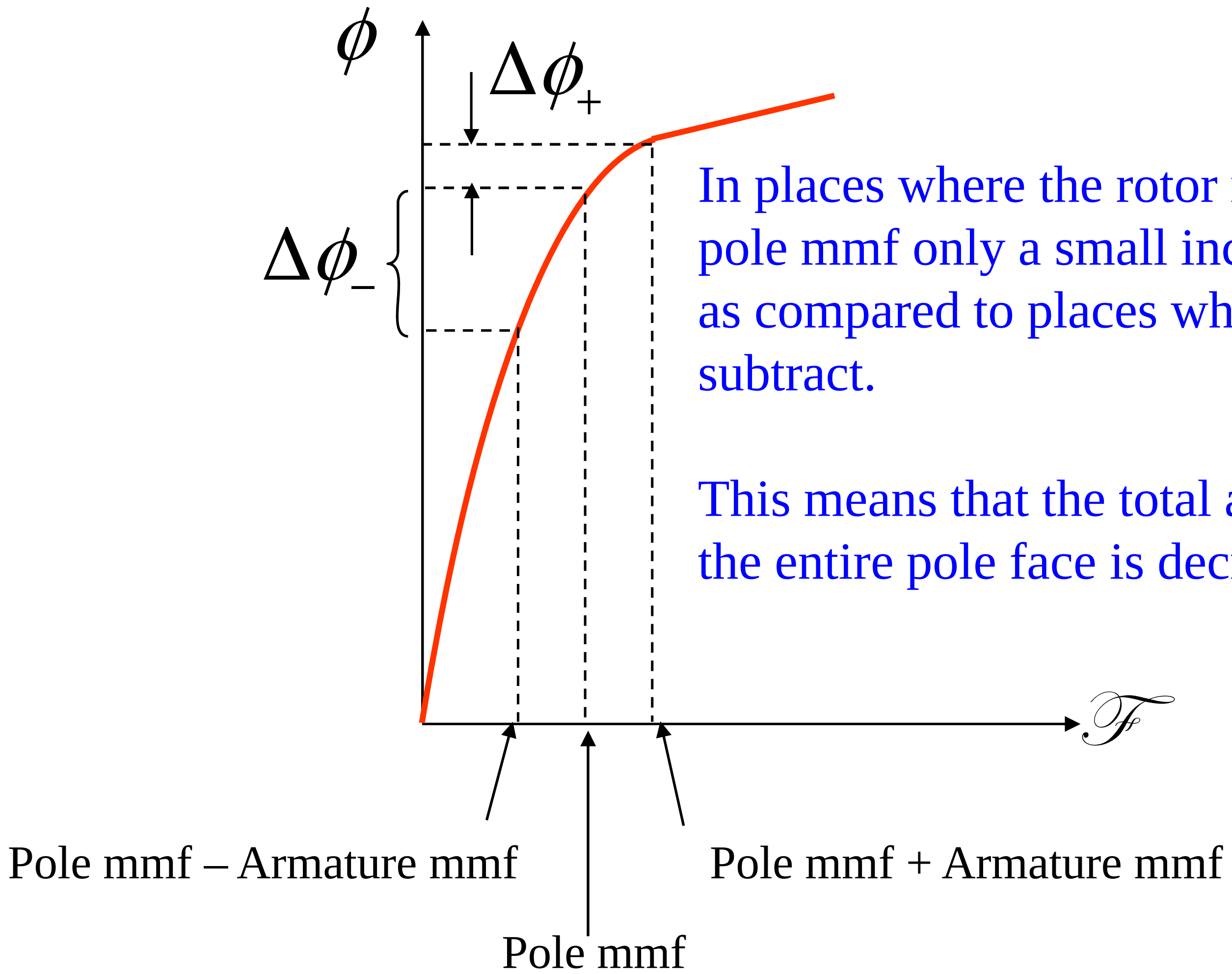
Armature Reaction

Another problem caused by armature reaction is called *flux weakening*.

Recall that to achieve maximum flux most machines operate near the saturation point of the magnetic material of the armature.



<u>**Armature Reaction**</u> *Flux Weakening*



In places where the rotor mmf adds to the pole mmf only a small increase in flux occurs as compared to places where the two mmf's subtract.

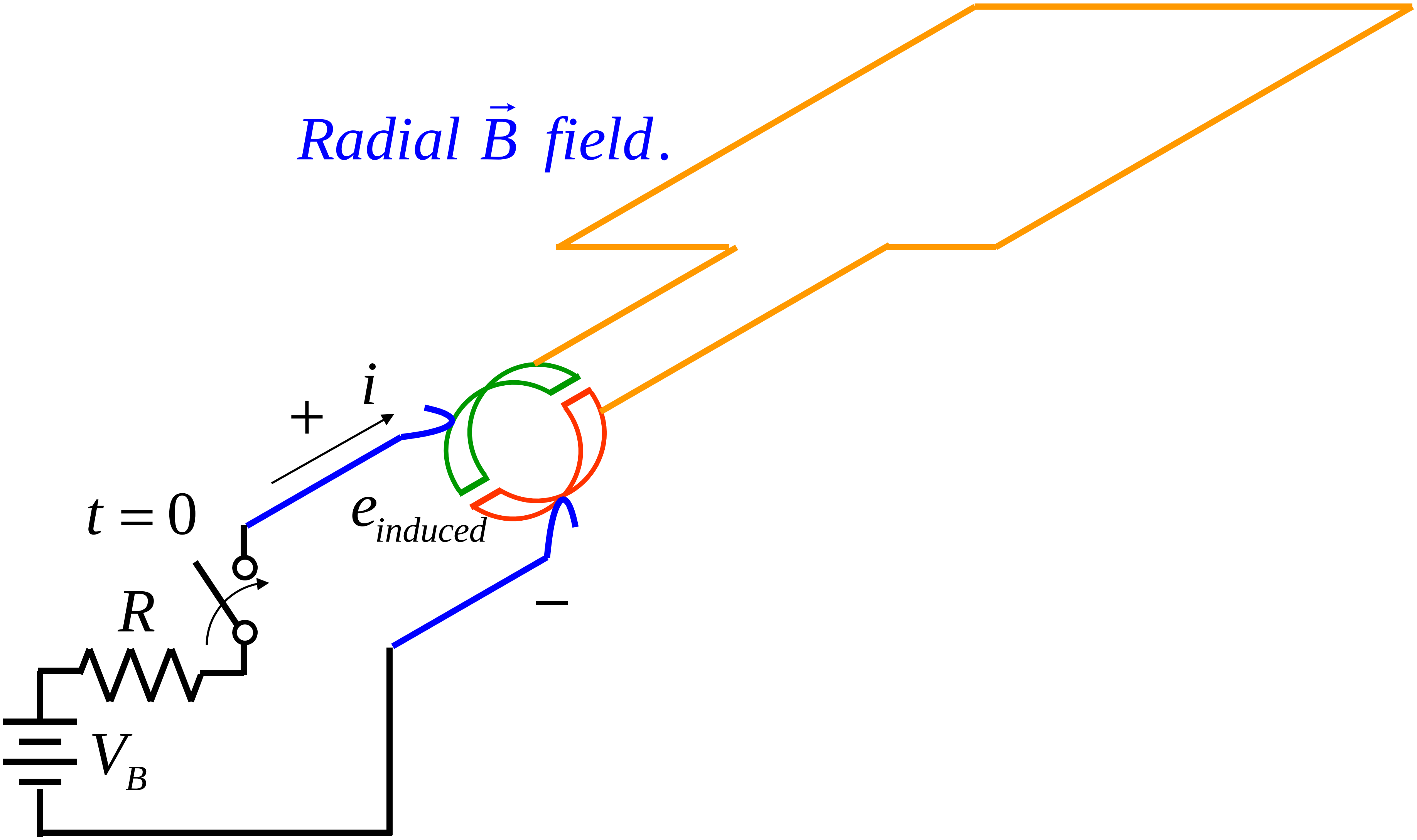
This means that the total average flux under the entire pole face is decreased.

Armature Reaction *Flux Weakening*

In a generator, a reduced flux reduces the generated voltage.

For a motor the situation is much more serious. Consider our simple motor form Slide 18...

Armature Reaction *Flux Weakening*



Armature Reaction *Flux Weakening*

First, assume that there is no load on the shaft. When the switch closes at $t = 0$, an current will flow in the loop,

$$i = \frac{V_B - e_{induced}}{R}$$

But since the loop is at first stationary,

$$e_{induced} = 0 \Rightarrow i = \frac{V_B}{R}$$

This current produces a torque,

$$\tau = \frac{2}{\pi}\phi i$$

Armature Reaction *Flux Weakening*

This torque causes the rotor to accelerate.

This acceleration clearly cannot continue forever. What causes the acceleration to decrease and a steady-state rotation to exist?

As the rotor accelerates, the moving loop now produces an induced voltage (generally termed the *counter electromotive force*) given by,

$$e_{induced} = \frac{2}{\pi}\phi\omega$$

Armature Reaction *Flux Weakening*

The induced voltage causes the current to drop since

$$i = \frac{V_B - e_{induced}}{R}$$

As the current decreases, the induced torque decreases since,

$$\tau = \frac{2}{\pi}\phi i$$

we eventually reach the steady-state operating conditions of,

$$\underbrace{\tau = 0}, \qquad e_{induced} = V_B,$$

$$i = 0,$$

$$i.e., \tau_{induced} = \tau_{load} = 0 \qquad \omega = \omega_{ss} \left(\text{a constant} \right)$$

What is ω_{ss} ?

Armature Reaction *Flux Weakening*

Since,

$$\tau = 0 \Rightarrow i = 0$$

$$\text{since} \quad \tau = \frac{2}{\pi}\phi i$$

$$\text{and} \quad \phi \neq 0$$

Since $i = 0$, $e_{induced} = V_B$, and since

$$e_{induced} = \frac{2}{\pi}\phi\omega \Rightarrow \omega_{ss} = \frac{\pi}{2}\frac{V_B}{\phi}$$

<u>**Armature Reaction**</u> *Flux Weakening*

$$\omega = \frac{\pi}{2}\frac{e_{induced}}{\phi} = \frac{\pi}{2}\frac{V_B}{\phi}$$

For a <u>fixed voltage</u>, as the flux *decreases* the speed *increases*.

But increasing the speed increases the load resulting in more flux weakening.

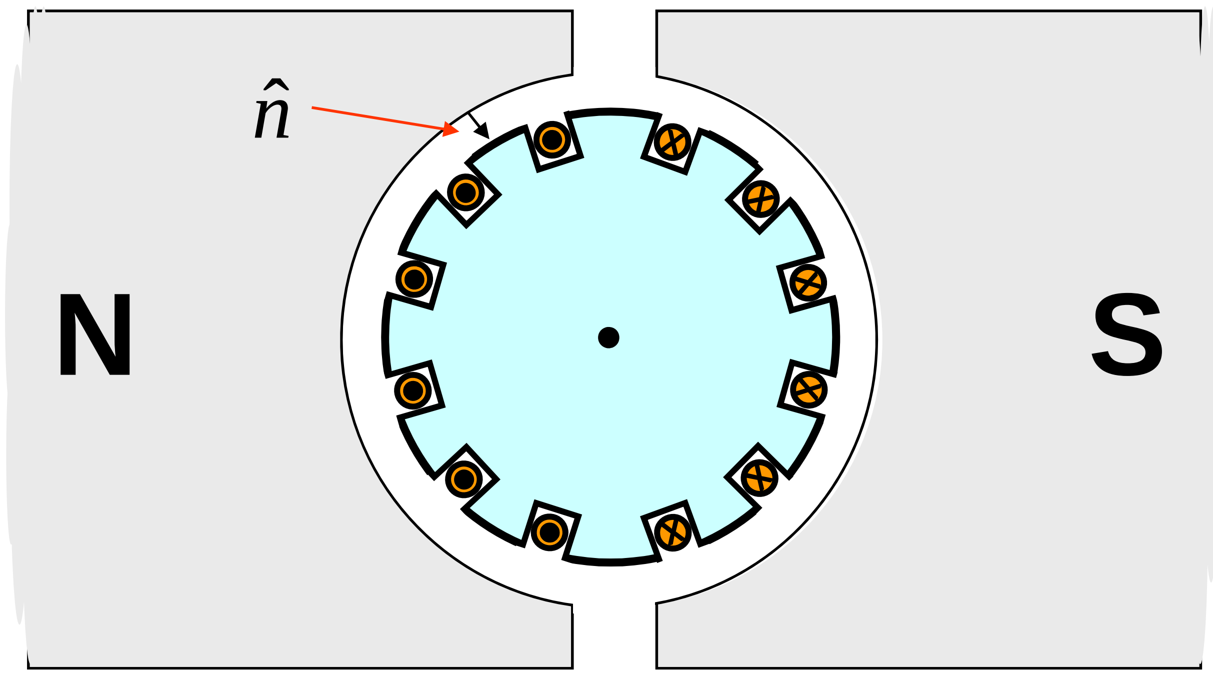
This can cause a run-away situation to occur.

This too poses a serious problem.

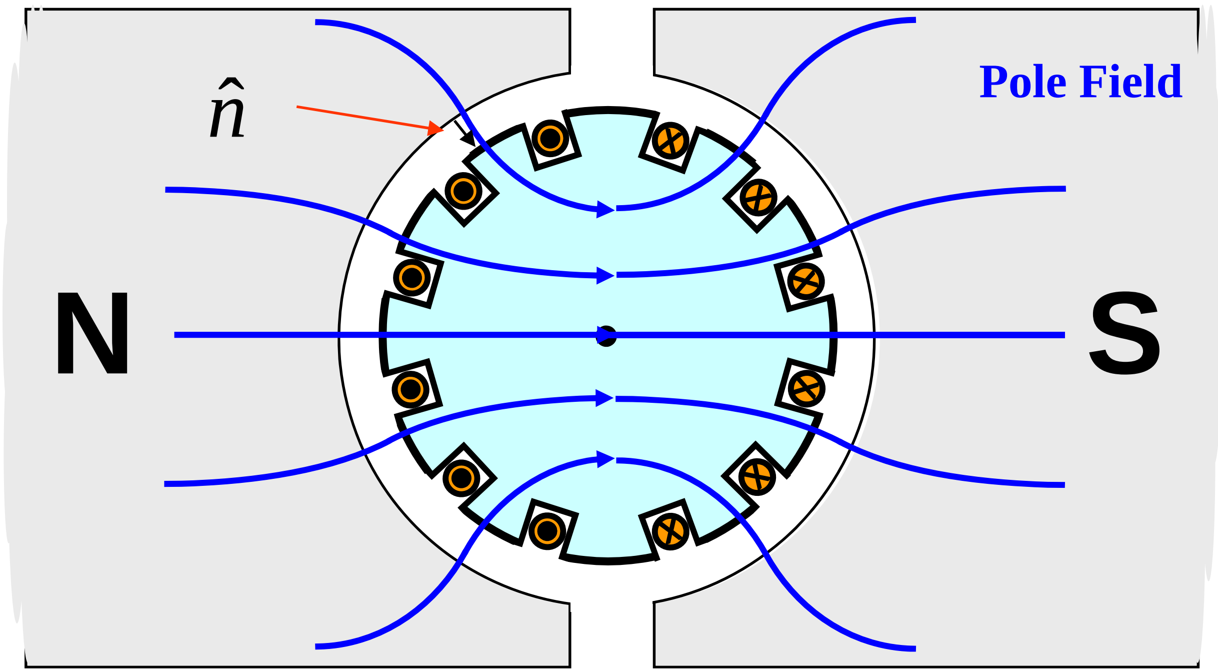
Armature Reaction *Flux Weakening*

Examine the mmf and the flux.

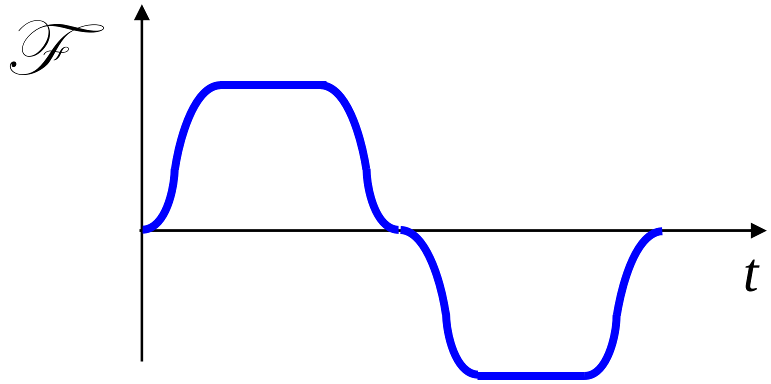
Recall: $$\mathscr{F} = \phi \mathscr{R}_{gap} = \vec{B} \bullet \vec{A} \mathscr{R}_{gap} = \vec{B} \bullet \hat{n} A \frac{\ell_{gap}}{\mu_o A}$$



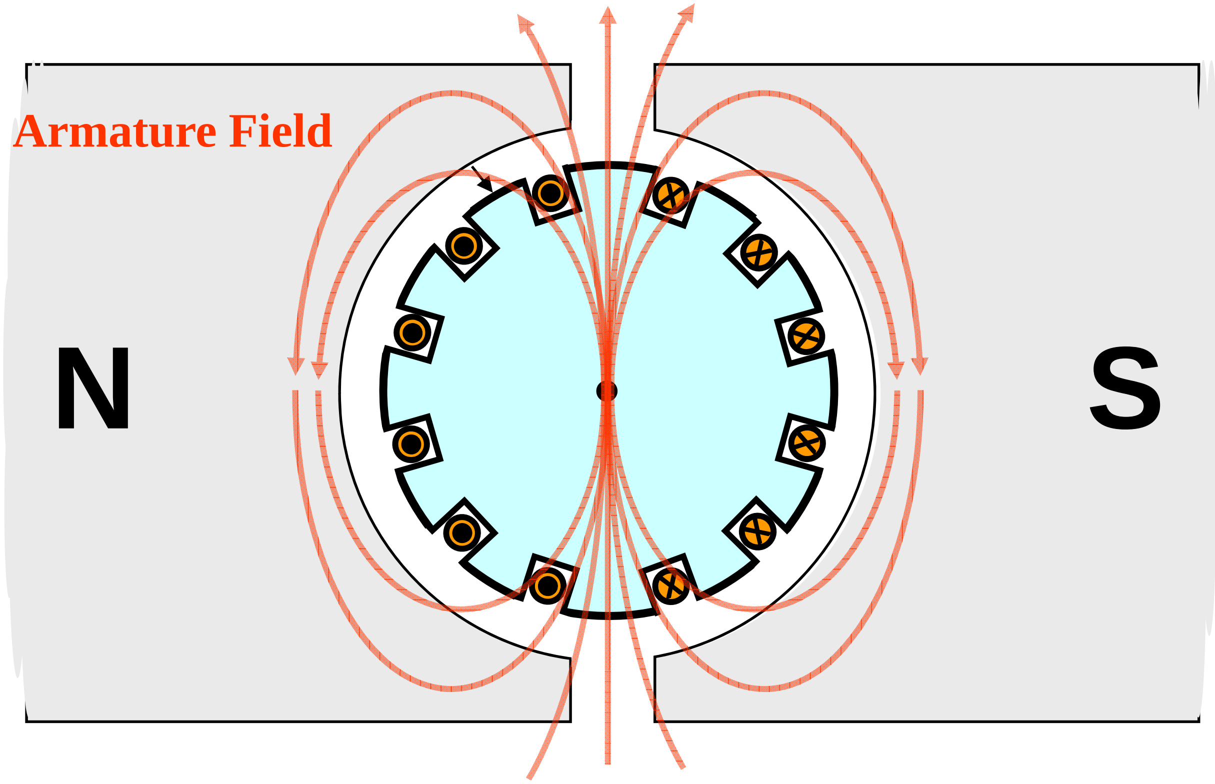
<u>Armature Reaction</u> *Flux Weakening*



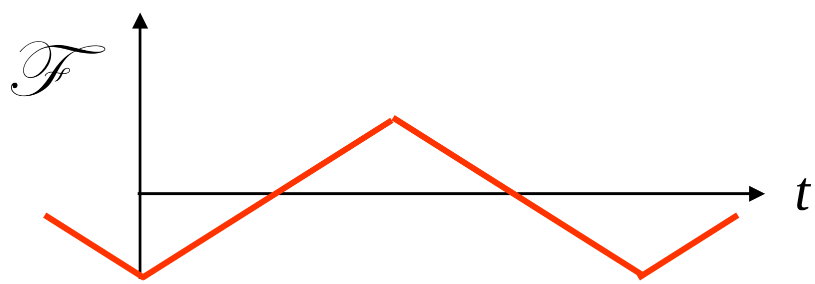
$$\mathscr{F} = \vec{B}\bullet\vec{A}\frac{\ell_{gap}}{\mu_o A}$$



Armature Reaction *Flux Weakening*

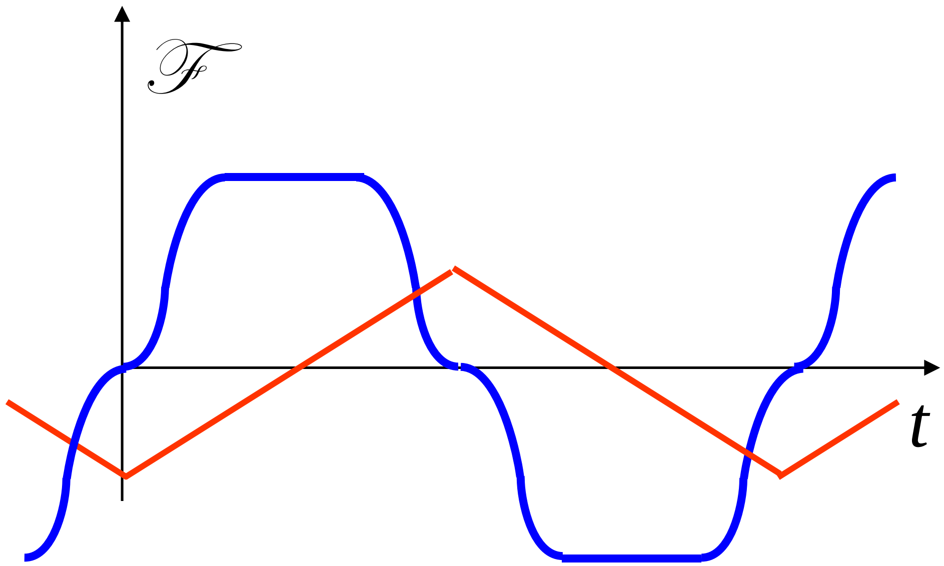


$$\mathscr{F} = \vec{B}\bullet\vec{A}\frac{\ell_{gap}}{\mu_o A}$$

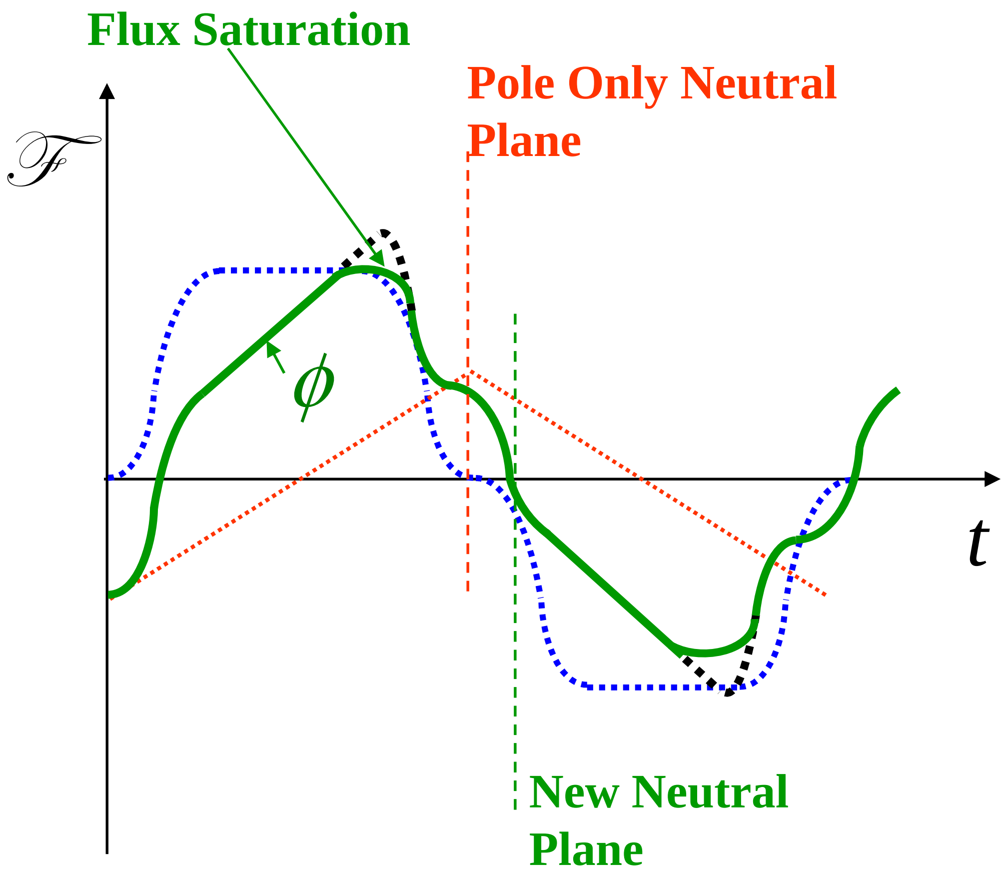


Armature Reaction *Flux Weakening*

Add these to get the net mmf:



Armature Reaction *Flux Weakening*



Armature Reaction *Flux Weakening*

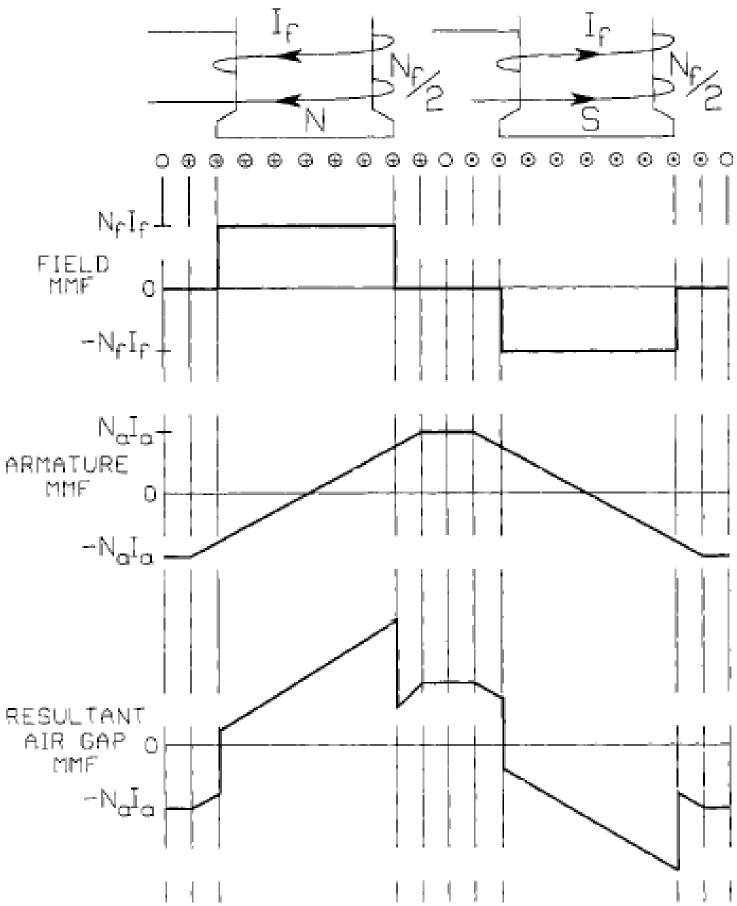


FIGURE 4.7 Flux distortion in the air gap.

W. H. Yeadon & A. W. Yeadon, HANDBOOK OF SMALL ELECTRIC MOTORS, McGraw-Hill 2001, Chapter 4, Direct Current Motors

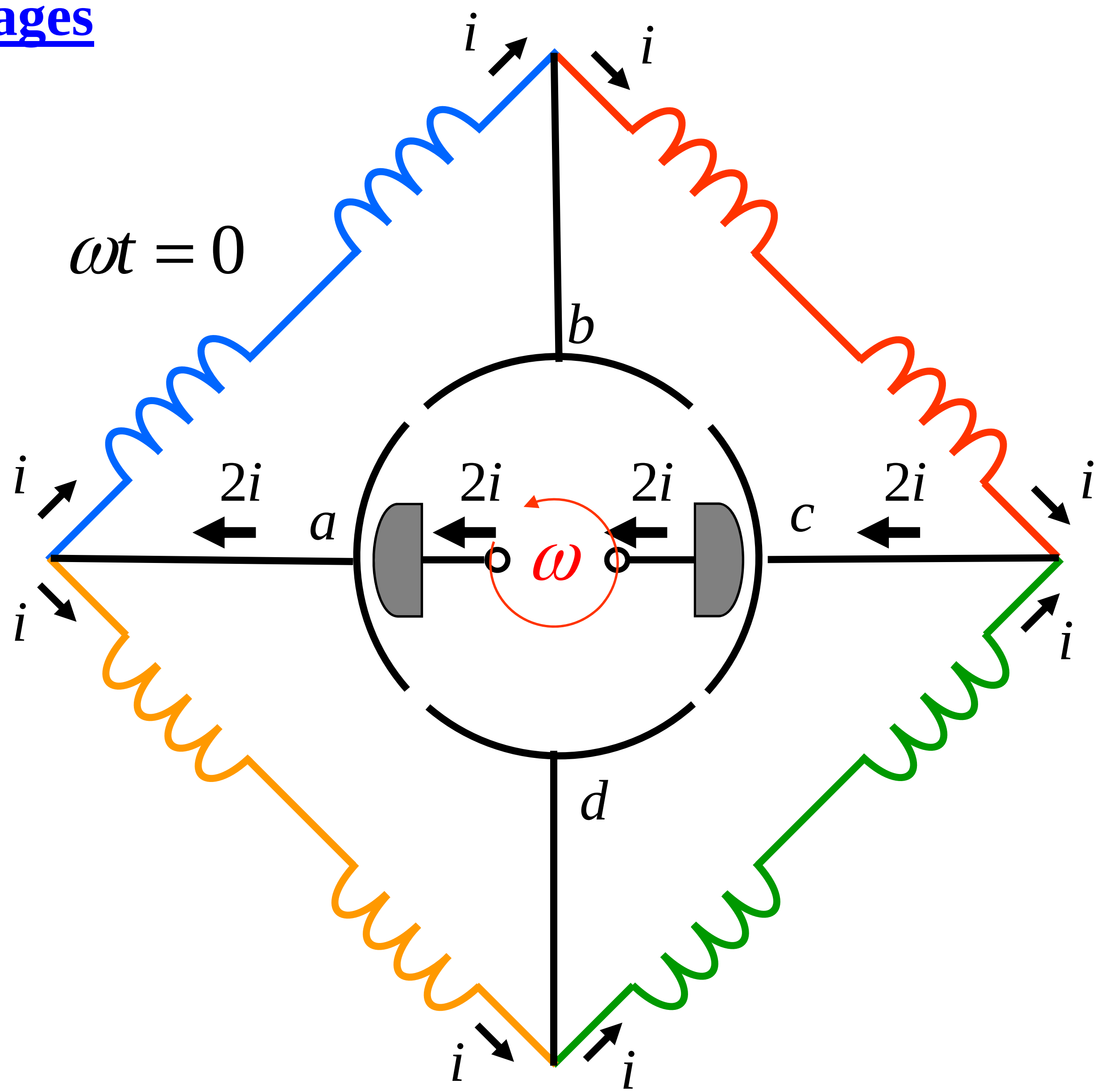
***Ldi/dt* Voltages**

The *Ldi*/*dt* voltage occurs in commutator segments being shorted out by the brushes.

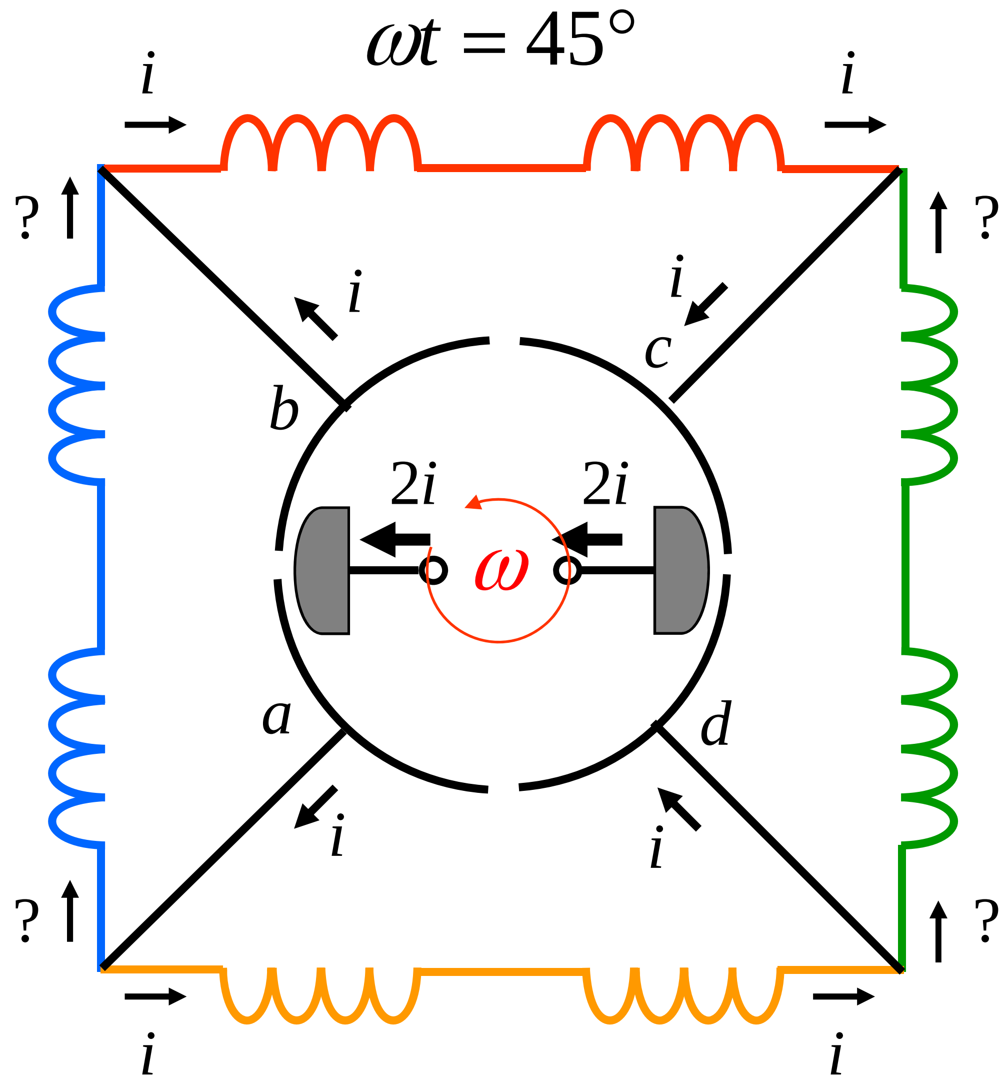
Consider the situation on the next three slides and notice that when a commutator segment is shorted out, the current flow through that commutator segment must reverse.

The question is, "How fast does this reversal in direction occur?"

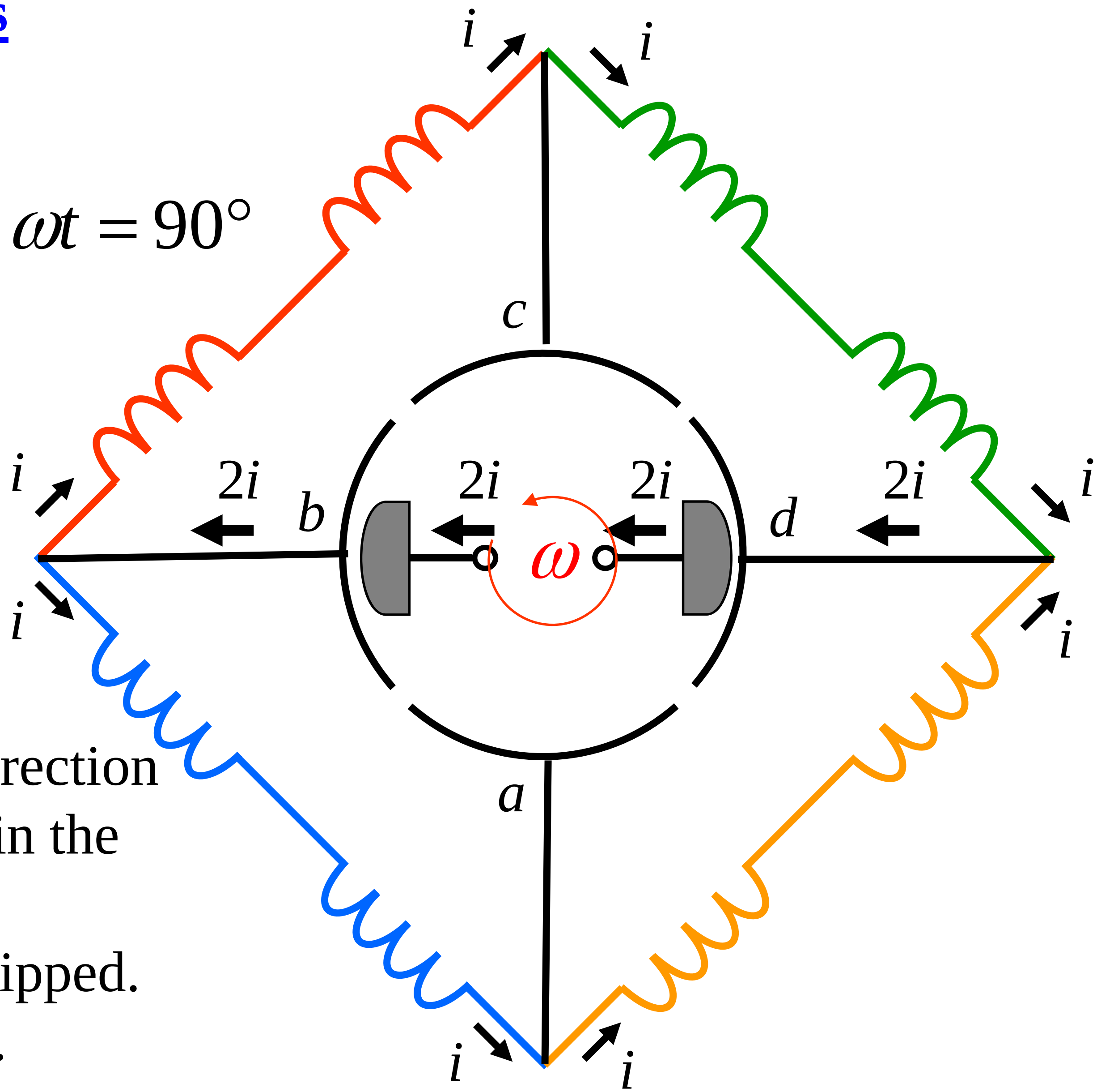
***Ldi/dt* Voltages**



***Ldi/dt* Voltages**



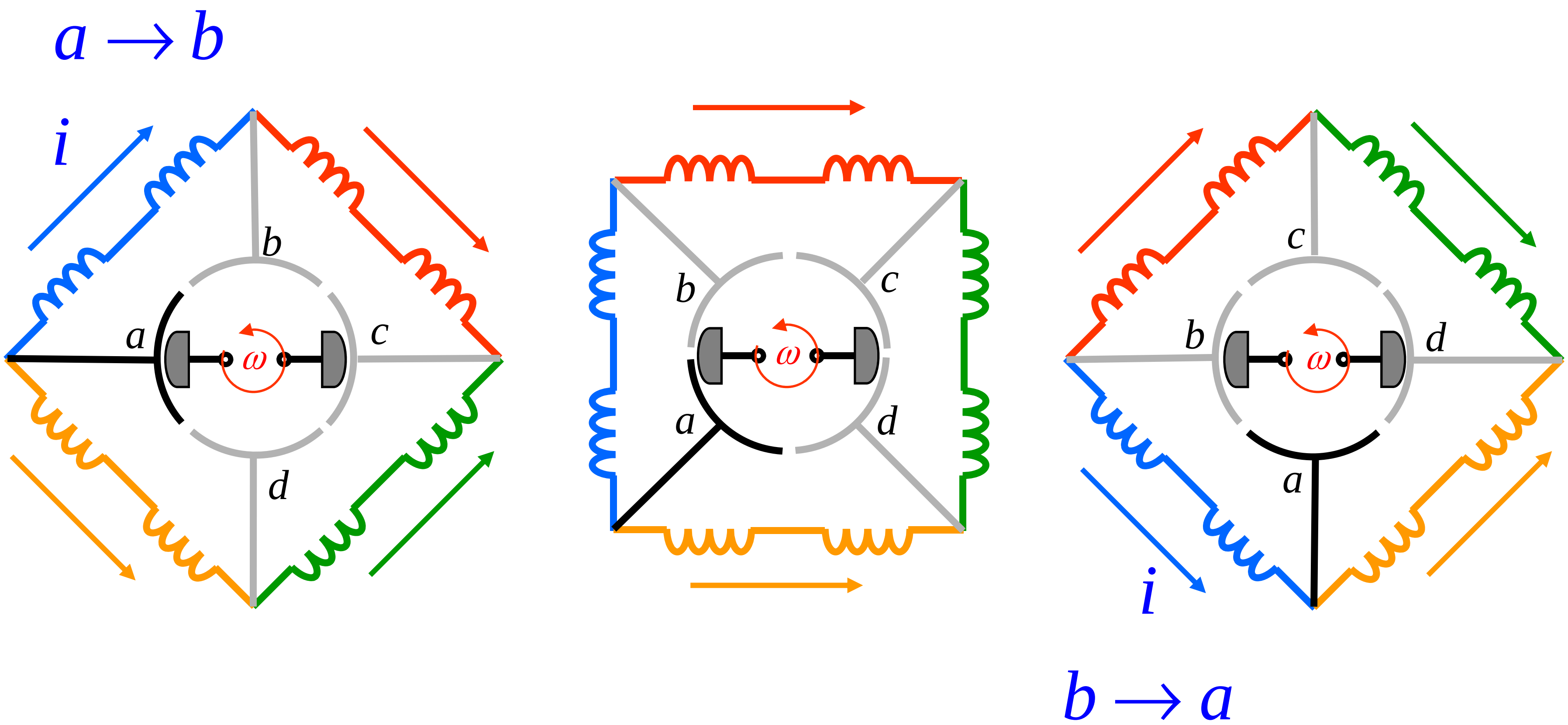
***Ldi/dt* Voltages**



Note that the direction of the currents in the blue and green segments has flipped. See next slide...

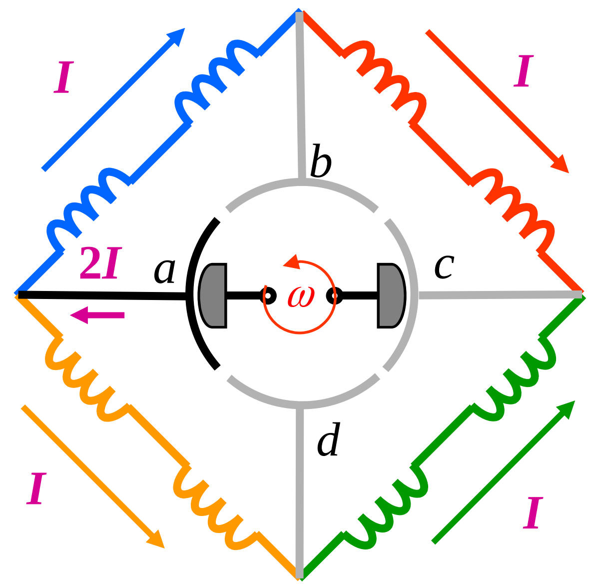
***Ldi/dt* Voltages**

$\omega t = 0°$ $\omega t = 45°$ $\omega t = 90°$



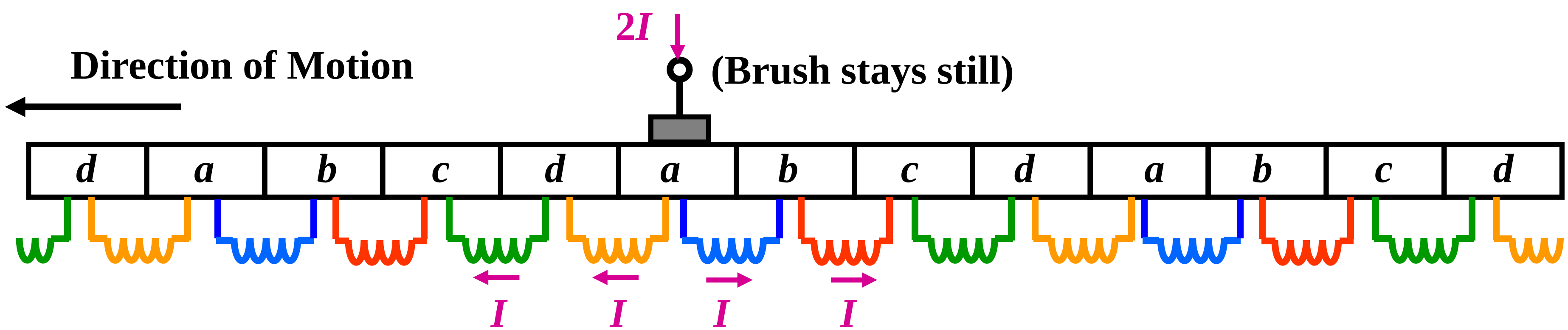
Let's 'unroll' this...

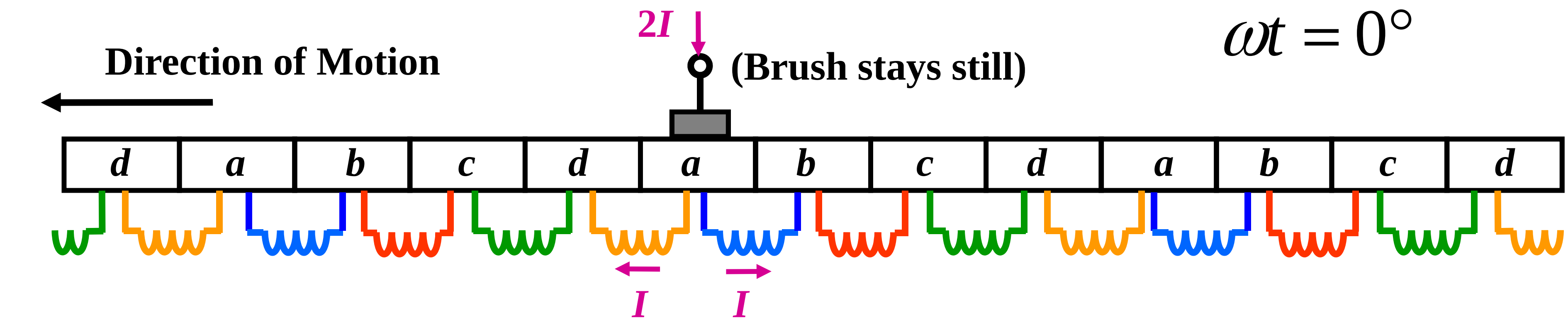
Ldi/dt Voltages

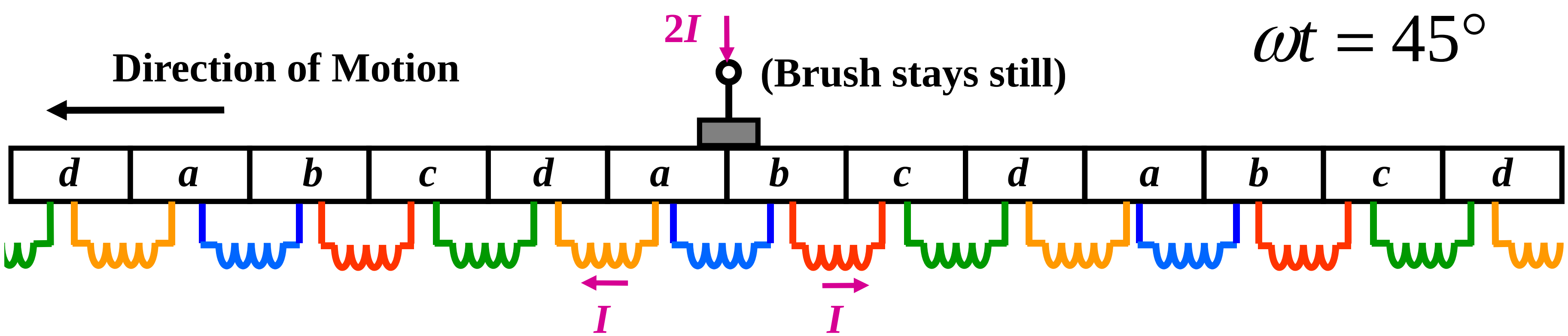


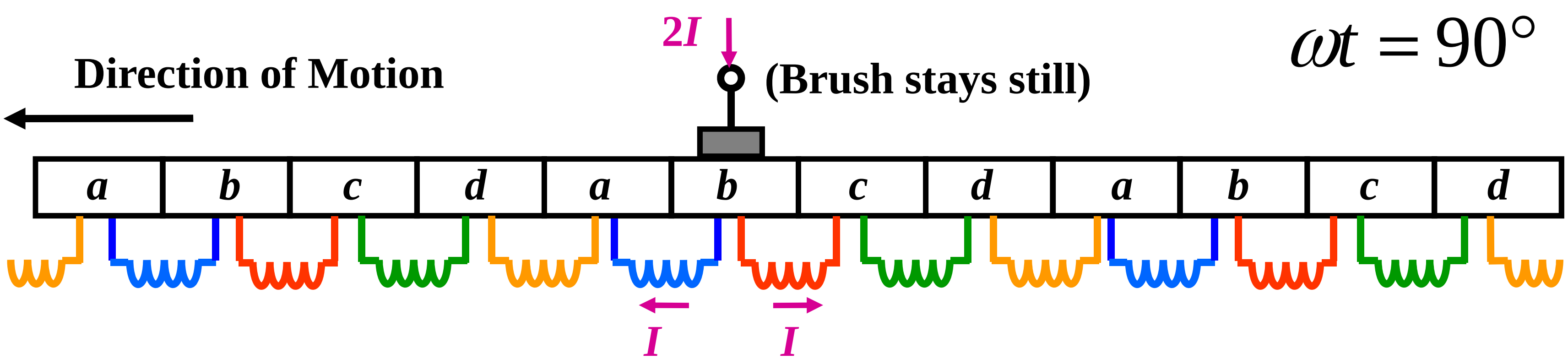
Let's 'unroll' this...

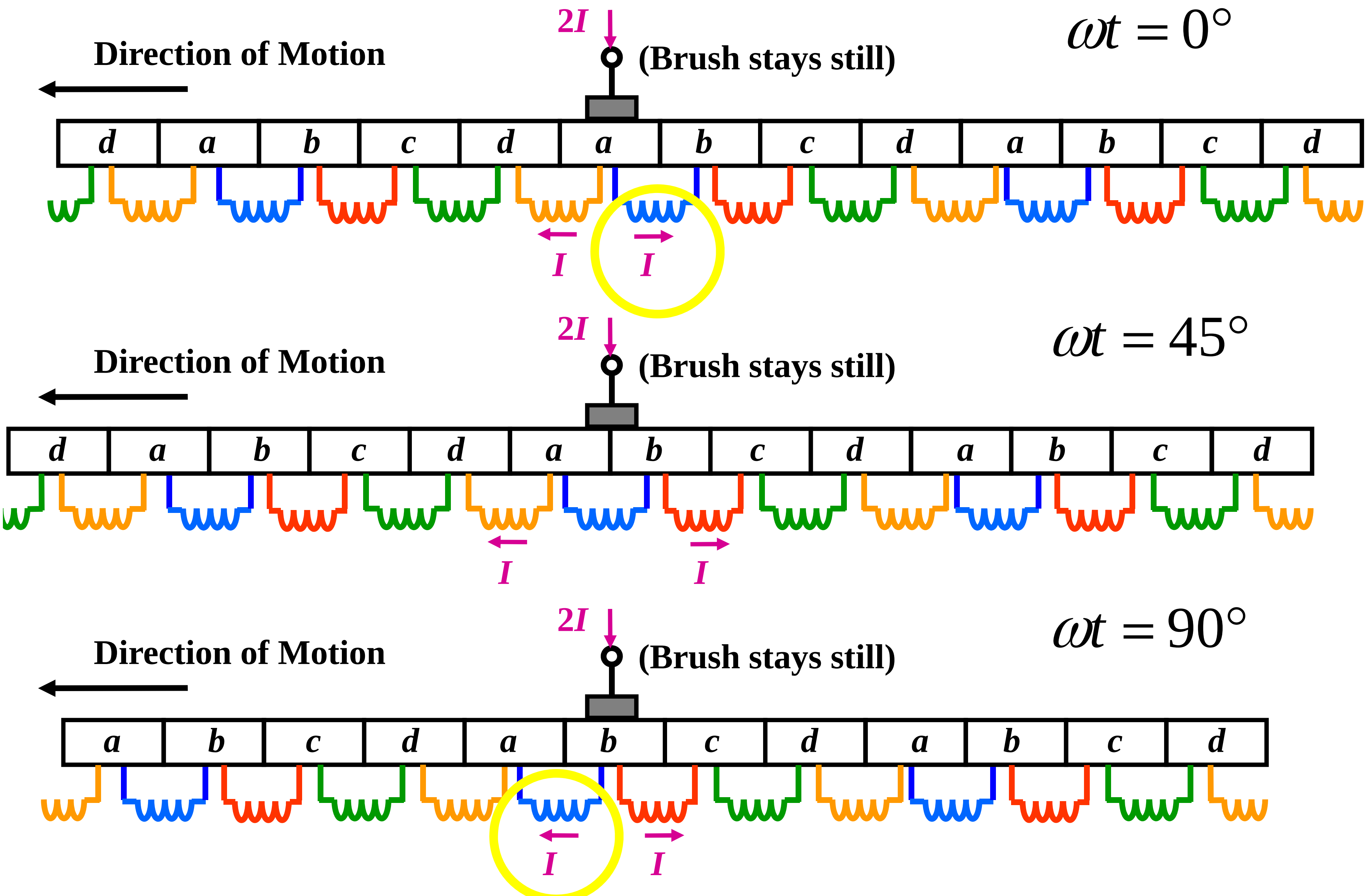
$$\omega t = 0°$$




2I
Direction of Motion
(Brush stays still)
ωt = 0°
d a b c d a b c d a b c d
I
I

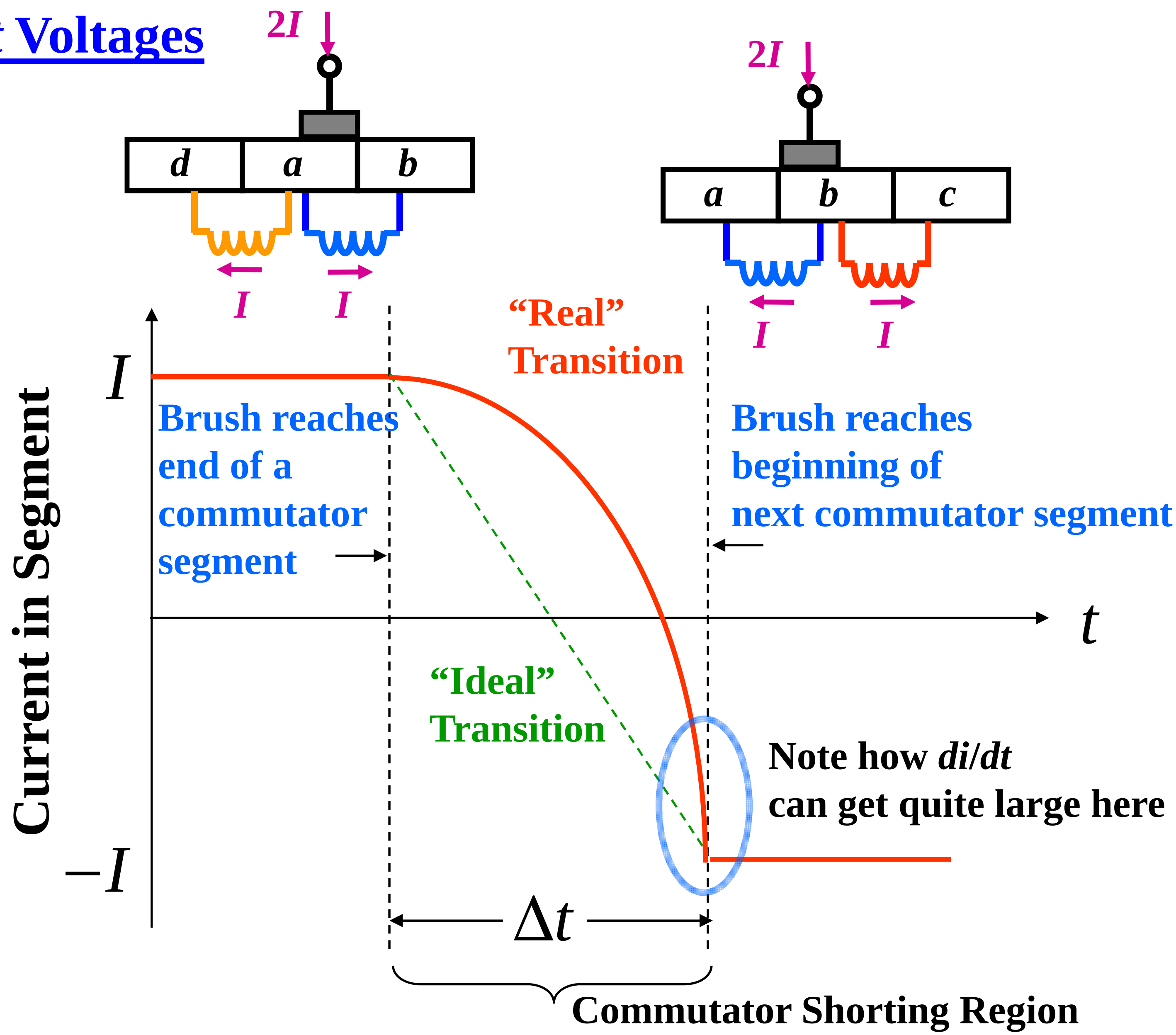

2I
Direction of Motion
(Brush stays still)
ωt = 45°
d a b c d a b c d a b c d
I
I


2I
Direction of Motion
(Brush stays still)
ωt = 90°
a b c d a b c d a b c d
I
I



2I
Direction of Motion
(Brush stays still)
$\omega t = 0°$
d a b c d a b c d a b c d
I
I
2I
Direction of Motion
(Brush stays still)
$\omega t = 45°$
d a b c d a b c d a b c d
I
I
2I
Direction of Motion
(Brush stays still)
$\omega t = 90°$
a b c d a b c d a b c d
I
I

Ldi/dt **Voltages**



***Ldi/dt* Voltages**

How much time elapsed while the current to changed directions?

Suppose the motor is turning at n_m rpm, and suppose that there are N commutator segments. Then,

$$n_m \frac{\text{rev}}{\text{min}}\text{rpm} \Rightarrow \frac{n_m}{60}\frac{\text{rev}}{\text{sec}} \Rightarrow \frac{60}{n_m}\frac{\text{sec}}{\text{rev}}$$

$$\Rightarrow \Delta t = \frac{1}{N}\frac{\text{rev}}{\text{segments}}\frac{60}{n_m}\frac{\text{sec}}{\text{rev}} = \frac{60}{Nn_m}\text{seconds}$$

***Ldi/dt* Voltages**

What voltage is induced? Take some reasonable numbers.

$$n_m = 1000 \text{ rpm}$$

$$N = 50$$

$$I = 100 \text{ amps}$$

$$\frac{dI}{dt} \sim \frac{\Delta I}{\Delta t} = \frac{2I}{60} N n_m = 166{,}667 \ \frac{\text{volts}}{\text{henry}}$$

Even with a small inductance, large induced voltages $\left(= L\frac{dI}{dt} \right)$ on the brushes can occur, and this is a problem.

Solutions to the Commutation Problem

There are two common techniques to fix (partially or completely) the problems associated with armature reaction and Ldi/dt voltages. These are,

1. *Commutating Poles* or *Interpoles*

2. *Compensating Windings*

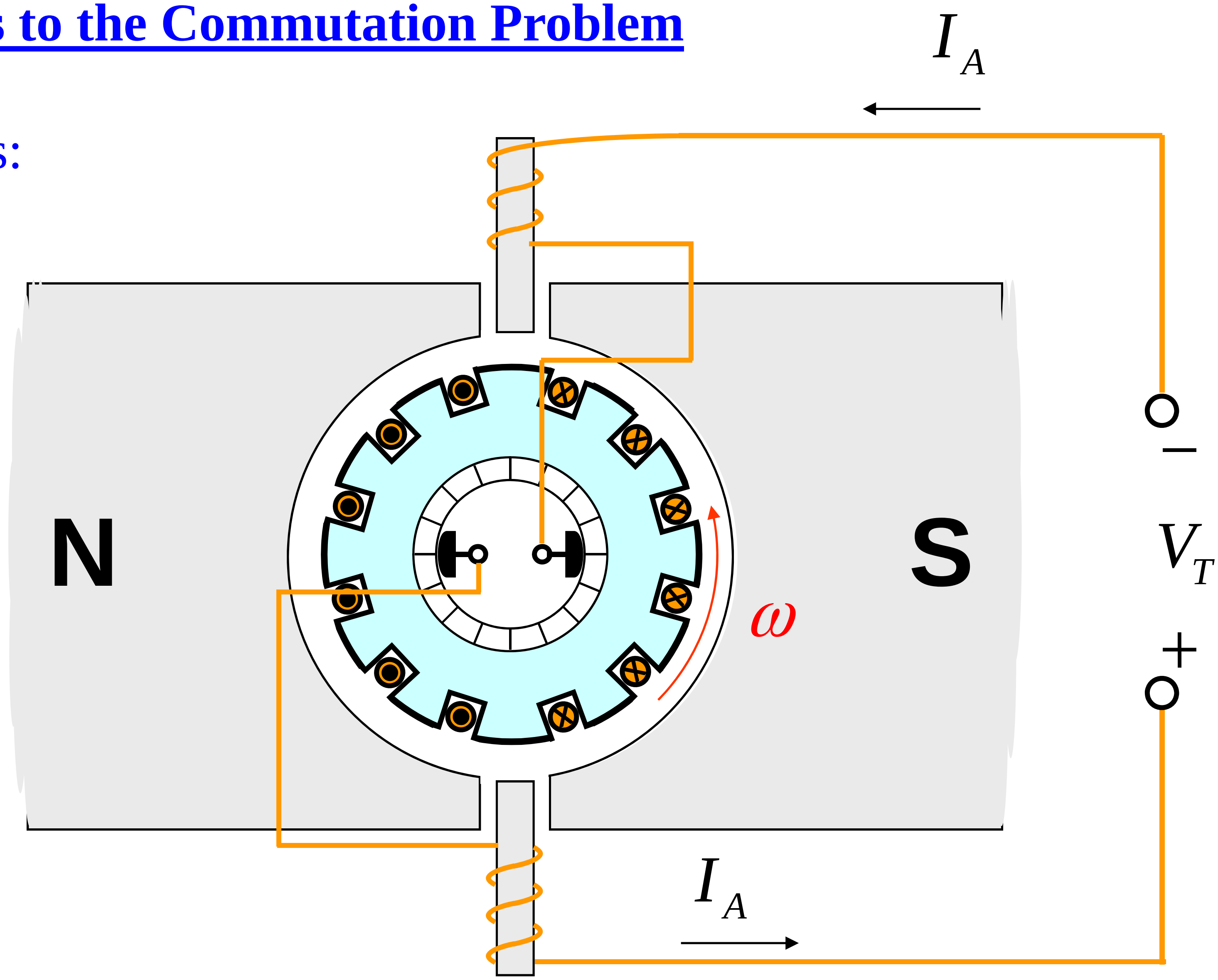
Solutions to the Commutation Problem

Commutating Poles – Basic idea: If the voltage in the wires undergoing commutation can be made zero, no sparking will occur.

To accomplish this, small poles are placed midway between the main poles as seen on the next slide...

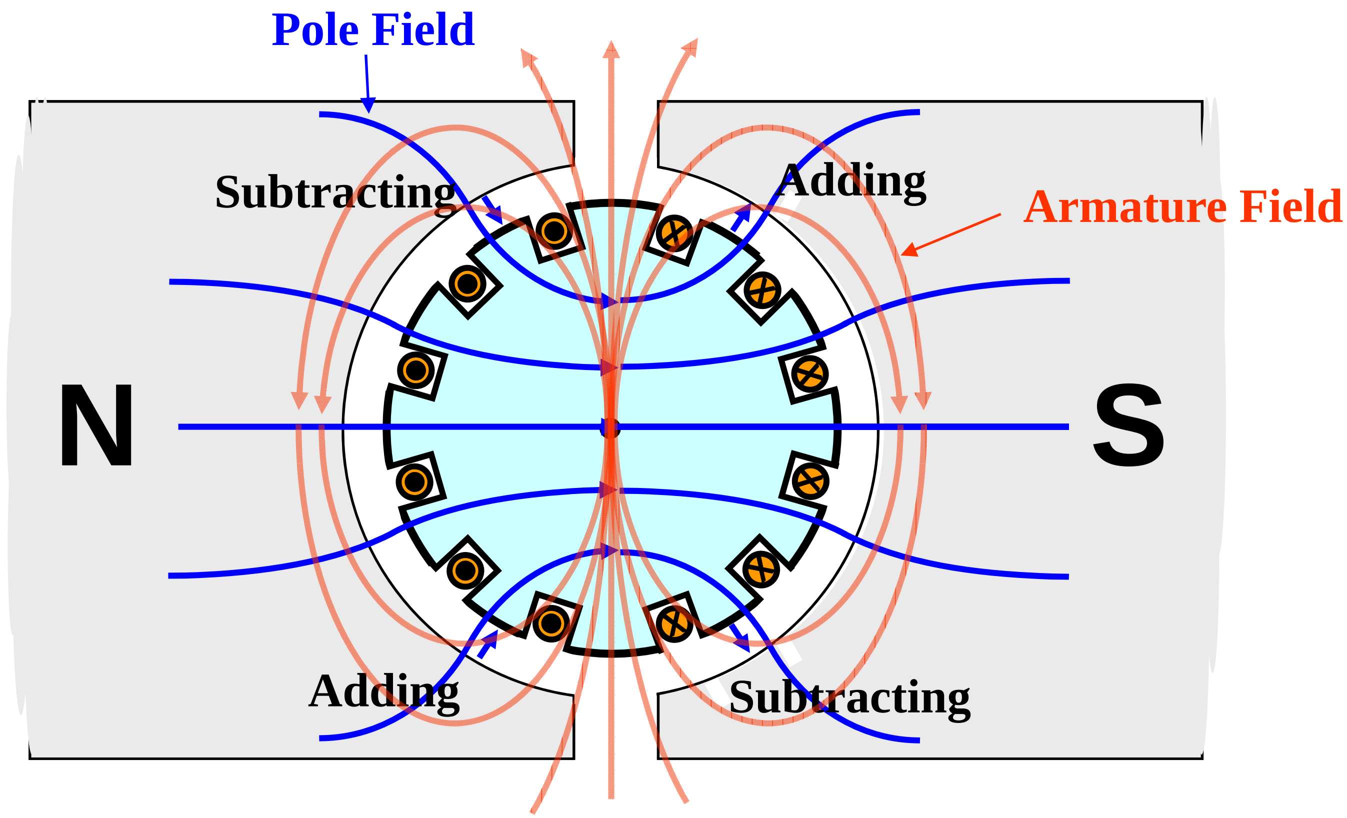
Solutions to the Commutation Problem

Interpoles:



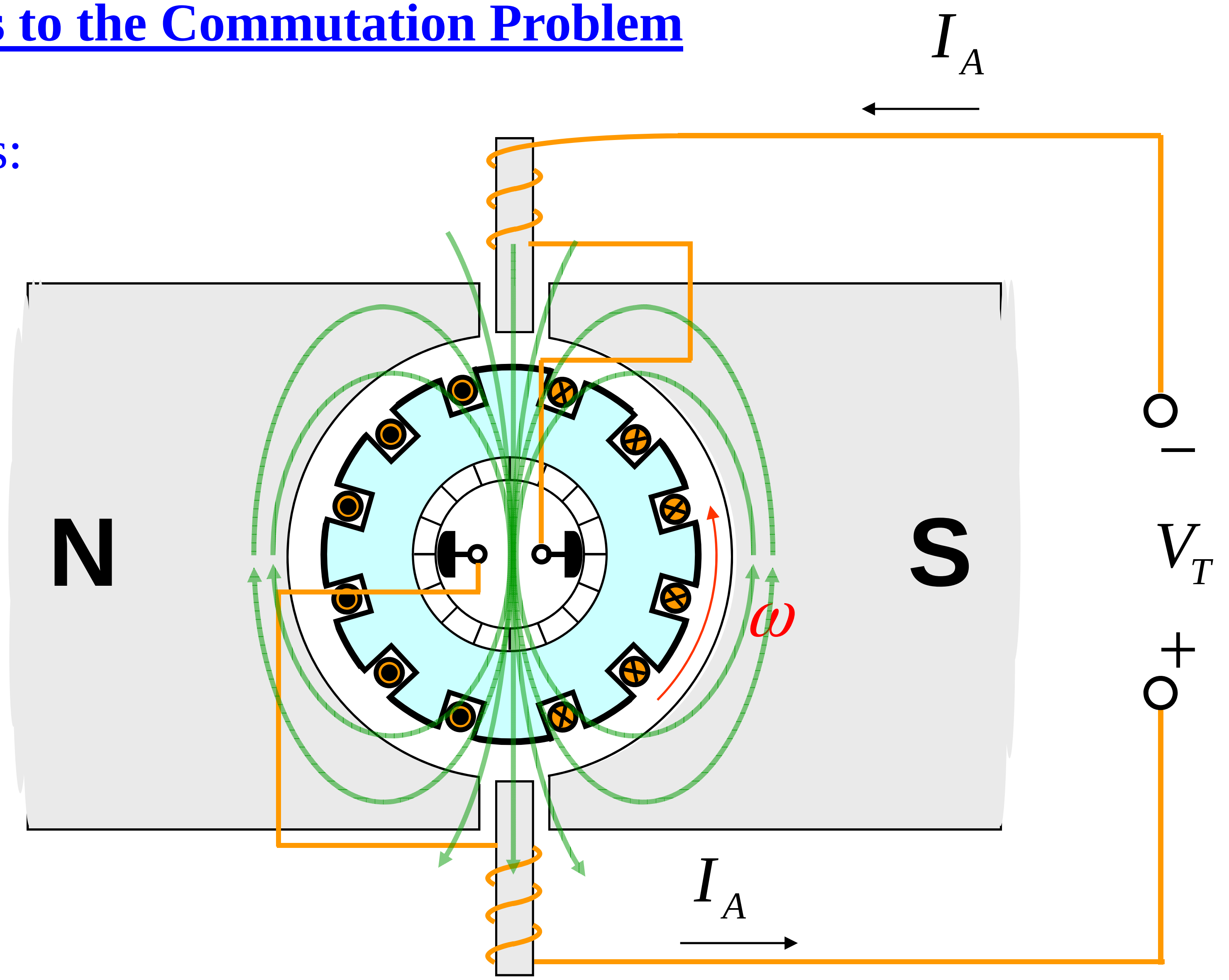
Solutions to the Commutation Problem

Q: What caused the problem? A: The armature field.



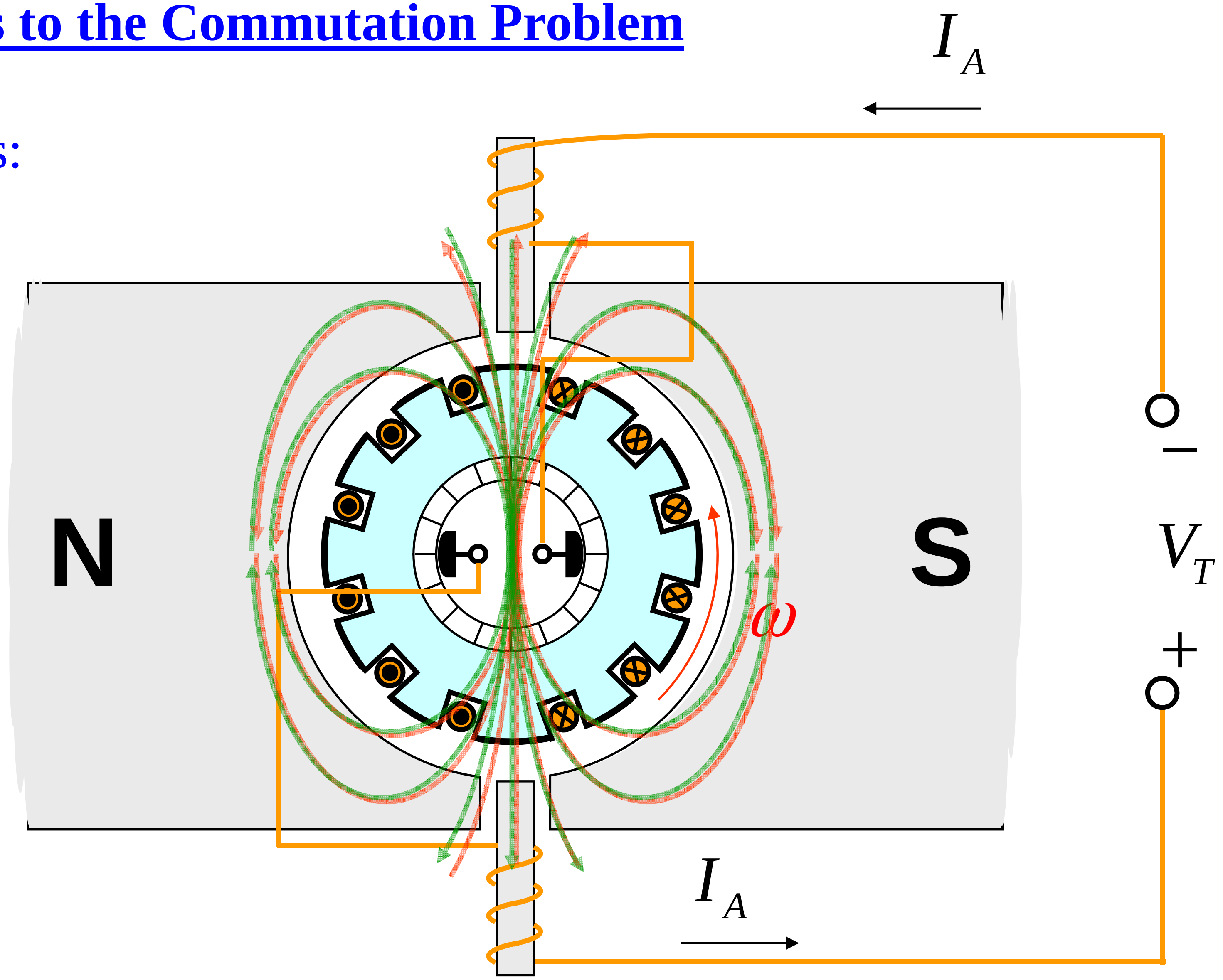
Solutions to the Commutation Problem

Interpoles:



Solutions to the Commutation Problem

Interpoles:



Solutions to the Commutation Problem

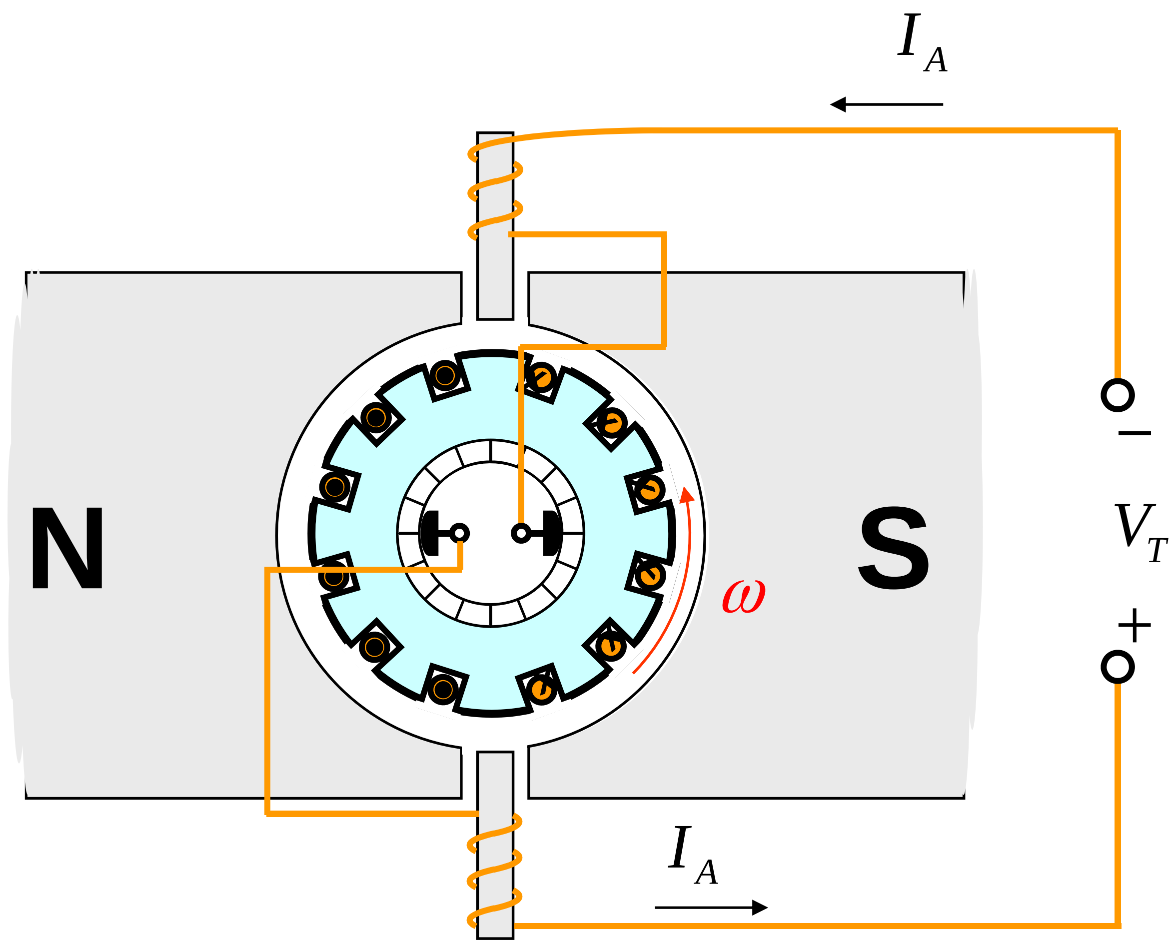
Commutating Poles – The commutating poles are placed directly over the conductors being commutated. These poles provide a flux that (ideally) exactly cancels the effect of that portion of the armature reaction which caused problems.

The commutating poles are so small that they effect only the few conductors about to undergo commutation.

Note that the armature reaction under the main pole faces are unaffected since the effects of the commutating poles do not extend that far.

Solutions to the Commutation Problem

Commutating Poles – Note that the commutating poles or interpoles are connected in series with the rotor winding.



Solutions to the Commutation Problem

Commutating Poles – As the load increases and the rotor current increases, the amount of shift in the neutral plane and the magnitude of the *Ldi*/*dt* voltage increases as well. However, the interpole flux also increases, producing a proportional voltage in the conductors that opposes the voltage causing the neutral plane shift.

The net result is a cancellation that is effective over a broad range of loads.

Solutions to the Commutation Problem

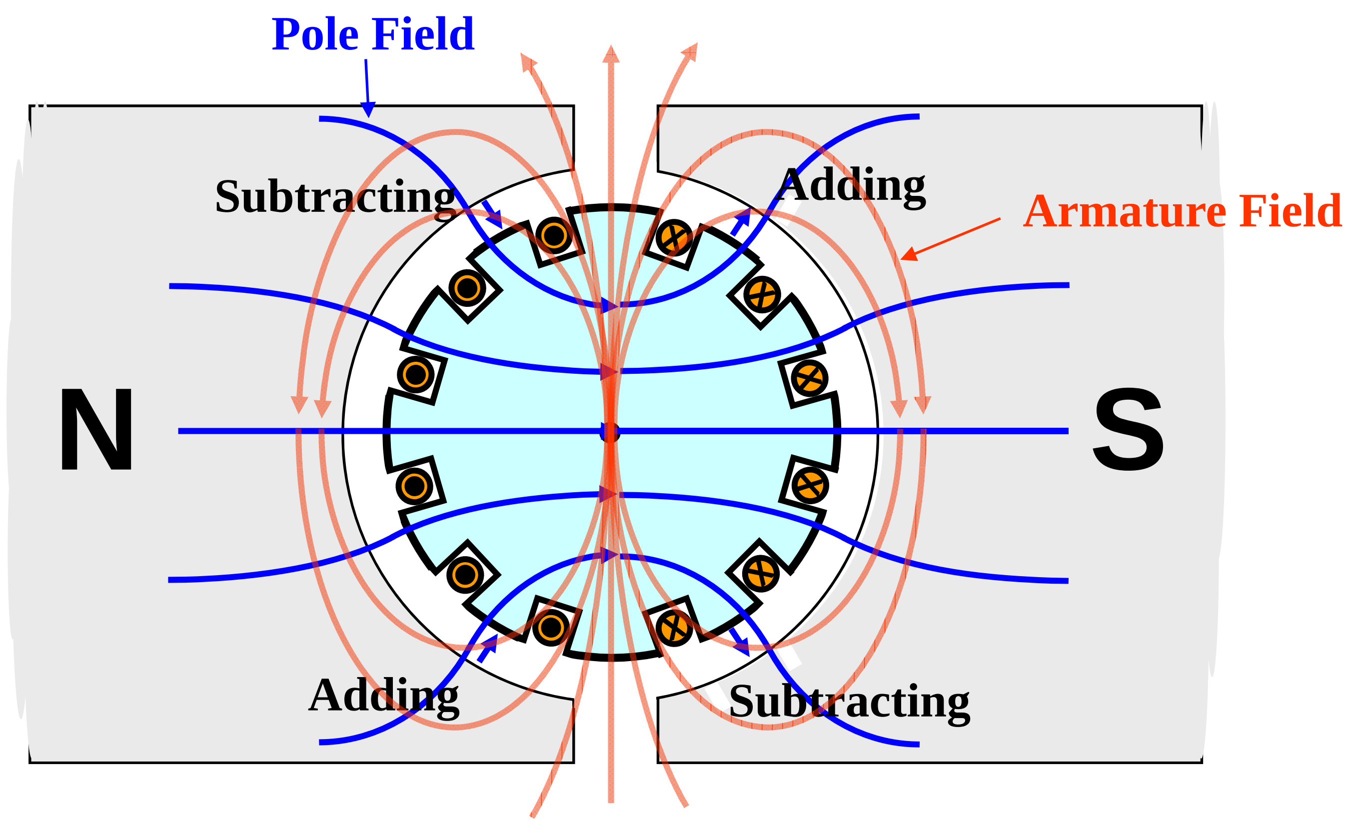
Compensating Windings: Can completely cancel armature reaction and eliminate neutral plane shift and flux weakening.

Compensating windings are windings placed in slots carved in the faces of the poles parallel to the rotor conductors.

These windings are connected is series with the rotor windings.

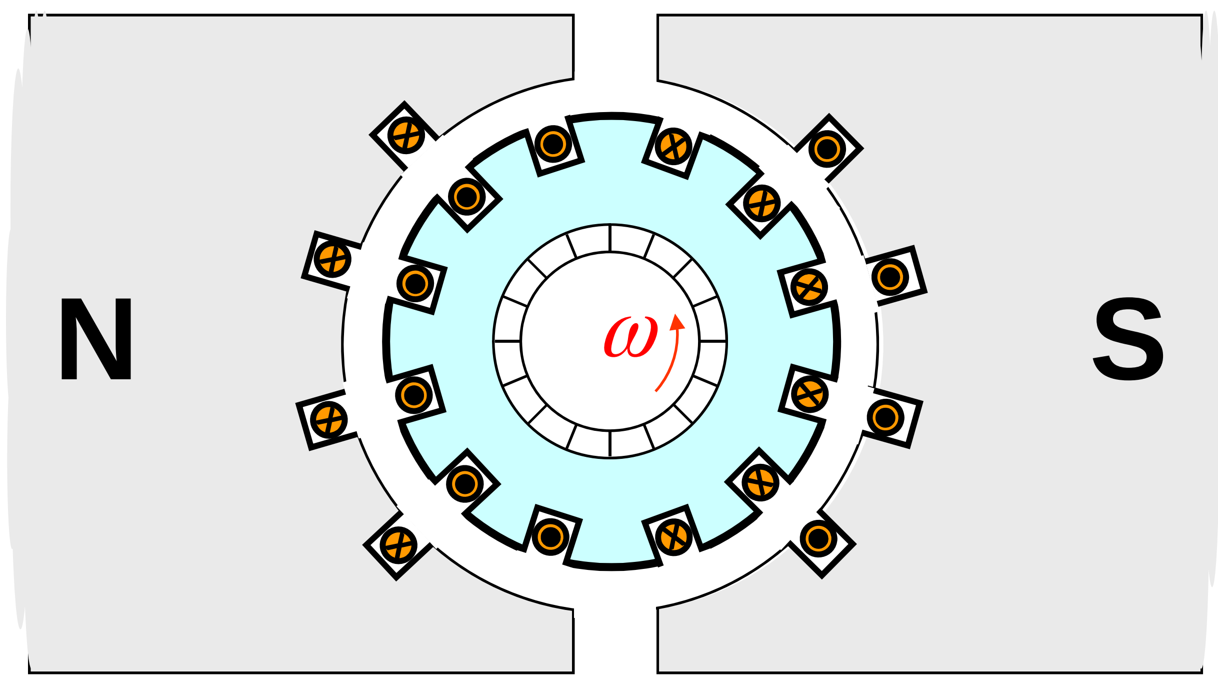
Solutions to the Commutation Problem

Q: What caused the problem? A: The armature field.



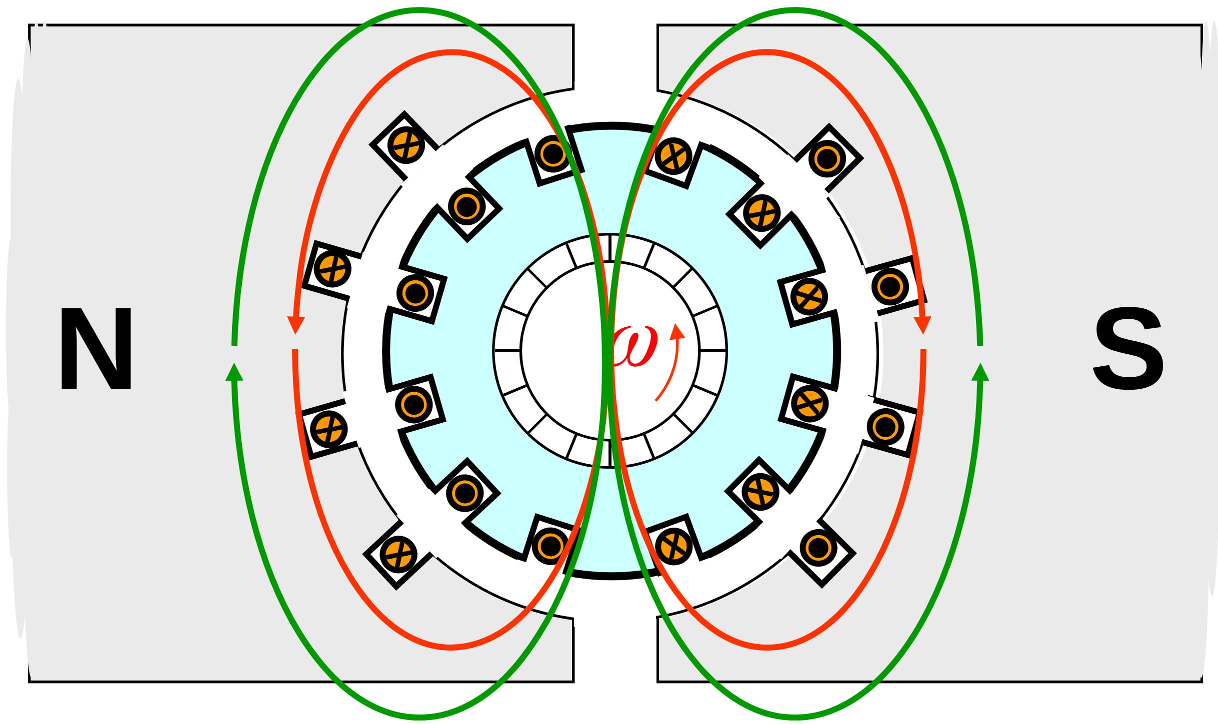
Solutions to the Commutation Problem

Compensating Windings:



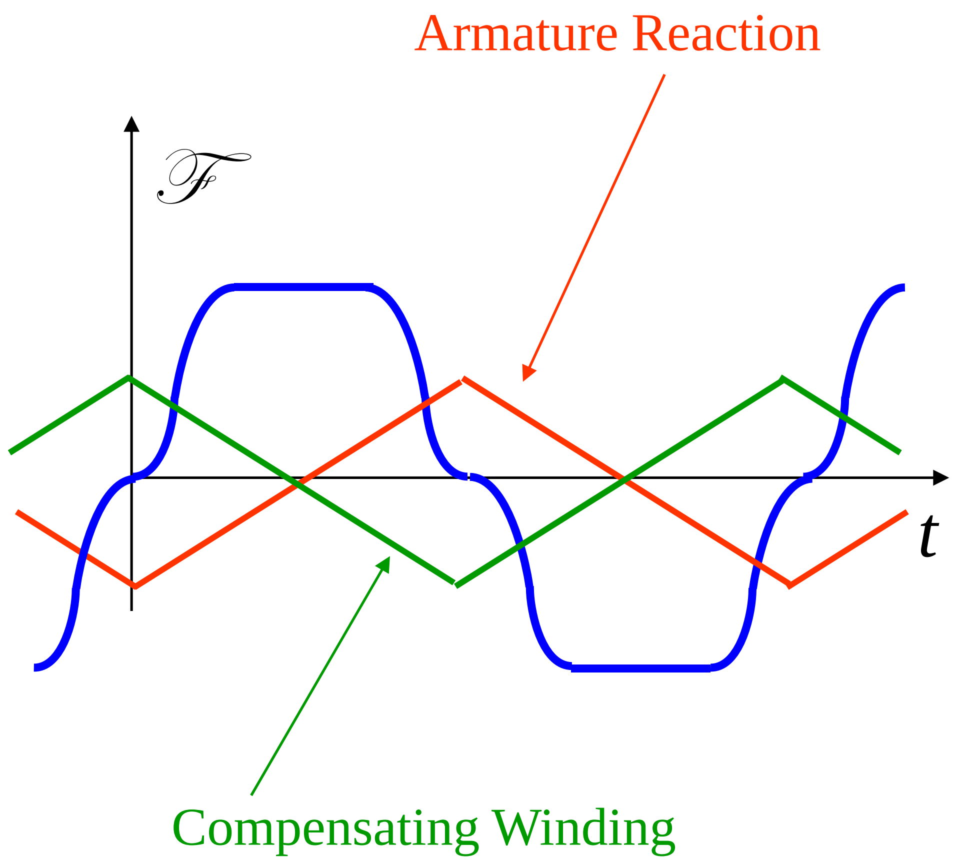
Solutions to the Commutation Problem

Compensating Windings:



Solutions to the Commutation Problem

For both cases:



Solutions to the Commutation Problem

Compensating Windings: Motors with compensating winding are more expensive to make.

Commutating poles or interpoles must still be used since compensating windings do not cancel *di*/*dt* effects though the interpoles can be smaller than before since they are only needed to cancel *di*/*dt* effects and not neutral plane shifts.

Because of the expense and complexity, dc motors that use compensating windings are used only in large motors when extreme load variations occur.

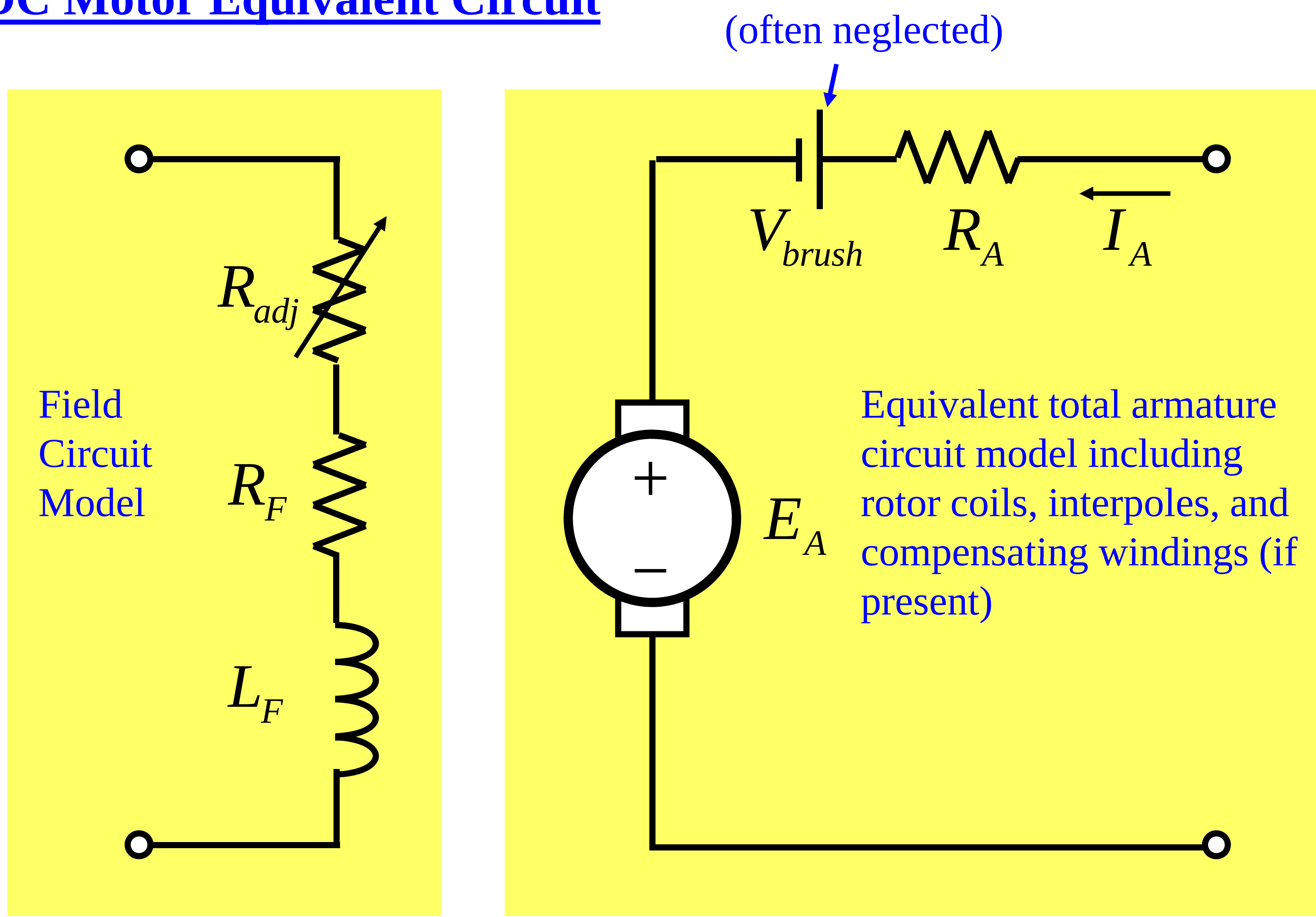
Types of DC Motors

There are five major types:

1. Separately excited DC motor.
2. Shunt DC motor
3. Permanent magnet DC motor
4. Series DC motor
5. Compounded DC motor

DC motors are assumed to be driven from a DC source whose voltage is assumed constant.

DC Motor Equivalent Circuit



DC Motor Equivalent Circuit

Recall from Slides 15 and 21 that:

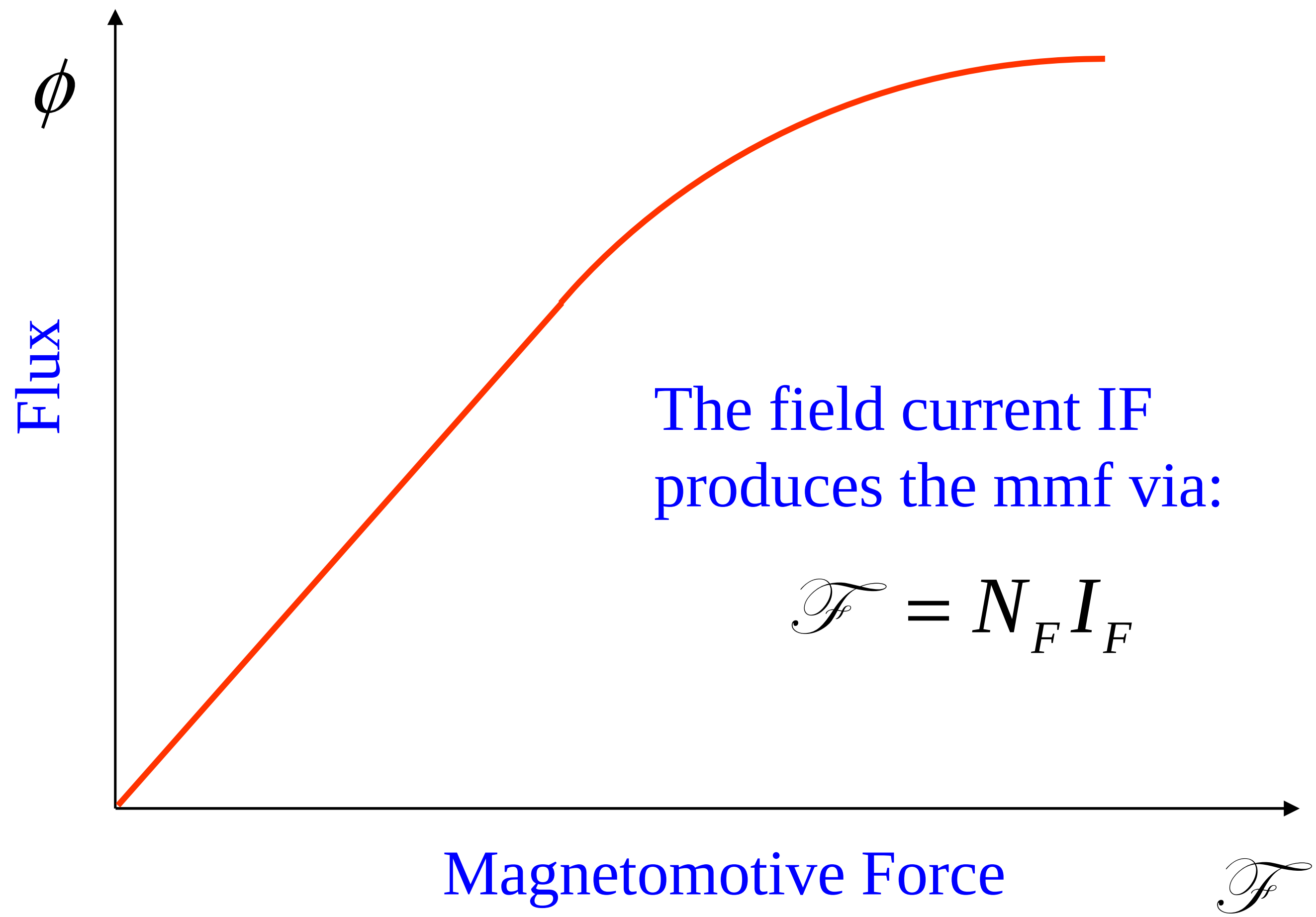
$$E_A = K\phi\omega$$

$$\tau_{induced} = K\phi I_A$$

Where K is a constant that depends on the machine construction. (For the case derived earlier, $K = \pi/2$).

These equations, the model, and the motor's magnetization curves are all that is needed to analyze a DC motor.

DC Motor Equivalent Circuit



The field current IF produces the mmf via:

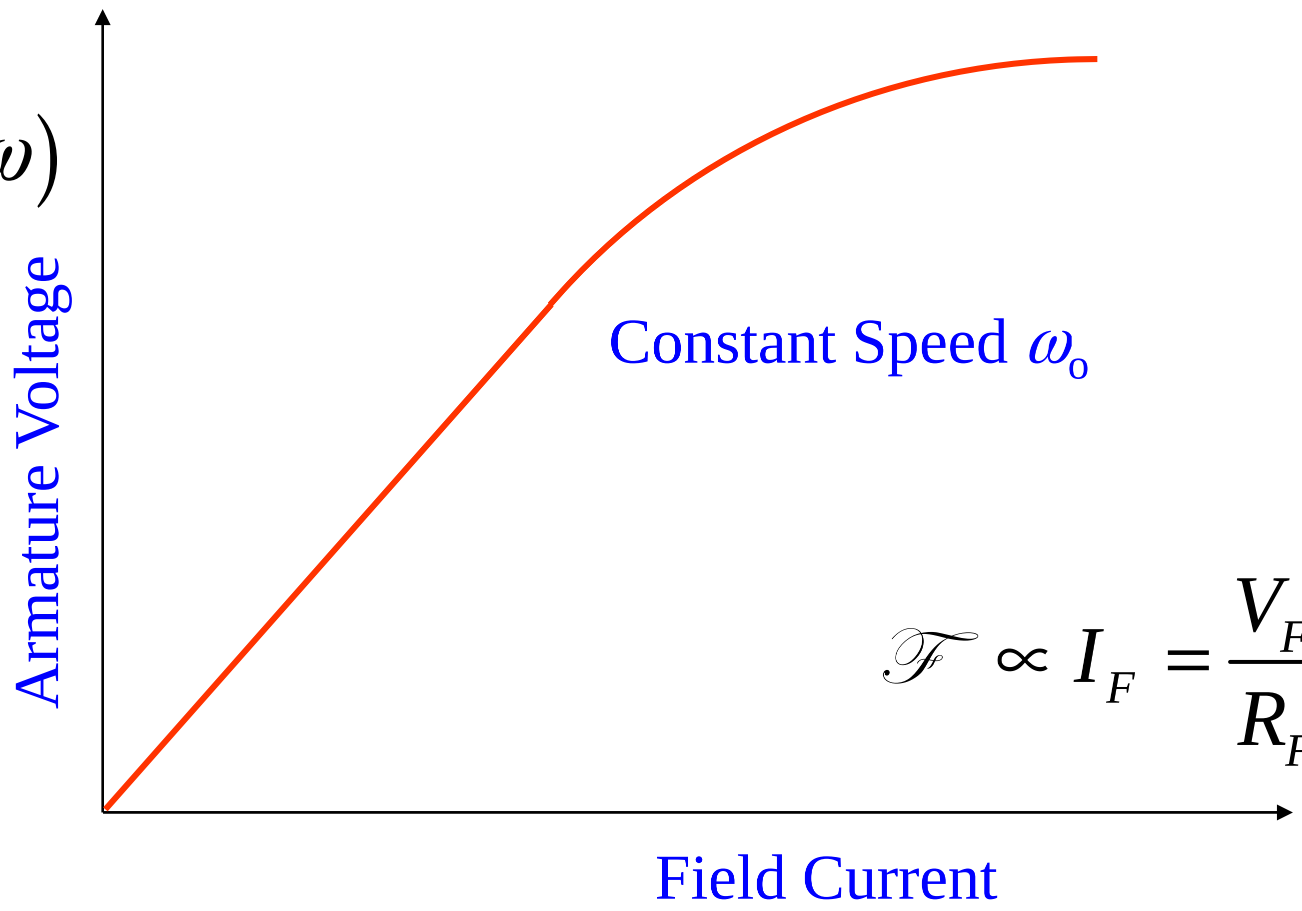
$$\mathscr{F} = N_F I_F$$

DC Motor Equivalent Circuit

Typical Curves:

$$\phi \propto E_A$$

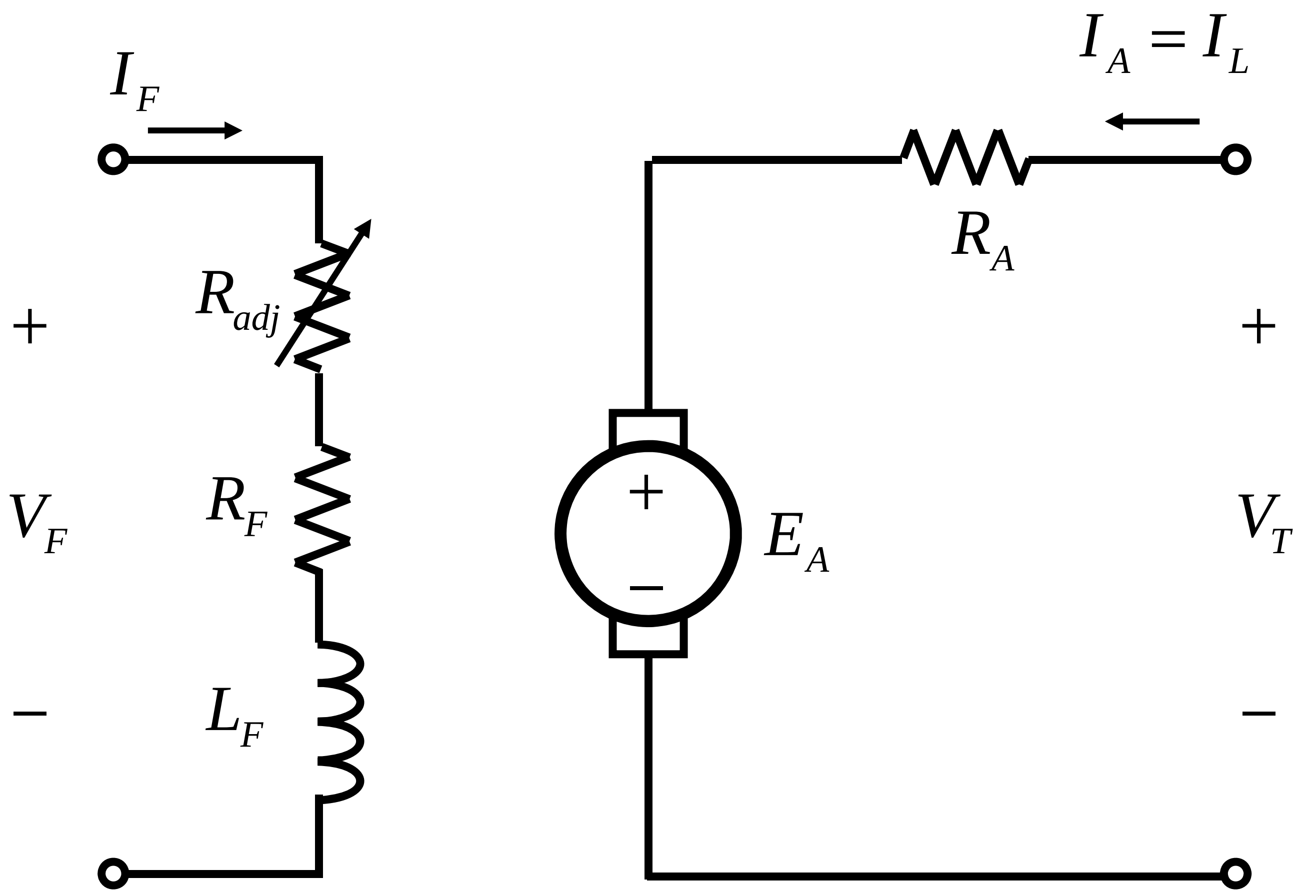
$$\left(E_A = K\phi\omega \right)$$



$$\mathscr{F} \propto I_F = \frac{V_F}{R_F}$$

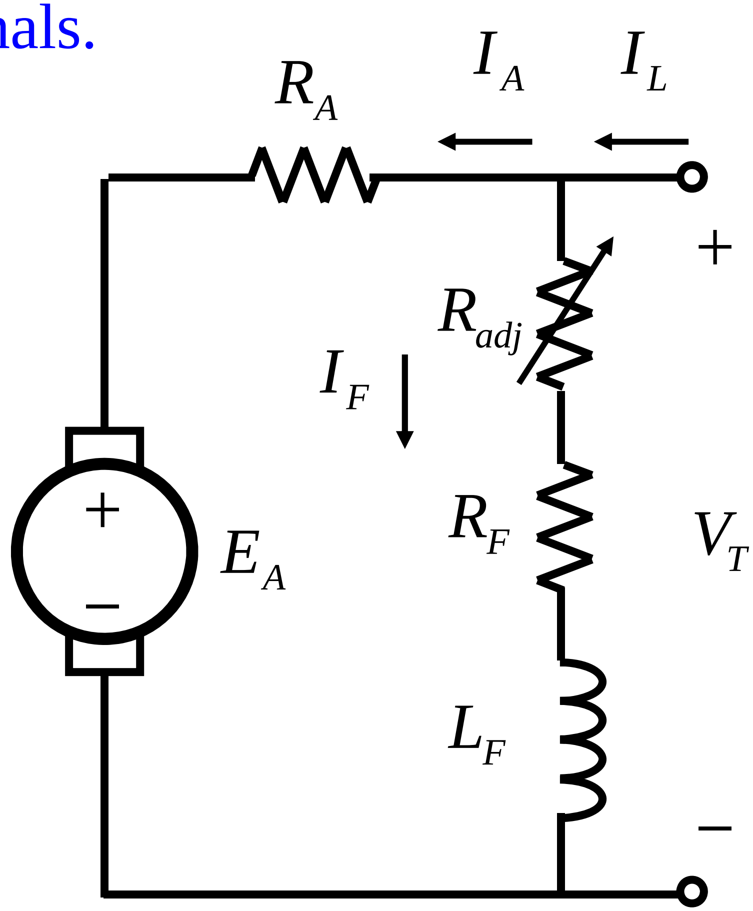
DC Motor Equivalent Circuit

Separately Excited DC Motor – one whose field current is supplied from a separate constant-voltage power supply.



DC Motor Equivalent Circuit

Shunt DC Motor – one whose field current is supplied from the armature terminals.



DC Motor Equivalent Circuit

There is no practical difference between a separately excited and a shunt DC motor.

How does a shunt DC motor respond to a load?

Shunt DC Motor

Suppose the load on the shaft of a shunt motor is increased.

Then the load torque will exceed the induced torque.

The motor will then start to slow down.

When it slows down, the internally generated voltage drops,

$$E_A = K\phi\omega$$

and the armature current increases, since

$$I_A = \frac{V_T - E_A}{R_A}$$

Shunt DC Motor

As the armature current increases, the induced torque increases,

$$\tau_{induced} = K\phi I_A$$

Finally, the induced torque will equal the load torque at a reduced speed of rotation.

The output characteristic (torque and speed) of a motor can be expressed as follows:

Shunt DC Motor

From the model, using $E_A = K\phi\omega$

$$
\begin{aligned}
V_T &= E_A + I_A R_A \\
&= K\phi\omega + I_A R_A
\end{aligned}
$$

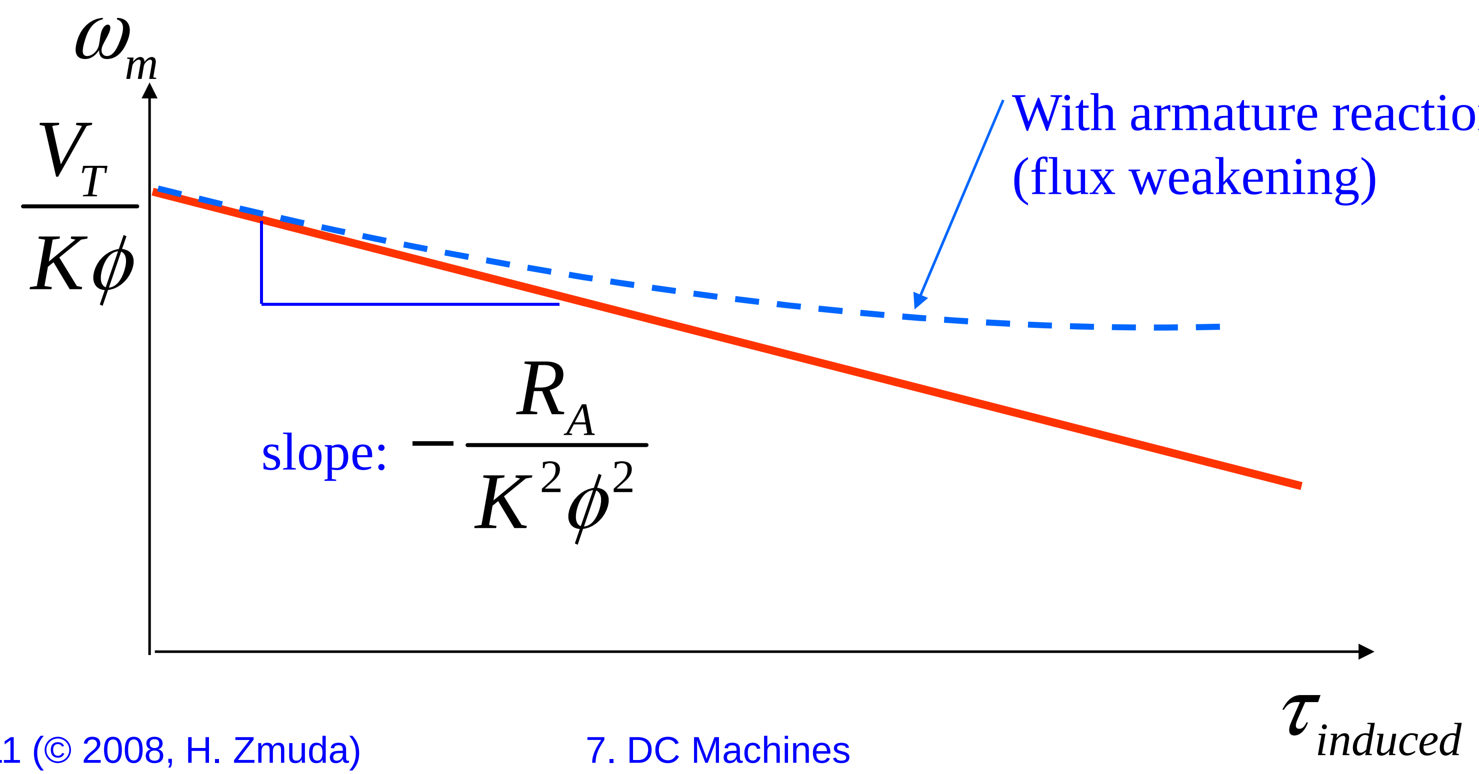
but $\tau_{induced} = K\phi I_A \Rightarrow I_A = \dfrac{\tau_{induced}}{K\phi}$

thus, $V_T = K\phi\omega + \dfrac{\tau_{induced}}{K\phi} R_A$

Shunt DC Motor

Solving for the motor's speed ω,

$$\omega_m = \frac{V_T}{K\phi} - \frac{R_A}{K^2\phi^2}\tau_{induced}$$



Shunt DC Motor Speed Control

The speed of a shunt DC motor is controlled in one of two ways:

1. – Adjusting the flux by adjusting the field resistance

2. – Adjusting the terminal voltage applied to the armature.

Shunt DC Motor Speed Control

Suppose the field resistance R_F is increased.

The field current decreases and so does the flux ϕ.

A decrease in flux causes a decrease in the internally generated voltage E_A, since

$$E_A = K\phi\omega$$

This causes a (large*) increase in the machine's armature current,

$$I_A = \frac{V_T - E_A}{R_A}$$

*Recall that since we are assuming that we are operating near saturation, a small change in flux can cause a large change in current.

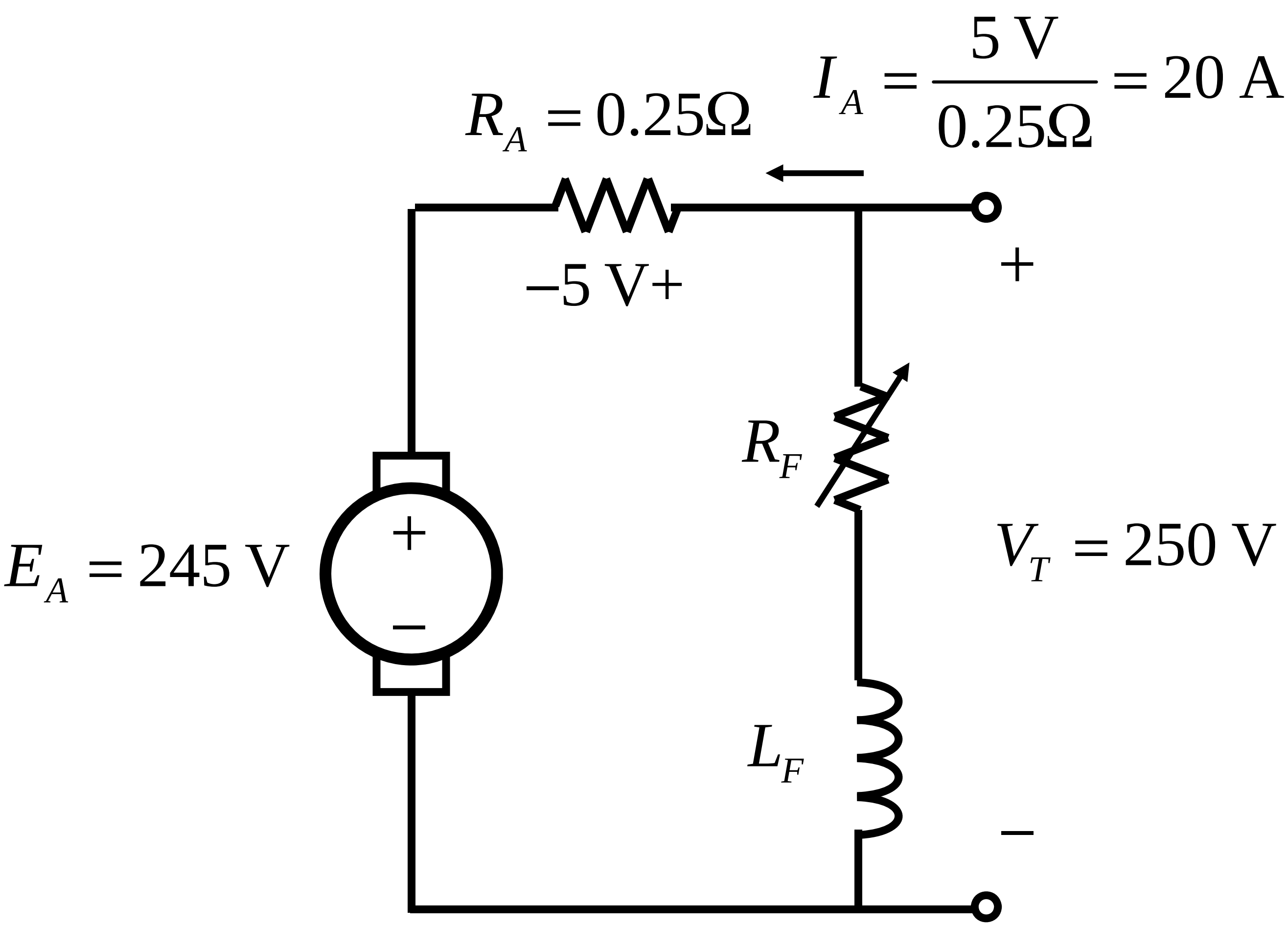
Shunt DC Motor Speed Control

Since $\tau_{induced} = K\phi I_A$

if I_A increases and ϕ decreases, what does he induced torque do?

Look at some typical model parameters:

Shunt DC Motor Speed Control



Shunt DC Motor Speed Control

Now suppose that 1% decrease in flux. Since $E_A = K\phi\omega$
E_A will also decrease by 1%.

$$E_A = 0.99 \cdot 245 = 242.55$$

The new armature current is

$$I_A = \frac{250 - 242.55}{0.25} = 29.8$$

A 1% decrease in flux produced a 49% increase in armature current.

Shunt DC Motor Speed Control

Since

$$\tau_{induced} = K\phi I_A$$

if I_A increases and ϕ decreases, we see that the torque increases, and so the motor speed up.

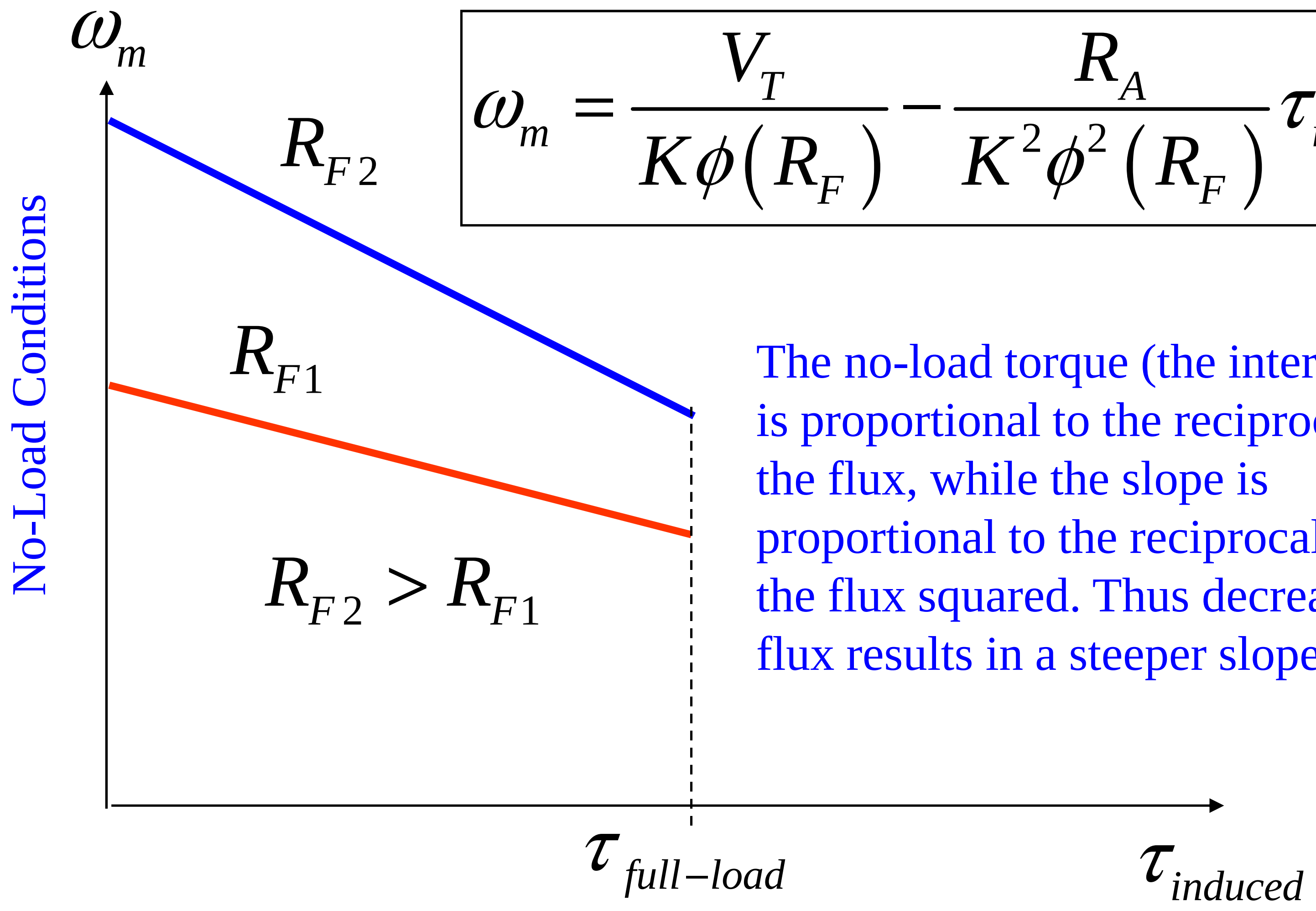
As the motor speed up the internally generated voltage E_A rises causing I_A to decrease.

As I_A decreases, so does the induced torque.

This continues until $\tau_{induced} = \tau_{load}$ but at a higher steady-state speed.

Shunt DC Motor Speed Control

$$\omega_m = \frac{V_T}{K\phi(R_F)} - \frac{R_A}{K^2\phi^2(R_F)}\tau_{induced}$$

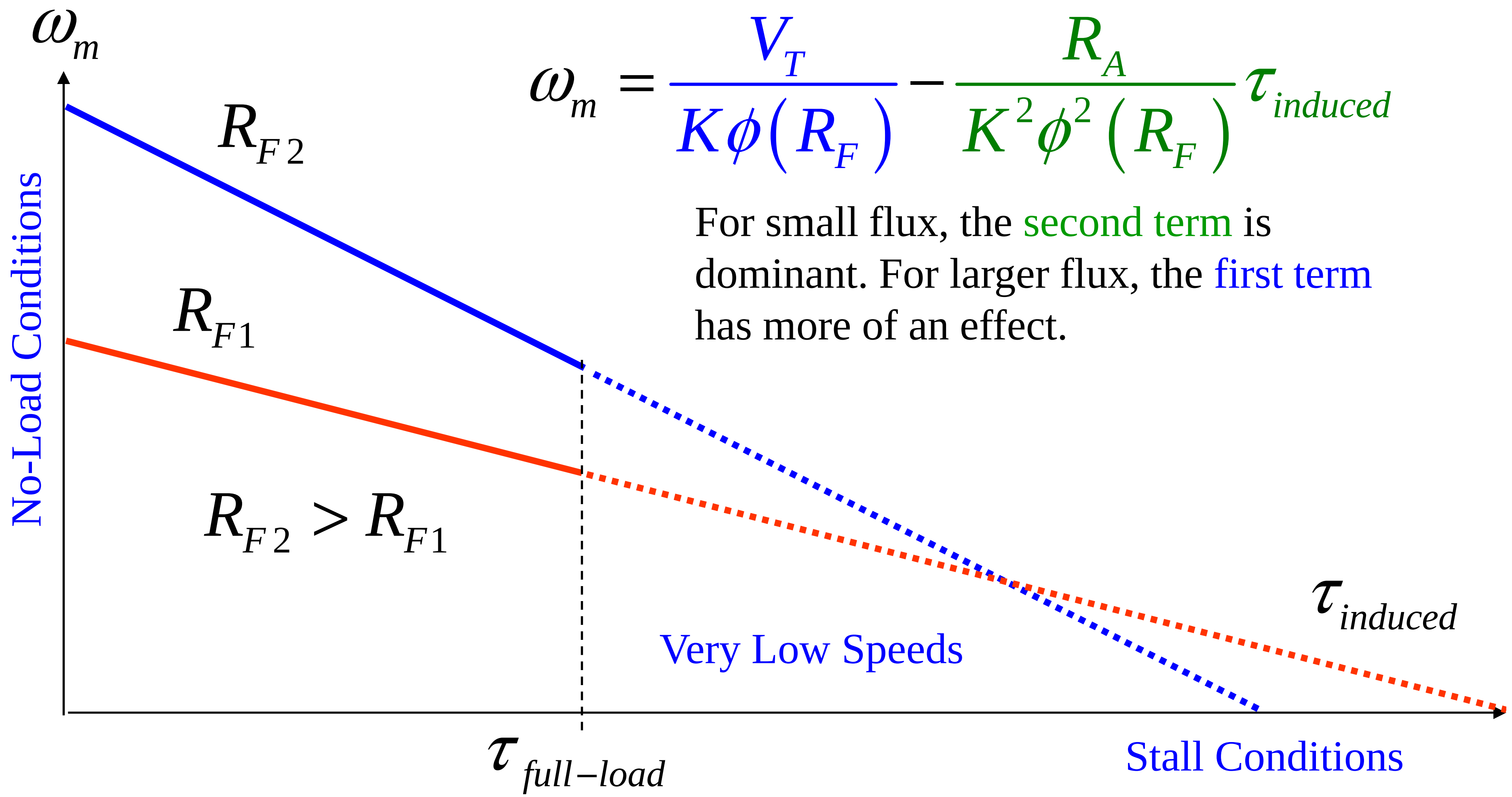


The no-load torque (the intercept) is proportional to the reciprocal of the flux, while the slope is proportional to the reciprocal of the flux squared. Thus decrease in flux results in a steeper slope.

Shunt DC Motor Speed Control

$$\omega_m = \frac{V_T}{K\phi(R_F)} - \frac{R_A}{K^2\phi^2(R_F)}\tau_{induced}$$

For small flux, the second term is dominant. For larger flux, the first term has more of an effect.



Shunt DC Motor Speed Control

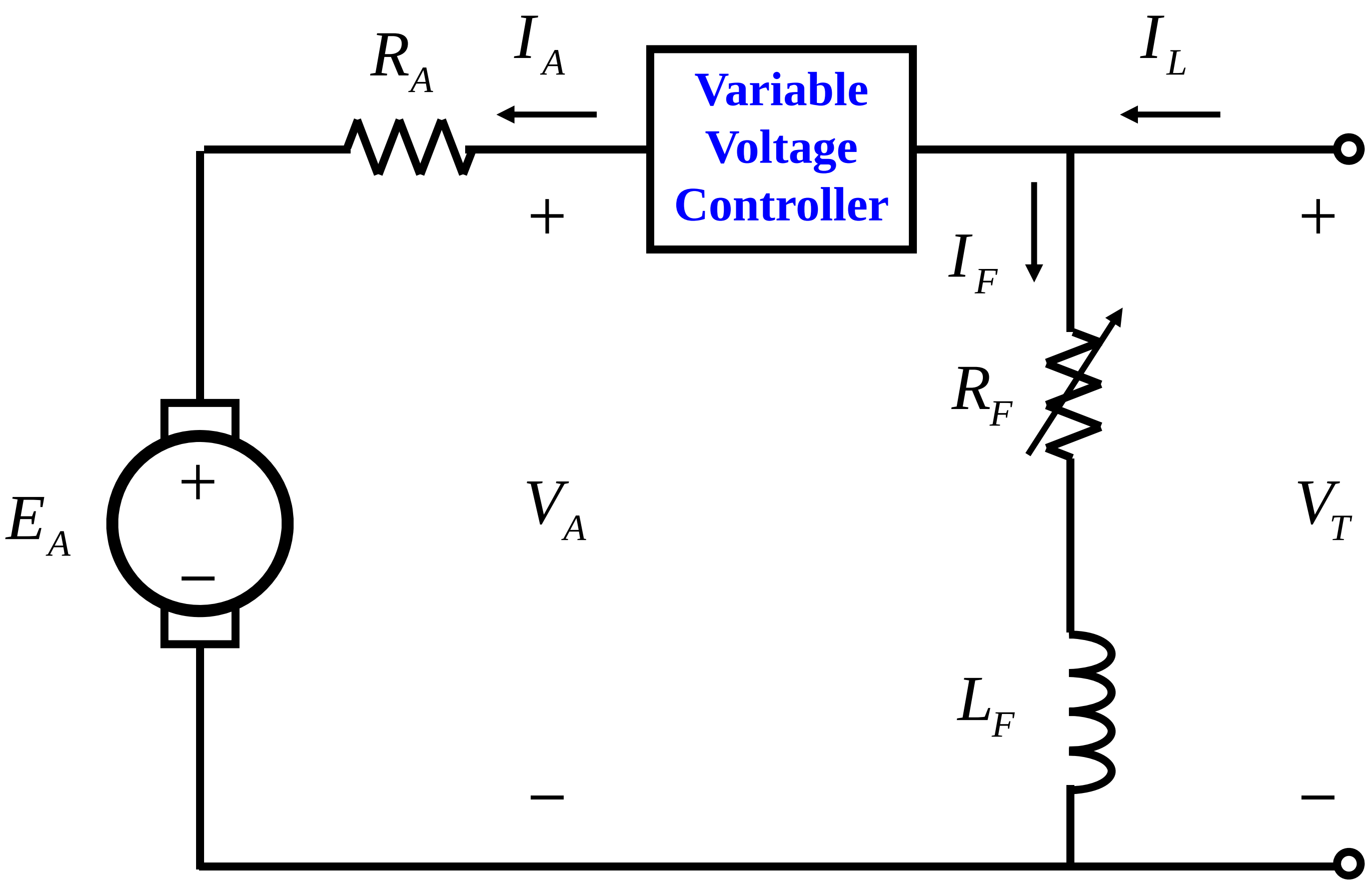
At very slow speeds, an increase in the field resistance can actually decrease the speed of the motor.

This is because at very low speeds the increase in armature current caused by the decrease in E_A is no longer large enough to compensate for the decrease in flux in the induced torque equation.

When the flux increase is larger than the armature current increase, the induced torque decreases, and the motor slows down.

If very low speeds are anticipated, this type of speed control should not be used. Instead, the armature voltage control method should be used.

Shunt DC Motor Speed Control - Armature Voltage Control – Controls the motor speed by changing the armature voltage *without* changing the voltage applied to the field circuit. In effect, this is a separately excited motor.



Shunt DC Motor Speed Control - Armature Voltage Control –

As V_A is increased, so is I_A, $I_A = \dfrac{V_A - E_A}{R_A}$

As I_A increases so does the induced torque, $\tau_{induced} = K\phi I_A$

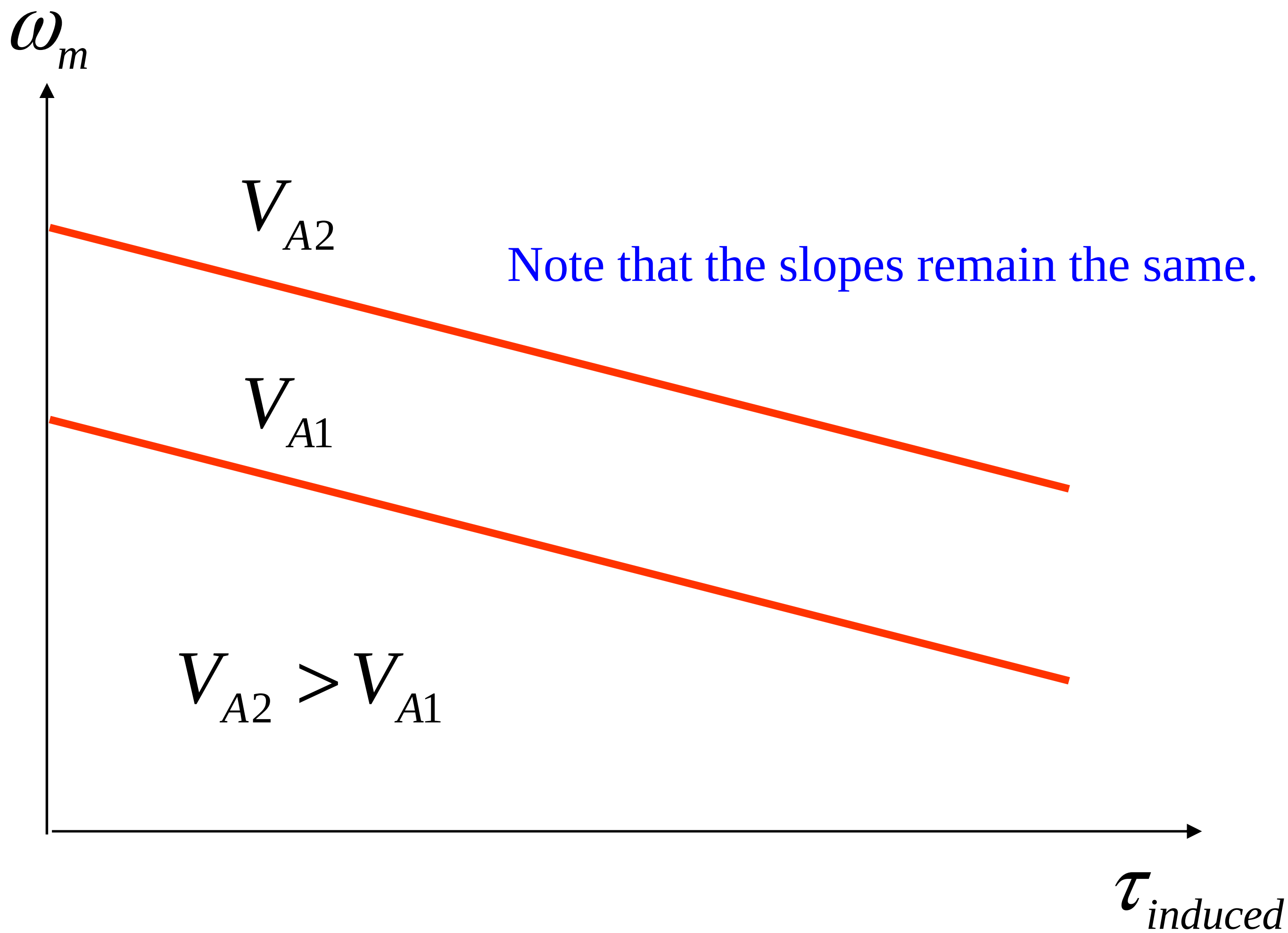
causing the motor to accelerate. As it does, so does the internally induced voltage E_A, $E_A = K\phi\omega$

causing the armature current to decrease, lowering the torque, and causing

$$\tau_{induced} = \tau_{load}$$

at a higher rotational speed ω.

Shunt DC Motor Speed Control



Note that the slopes remain the same.

Shunt DC Motor Speed Control

In *field resistance control*, the lower field current in a shunt or separately excited DC motor, the faster it turns, and the higher the field current the slower it turns.

The minimum achievable speed occurs when the field circuit has the maximum possible current flowing through it.

A motor operating at rated terminal voltage, power, and field current will be running at rated speed. This is known as the *base speed*.

Field resistance control circuits can control the speed of the motor for speeds *above* the base speed but *not below*, since speed below the base speed often result in excessive field currents.

Shunt DC Motor Speed Control

For *armature voltage control*, the lower the armature voltage the slower the motor turns, and the higher the armature voltage the faster it turns.

Since an increase in armature voltage causes an increase in speed, there is a maximum achievable speed which occurs when the armature voltage reaches its maximum permissible level.

Armature control can be used to control the speed of a motor below the base speed but not above, since faster speeds could result in excessive armature voltage.

Shunt DC Motor Speed Control

Clearly these two forms of speed control are complementary.

Motors which combine these two methods of speed control can have a very wide range of speed variation.

The limiting factor, however, is always the heating to the armature conductors, which places a limit on the magnitude of the armature current I_A.

Shunt DC Motor Speed Control

Note that for *armature voltage speed control*, the flux in the motor is constant, so the maximum torque of the motor is:

$$\tau_{\max} = K\phi I_{A_{\max}}$$

The maximum torque is constant regardless of the speed of the motor.

But, since

$$P_{\max} = \tau_{\max}\omega$$

the maximum power out of the motor is directly proportional to its operating speed under armature voltage control.

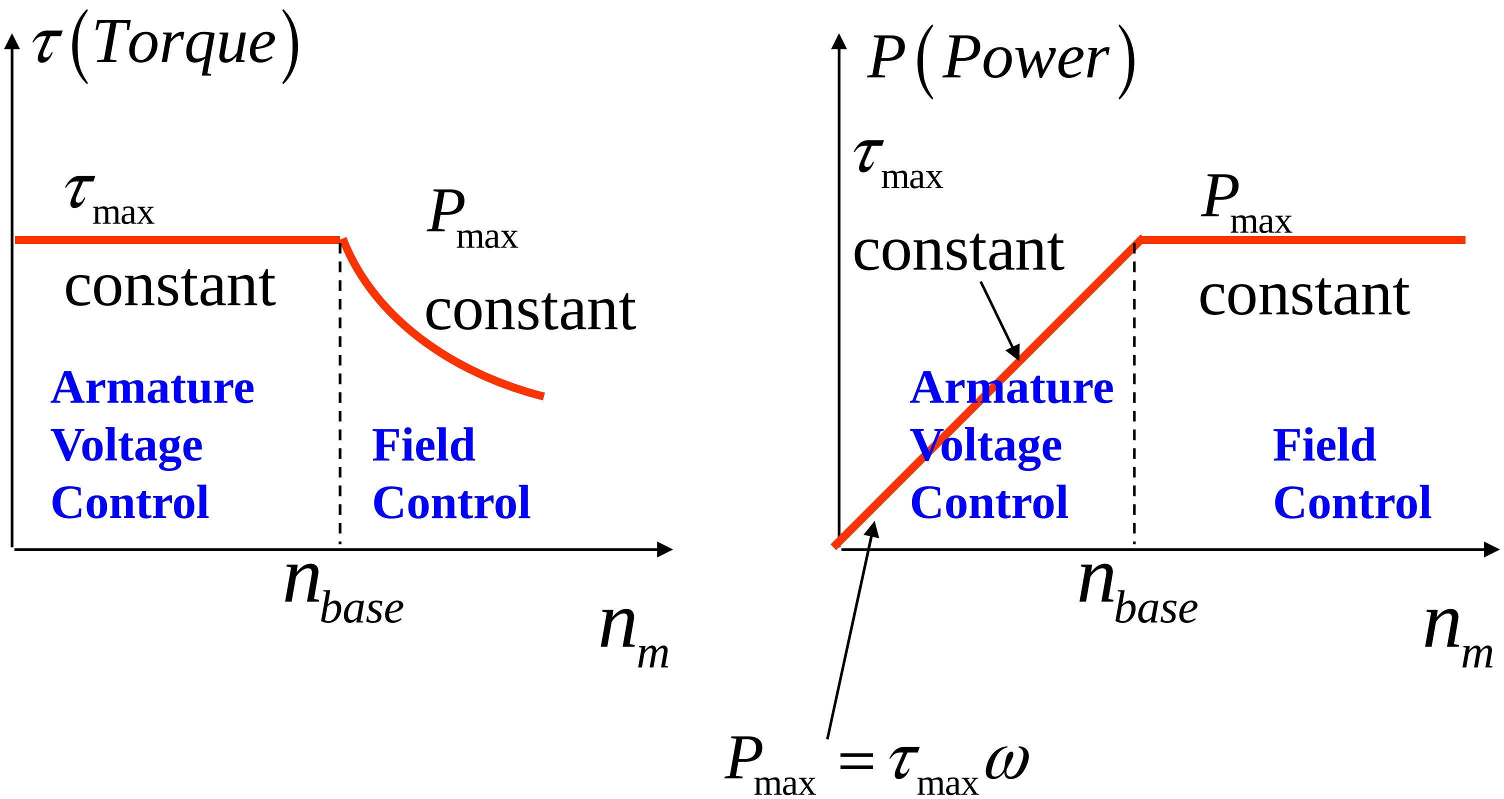
Shunt DC Motor Speed Control

Note that for *field resistance speed control*, the flux in the motor changes. An increase in speed is caused by a decrease in flux. For the armature current limit not to be exceeded, the induced torque limit must decrease as the speed of the motor increases. Since

$$P = \tau\omega$$

and the torque limit decreases as the speed of the motor increases, *the maximum power out of a DC motor under field current control is constant, while the maximum torque varies as the reciprocal of the motor speed.*

Shunt DC Motor Speed Control



Permanent Magnet DC Motors (PMDC)

Advantages:

- No external field circuit required
- No field circuit copper losses
- Can be smaller than motors with field windings
- Often the least expensive

Disadvantages:

- Permanent magnets (PM) cannot provide as high a flux density as an externally applied shunt field (though this is changing), so PMDC motors have a lower induced torque per amp of armature current compared with a shunt motor of the same size and construction
- A PM can become demagnetized. This can happen through heating, bumping, or, most of all, armature reaction. Again, new developed materials help minimize this effect.

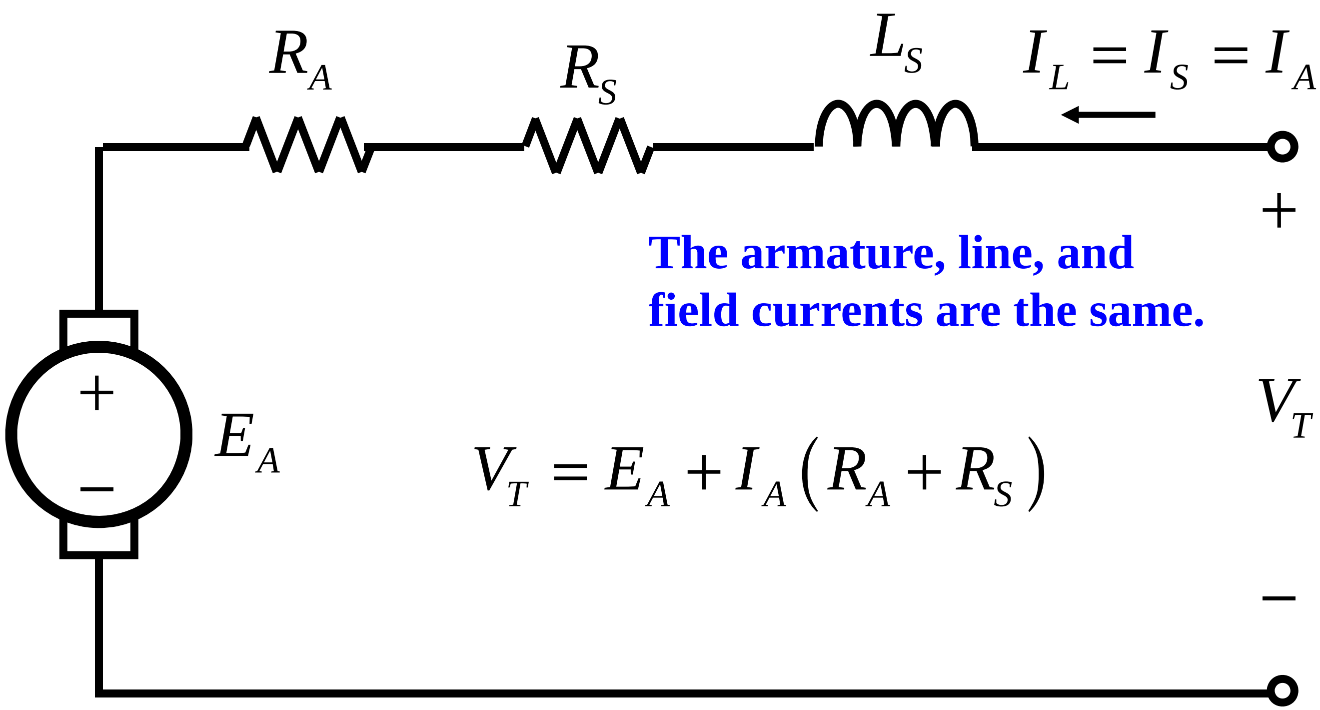
Permanent Magnet DC Motors (PMDC)

A PMDC motor is essentially the same as shunt DC motor expect that the *flux of a PMDC motor is fixed*. Consequently it is not possible to control the speed using the field control method.

The only way to control the speed of a PMDC motor is to vary the armature voltage or armature resistance.

Series DC Motors

A series DC motor is one whose field winding consists of a relatively few turns connected in series with the armature circuit.



Series DC Motors

Since the flux is directly proportional to the armature current (until saturation is reached).

This makes the behavior of a series DC motor quite different than what we have considered thus far.

As the load on the motor increases, its flux increases too. But recall that an increase in flux causes a decrease in speed.

Consequently the series DC motor has a sharply decreasing torque-speed characteristic.

Series DC Motors – Terminal Characteristics

Recall again that the induced torque is given by,

$$\tau_{induced} = K\phi I_A$$

but the flux is proportional to I_A, $\phi \propto I_A \Rightarrow \phi = cI_A$

thus,

$$\tau_{induced} = cKI_A{}^2$$

The torque is proportional to the square of its armature current.

As a consequence, the series DC motor gives more torque per ampere than any other DC motor.

Series DC Motors – Terminal Characteristics

Neglecting saturation, let $\phi = cI_A$

Now $V_T = E_A + I_A\left(R_A + R_S\right)$

and since $\tau_{induced} = cKI_A^{\,2} \Rightarrow I_A = \sqrt{\dfrac{\tau_{induced}}{cK}}$

$$\Rightarrow V_T = E_A + \sqrt{\frac{\tau_{induced}}{cK}}\left(R_A + R_S\right)$$

$$E_A = K\phi\omega$$

$$\Rightarrow V_T = K\phi\omega + \sqrt{\frac{\tau_{induced}}{cK}}\left(R_A + R_S\right)$$

Series DC Motors – Terminal Characteristics

Now express $I_A = \dfrac{\phi}{c}$

along with $\tau_{induced} = cKI_A{}^2$

thus

$$\tau_{induced} = cKI_A{}^2 = cK\left(\frac{\phi}{c}\right)^2 = \frac{K}{c}\phi^2$$

and

$$\phi = \sqrt{\frac{c}{K}\tau_{induced}}$$

Series DC Motors – Terminal Characteristics

Substituting: $$V_T = K\phi\omega + \sqrt{\frac{\tau_{induced}}{cK}}\left(R_A + R_S\right)$$

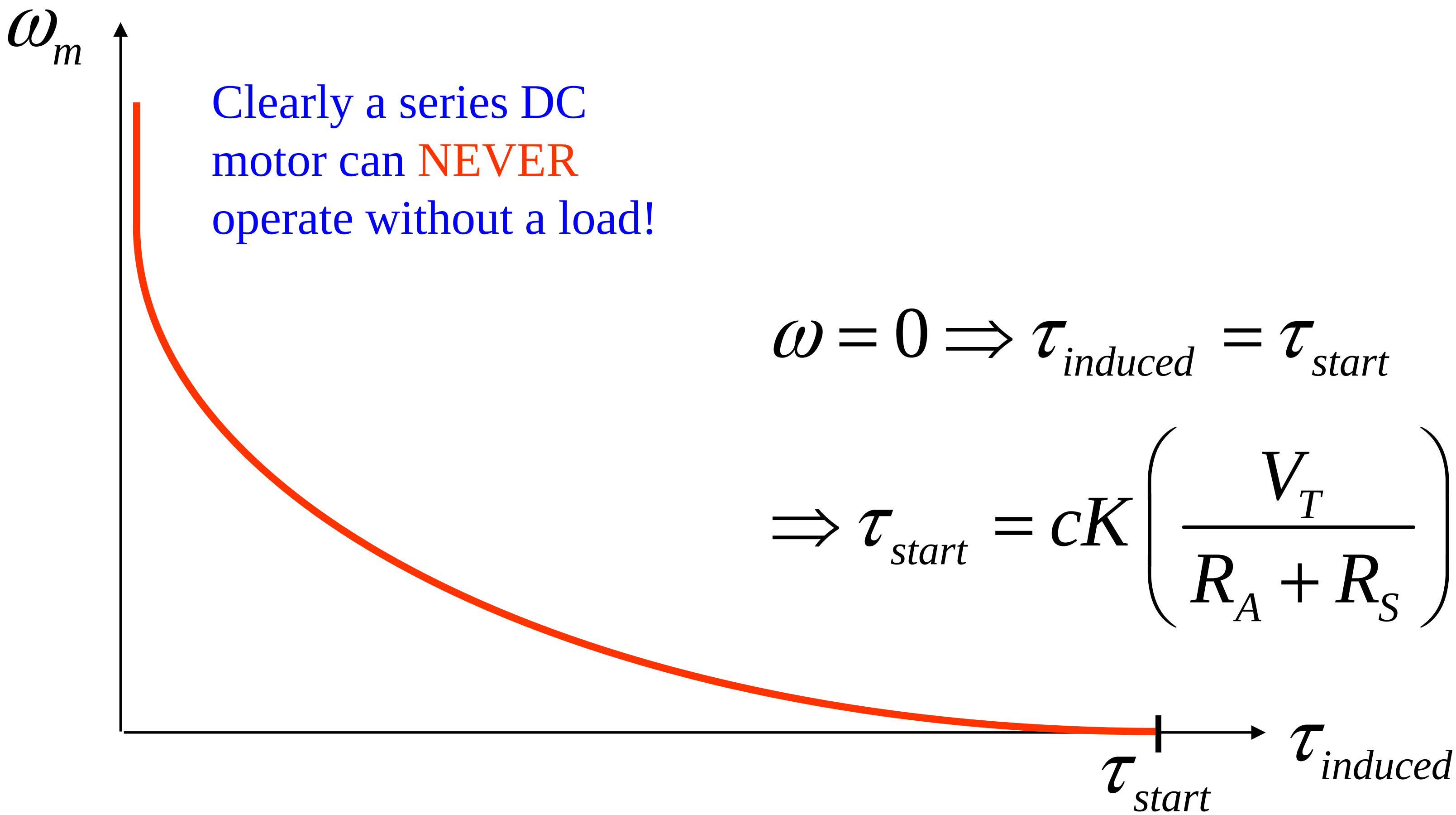
$$\tau_{induced} = \frac{K}{c}\phi^2$$

$$\Rightarrow V_T = K\sqrt{\frac{c}{K}\tau_{induced}}\,\omega + \sqrt{\frac{\tau_{induced}}{cK}}\left(R_A + R_S\right)$$

$$\omega = \frac{V_T}{\sqrt{cK}\sqrt{\tau_{induced}}} - \frac{R_A + R_S}{cK}, \; or \qquad \omega \propto \frac{1}{\sqrt{\tau}}$$

Series DC Motors – Terminal Characteristics

$$\tau \rightarrow 0 \Rightarrow \omega \rightarrow \infty$$



Clearly a series DC motor can NEVER operate without a load!

$$\omega = 0 \Rightarrow \tau_{induced} = \tau_{start}$$

$$\Rightarrow \tau_{start} = cK\left(\frac{V_T}{R_A + R_S}\right)^2$$

Series DC Motors – Speed Control

Clearly the only way to vary the speed of a series DC motor is to change V_T, since

$$\omega = \frac{V_T}{\sqrt{cK}\sqrt{\tau_{induced}}} - \frac{R_A + R_S}{cK}$$

As V_T is increased so is the speed for any given torque.

Compounded DC Motor

A DC motor with both a shunt and a series field.

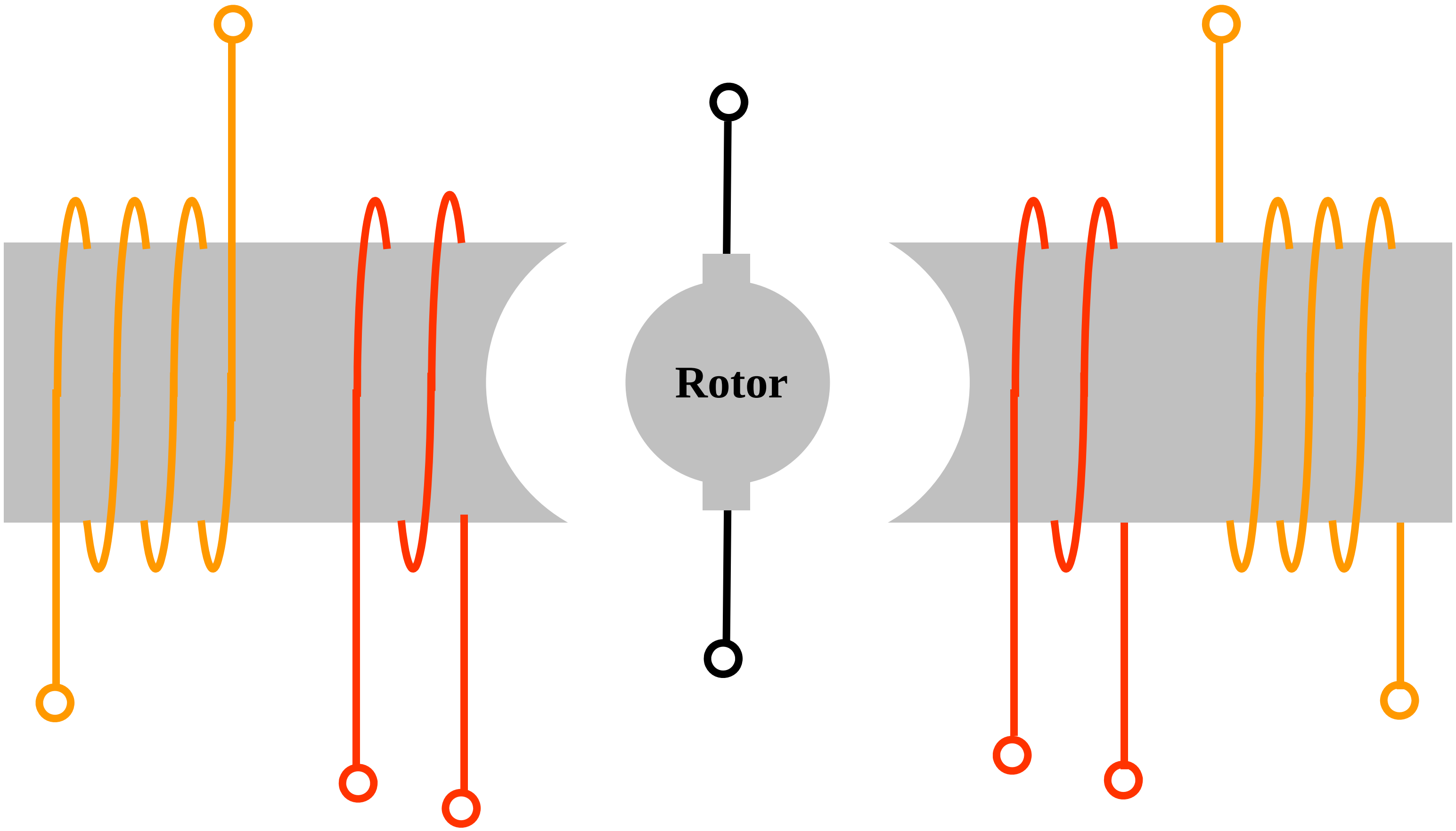
The compounded DC motor combines the best features of the shunt and series motors.

The compounded DC motor has a higher starting torque than a shunt motor (whose flux is constant) but a lower starting torque than a series motor (whose flux is proportional to armature current).

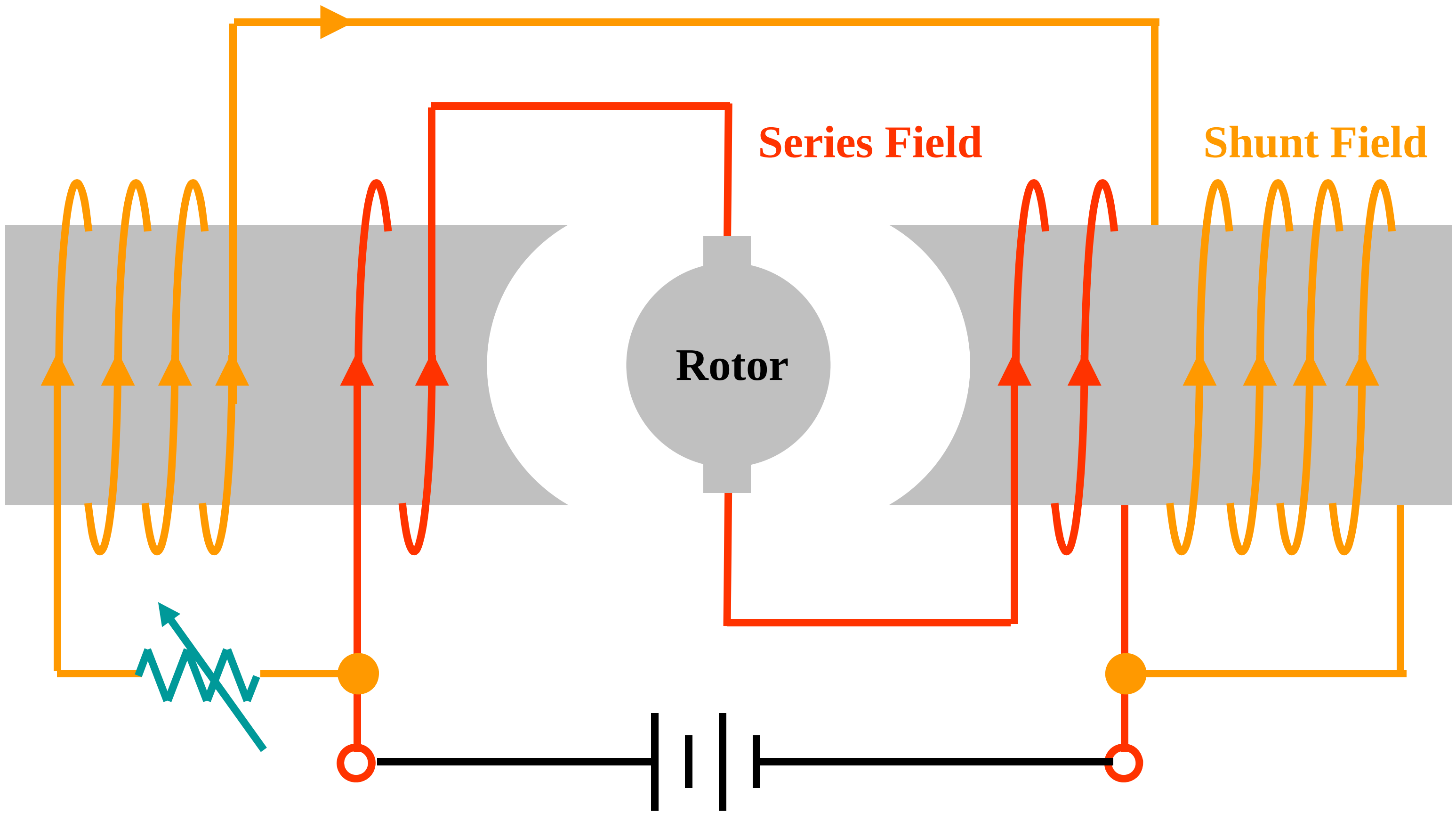
Like a series motor, it has extra torque for starting but does not overspeed under no-load conditions.

Needs two field windings.

Compounded DC Motors – Needs two field windings

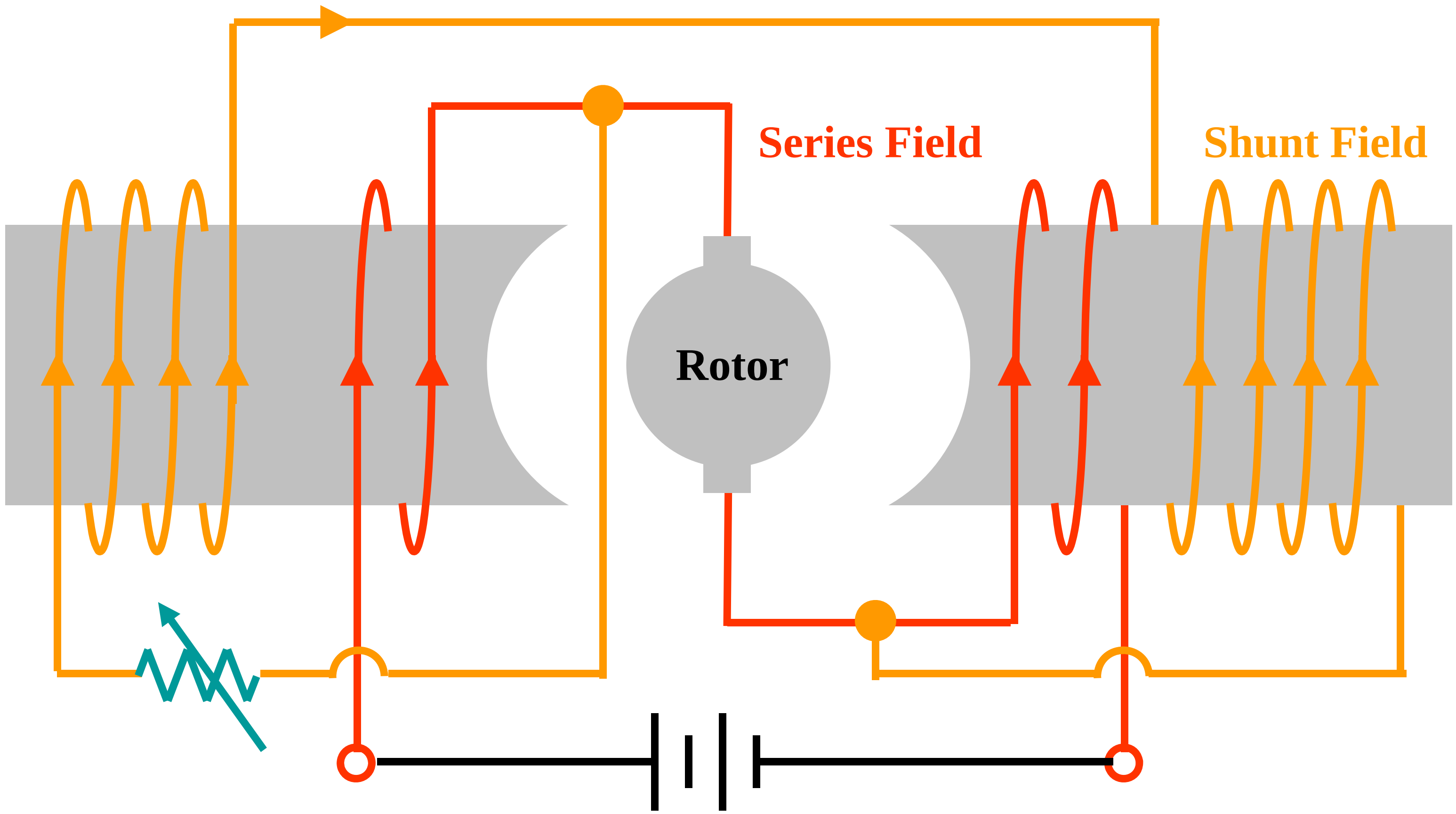


Compounded DC Motors



Short-Shunt Connection

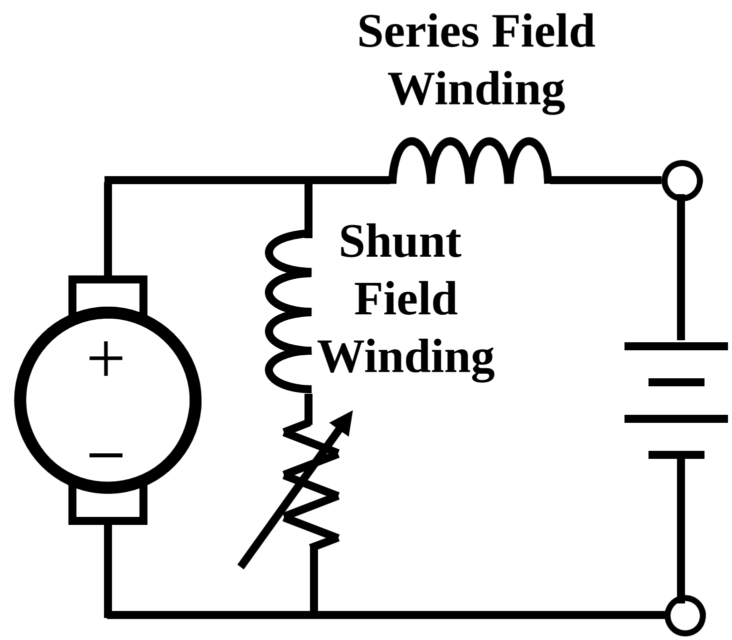
Compounded DC Motors

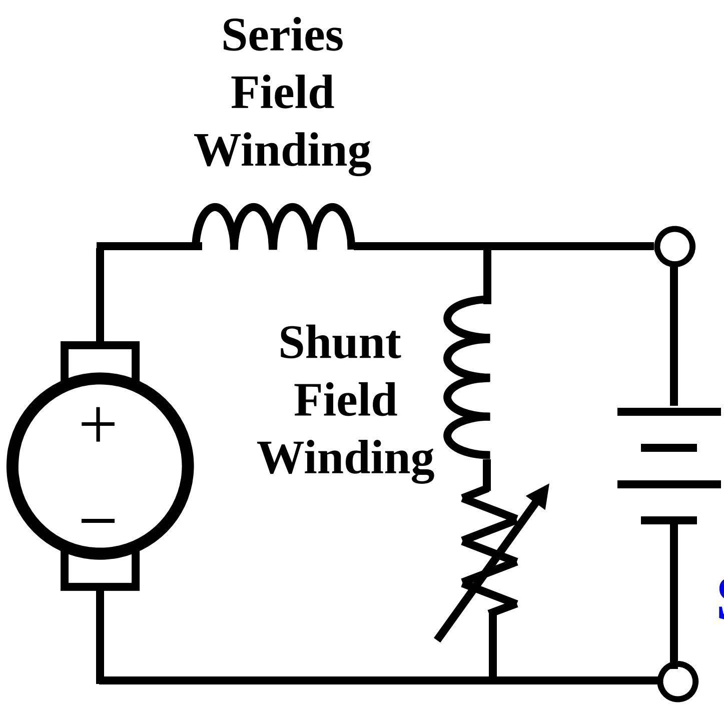


Long-Shunt Connection

Compounded DC Motors

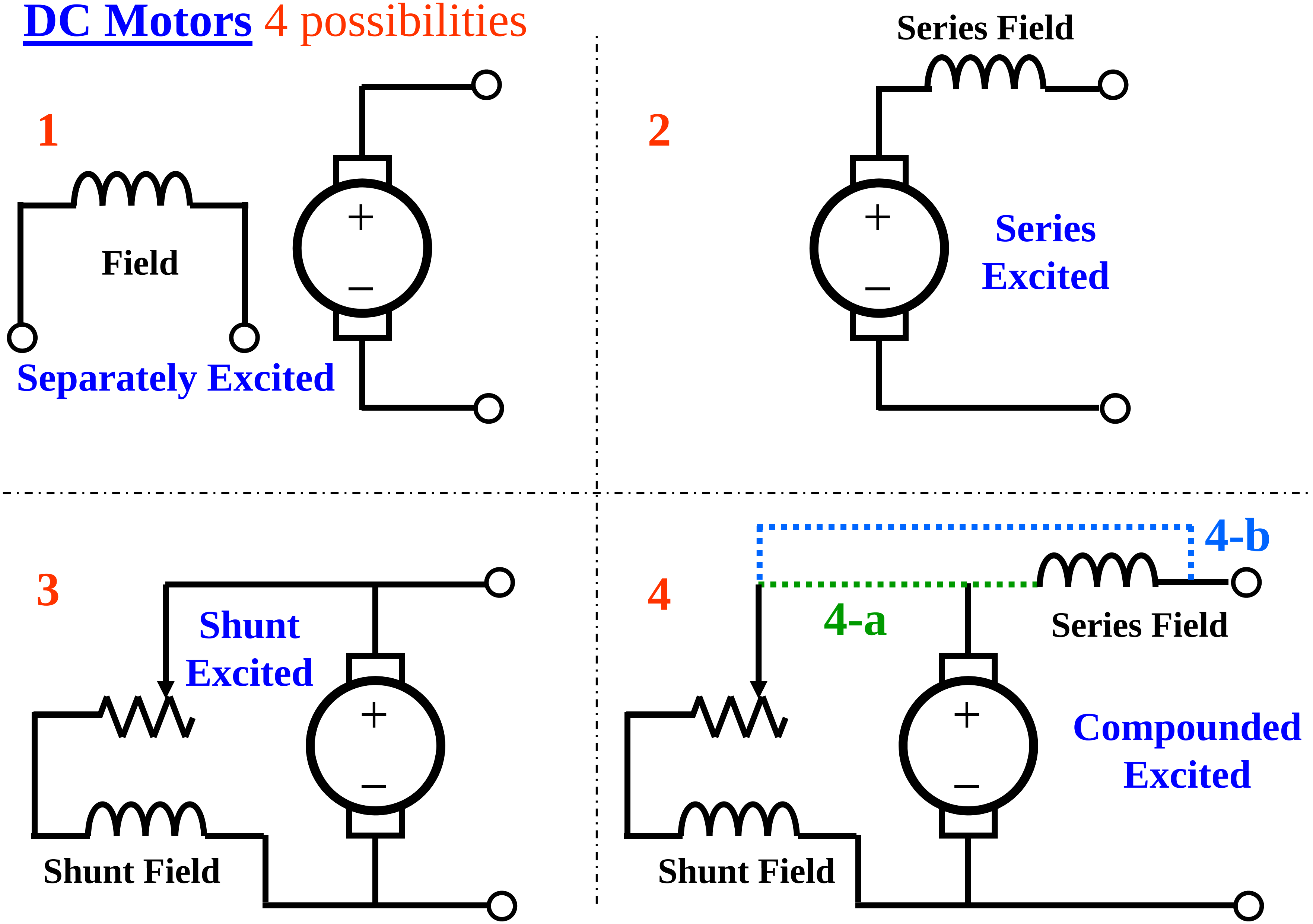
Long-Shunt Connection





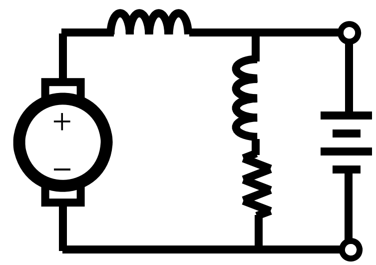
Short-Shunt Connection

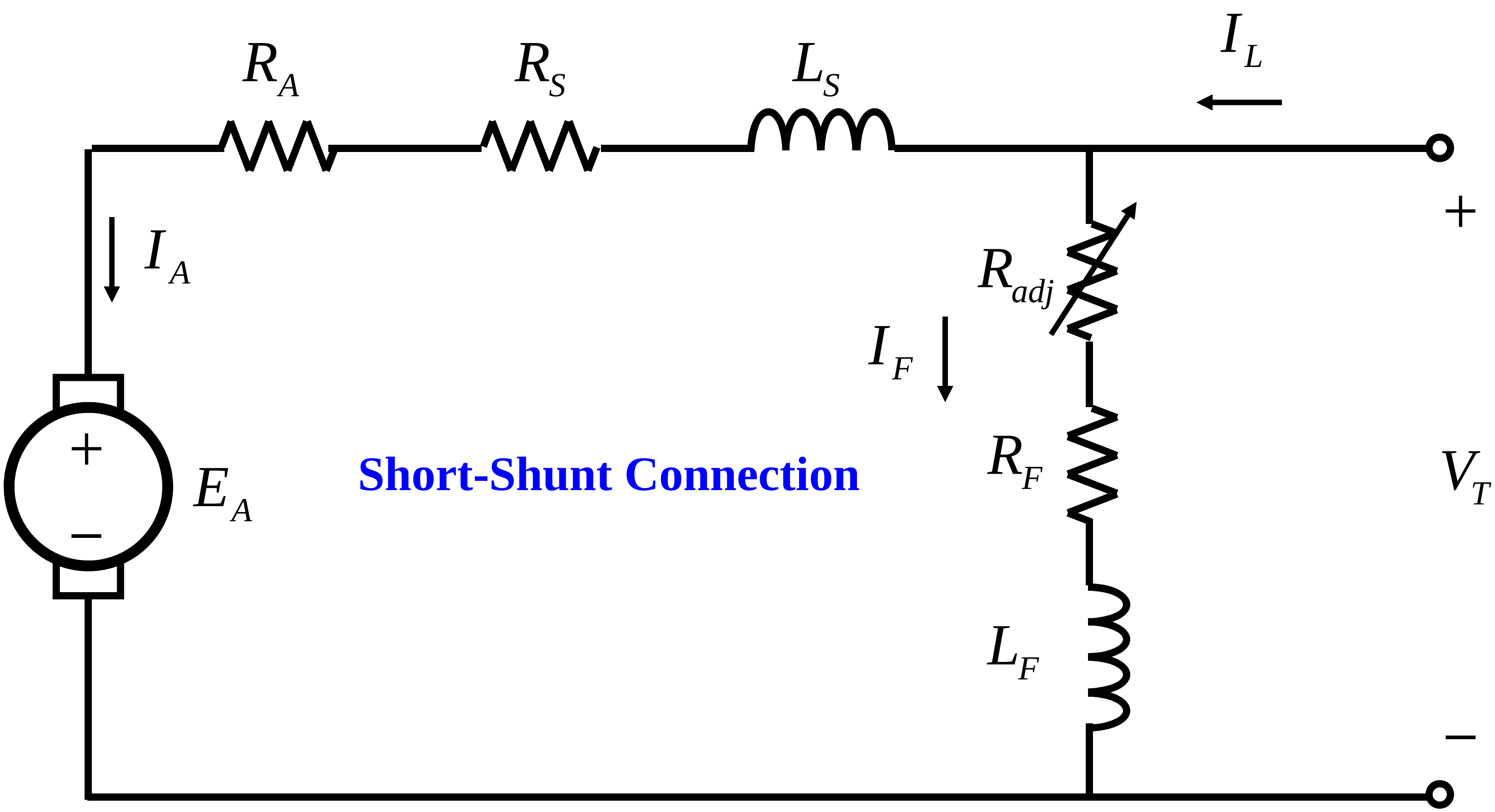
DC Motors 4 possibilities



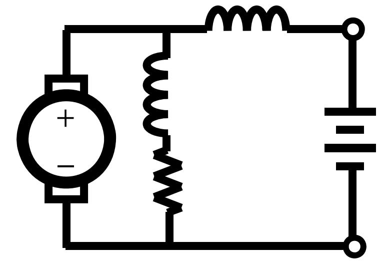
Compounded DC Motor

Model:

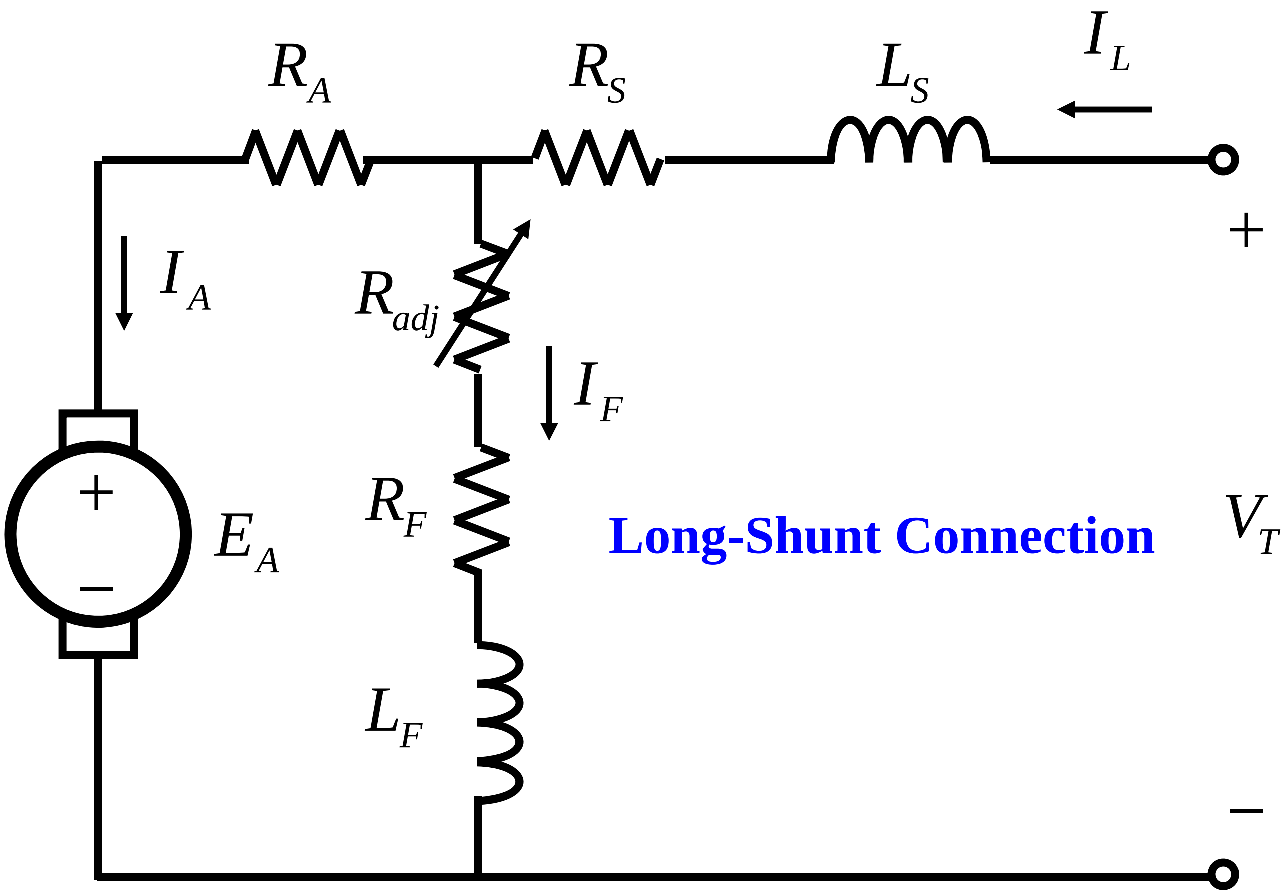




Compounded DC Motor

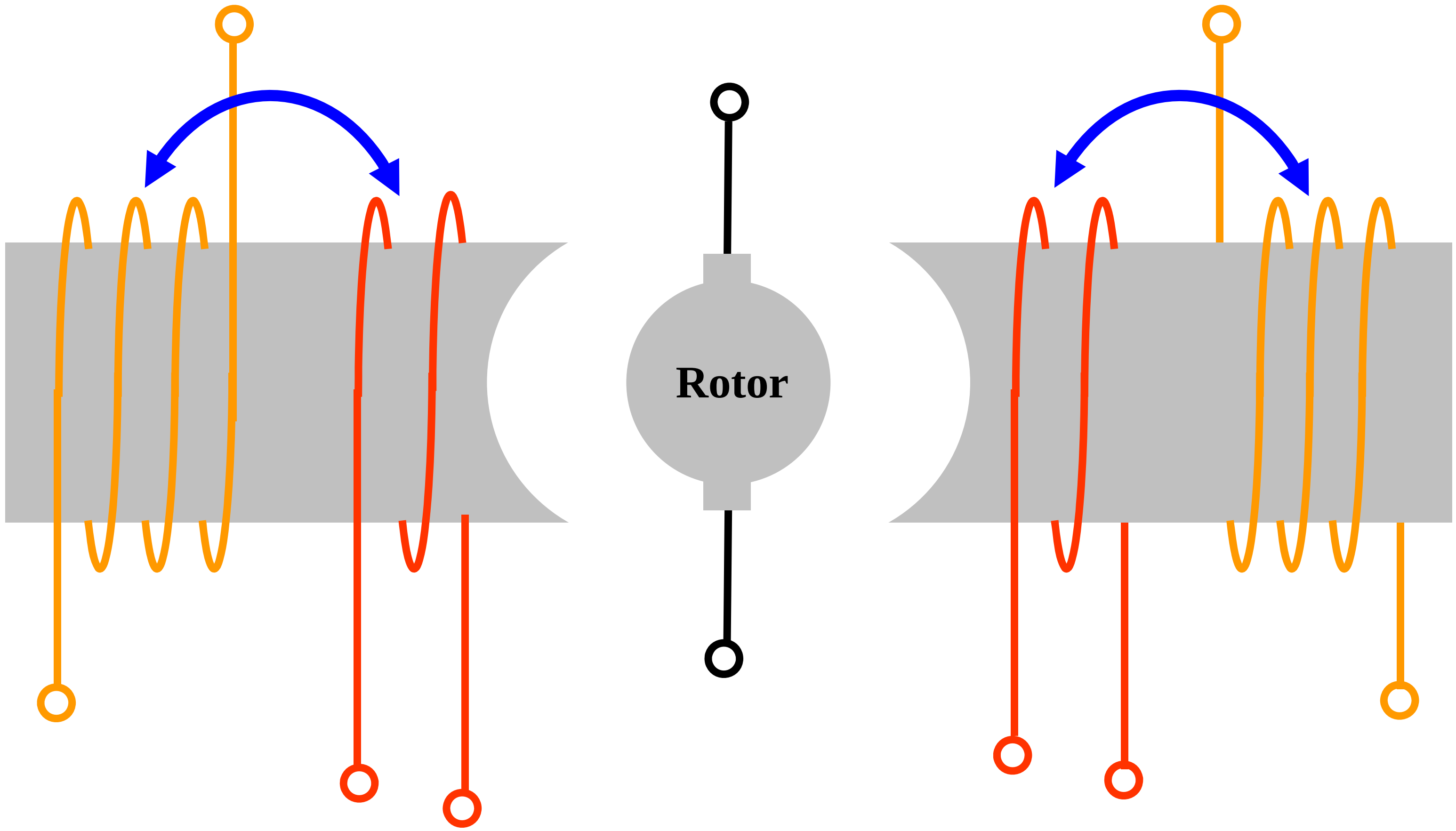


Model:

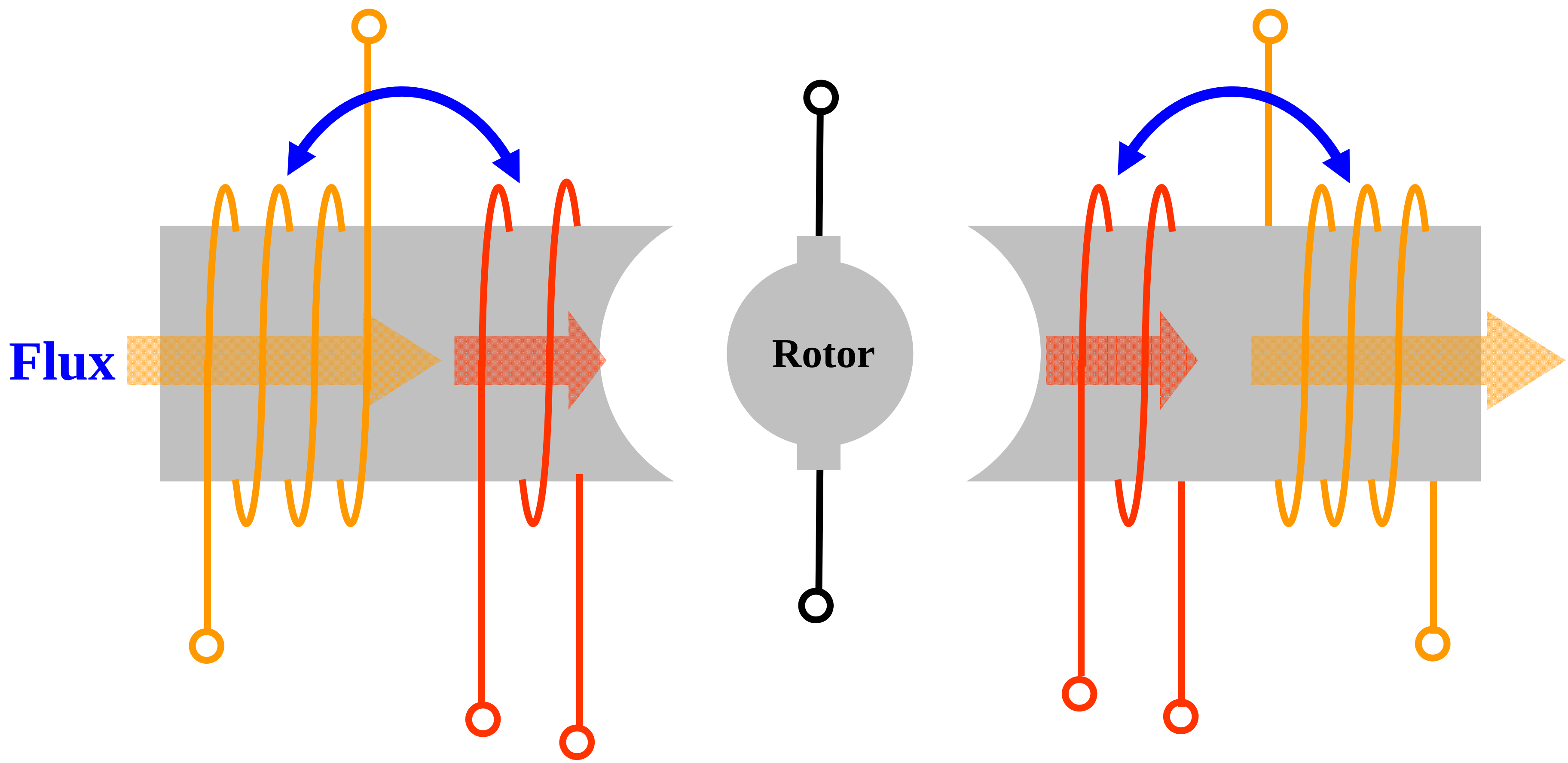


Long-Shunt Connection

Compounded DC Motors – These are coupled!

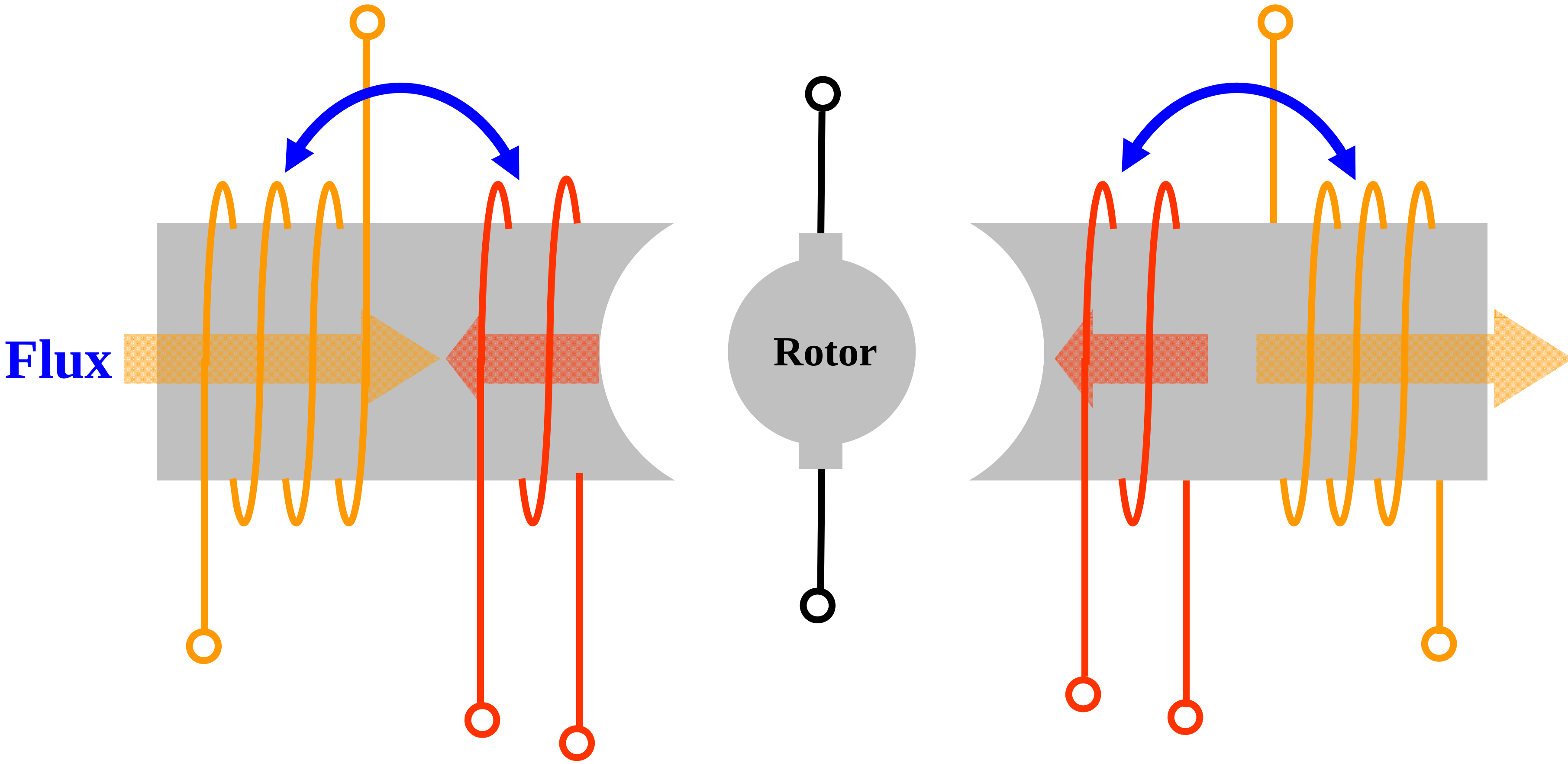


Compounded DC Motors – These are coupled!



Cumulatively Compounded

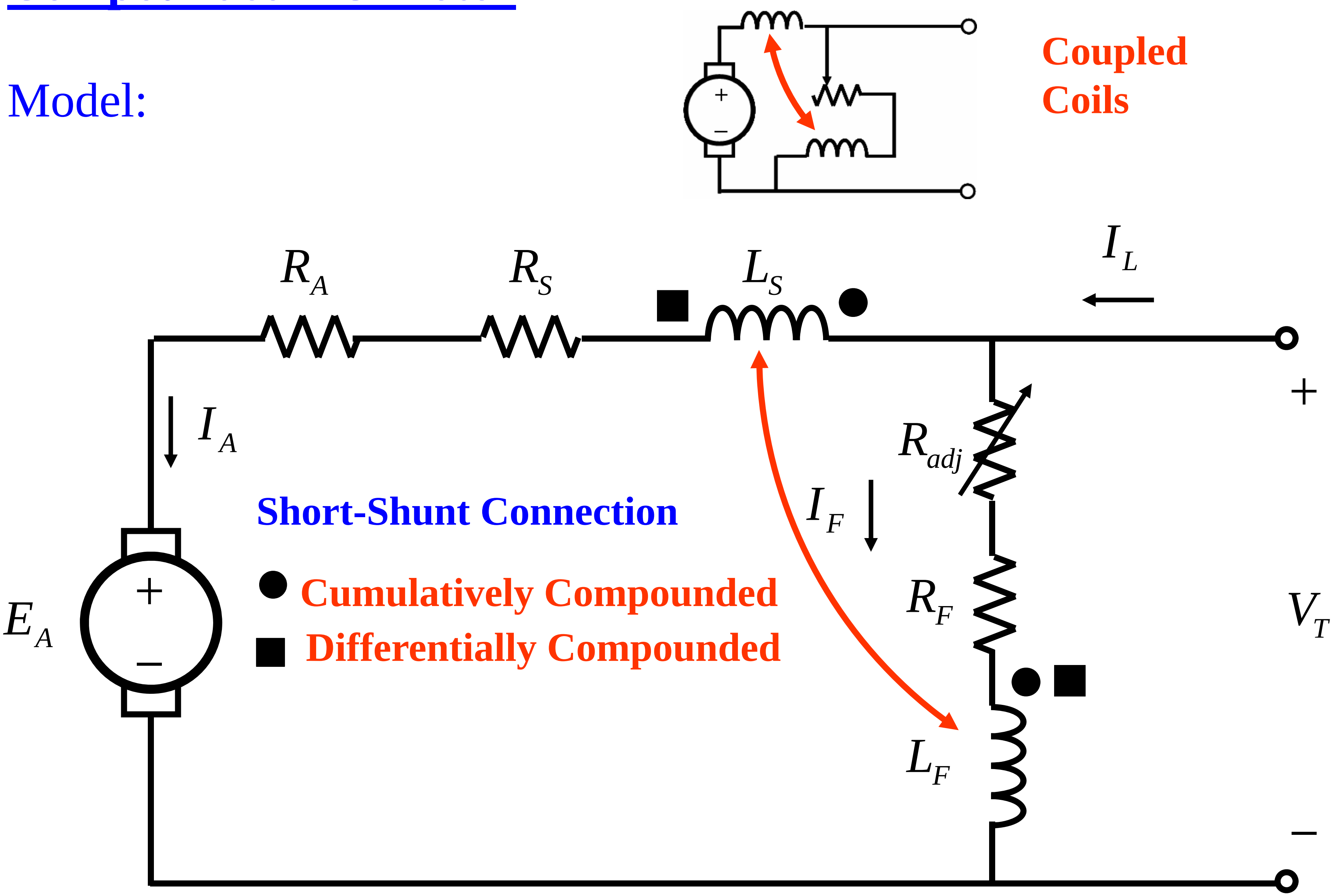
Compounded DC Motors – These are coupled!



Differentially Compounded

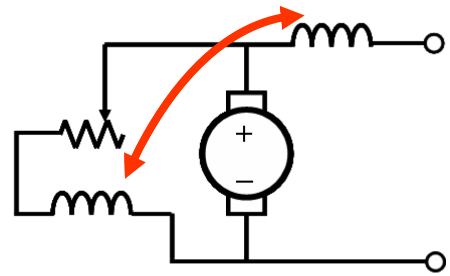
Compounded DC Motor

Model:

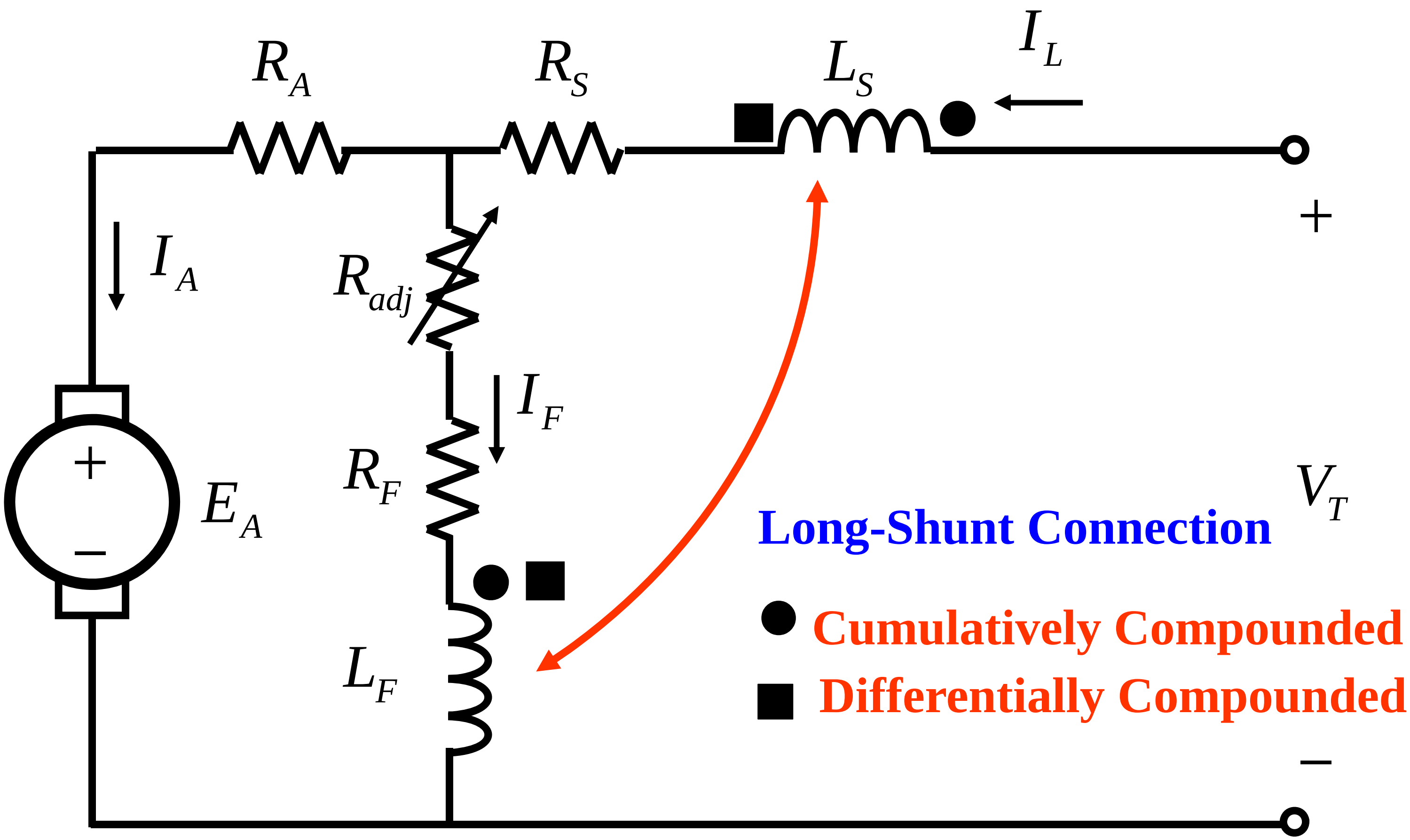


Compounded DC Motor

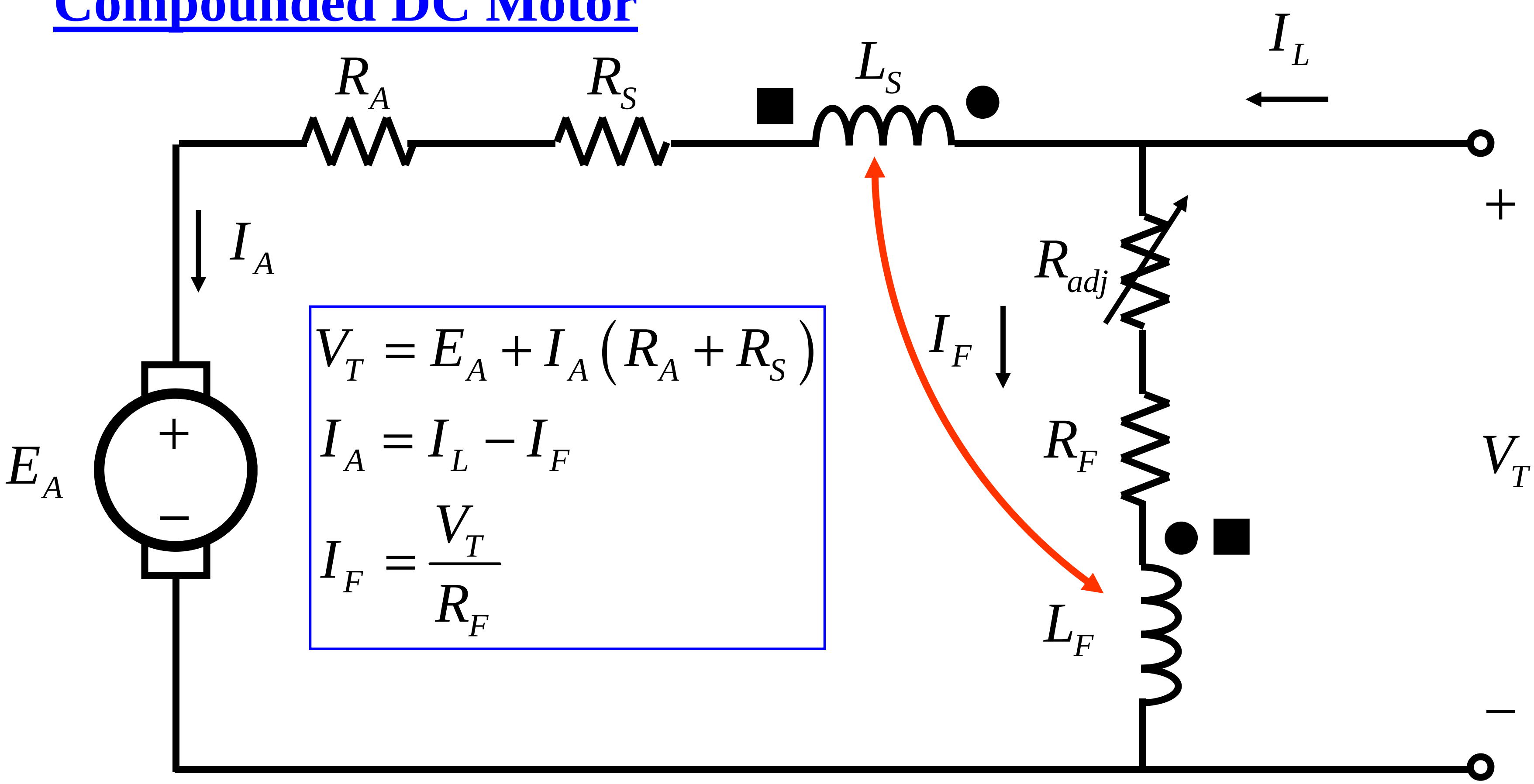
Model:



Coupled Coils



Compounded DC Motor



$$V_T = E_A + I_A\left(R_A + R_S\right)$$

$$I_A = I_L - I_F$$

$$I_F = \frac{V_T}{R_F}$$

$$\mathscr{F}_{net} = \mathscr{F}_F \pm \mathscr{F}_S - \mathscr{F}_{\substack{Armature\\Reaction}}, \pm \Rightarrow \begin{cases} + \text{Cumulative Compounding} \\ - \text{Differential Compounding} \end{cases}$$

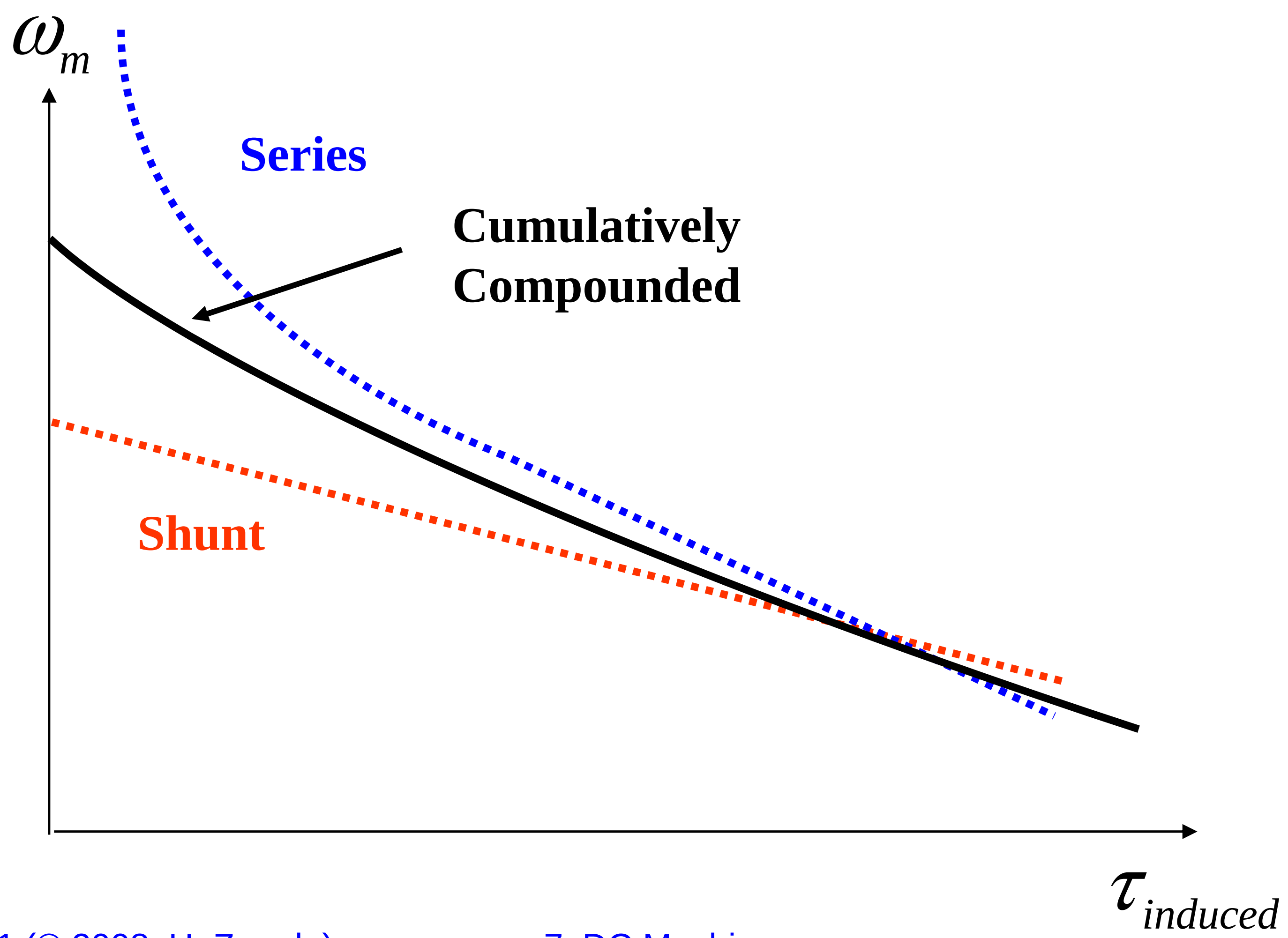
Cumulatively Compounded DC Motor Torque-Speed Characteristics

In a compounded DC motor there is a component of flux that is constant and a component that is proportional to the armature current and thus to the load.

At light loads, the series field has a small current and hence very little effect, and the motor behaves as a shunt DC motor.

As the load gets very large, the series current/flux becomes significant and the motor behaves more like a series DC motor.

Cumulatively Compounded DC Motor Torque-Speed Characteristics



Differentially Compounded DC Motor Torque-Speed Characteristics

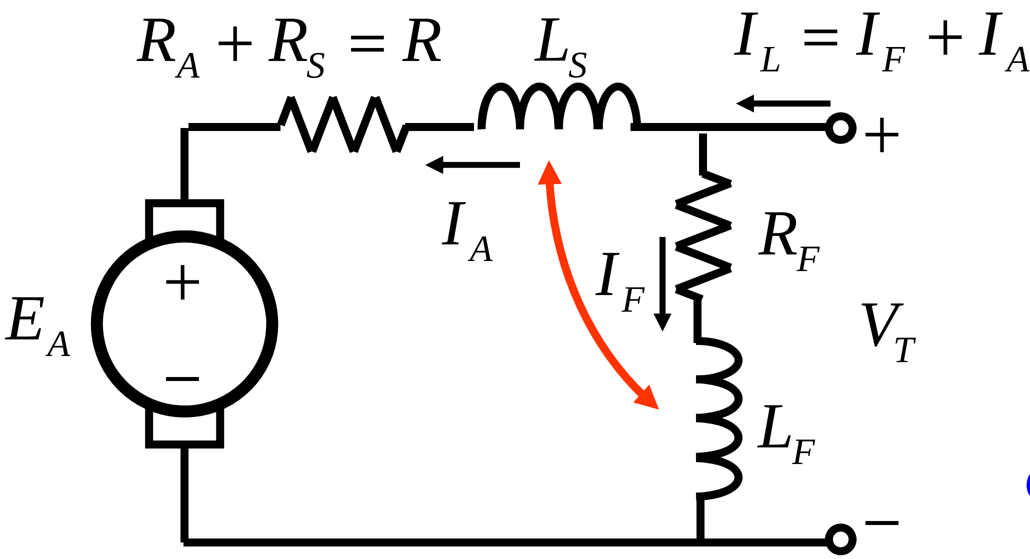
For a differentially compounded DC motor, the shunt and series magnetomotive forces subtract from each other since

$$\mathscr{F}_{net} = \mathscr{F}_{shunt} - \mathscr{F}_{series} = N_{shunt} I_F - N_{series} I_A$$

As the load increases, I_A increases, the motor flux actually decreases because of the differencing. But as the flux decreases the speed increases which causes a further increase in the load, further increasing I_A, further decreasing the flux, and increasing the speed again. This is a run-away condition.

Differentially compounded DC motors are extremely unstable and are never used.

Compounded DC Motor



Basic Motor Equations:

$$E_A \propto \phi\omega$$

$$\tau \propto \phi I_A$$

Cumulative Compounding (upper sign)
Differential Compounding (lower sign)

Flux: $\phi = k_F N_F I_F \pm k_A N_A I_A$

Circuit Equations:

$$V_T = E_A + RI_A$$

$$I_A = I_L - I_F$$

$$I_F = \frac{V_T}{R_F}$$

Solve for the speed as a function of torque... algebra...

Compounded DC Motor

Cumulative Compounding

$$V_T = E_A + RI_A \qquad E_A \propto \phi\omega \qquad \phi = k_F N_F I_F + k_A N_A I_A$$

$$I_F = \frac{V_T}{R_F} \qquad \tau \propto \phi I_A \qquad = k_F N_F \frac{V_T}{R_F} + k_A N_A I_A$$

$$\Rightarrow V_T = E_A + RI_A$$

$$= \left(k_F N_F \frac{V_T}{R_F} + k_A N_A I_A \right)\omega + RI_A \Rightarrow \text{Solve for } \omega$$

$$\Rightarrow \omega = \frac{V_T - RI_A}{k_F N_F \frac{V_T}{R_F} + k_A N_A I_A}$$

$$\Rightarrow \tau \propto \phi I_A = \left(k_F N_F \frac{V_T}{R_F} + k_A N_A I_A \right) I_A \Rightarrow \text{Solve for } I_A$$

Compounded DC Motor

Cumulative Compounding

$$\Rightarrow \tau = \left(k_F N_F \frac{V_T}{R_F} + k_A N_A I_A \right) I_A \Rightarrow \text{Solve for } I_A$$

$$0 = -\tau + k_F N_F \frac{V_T}{R_F} I_A + k_A N_A I_A{}^2$$

$$I_A = -\frac{k_F N_F}{2k_A N_A} \frac{V_T}{R_F} \pm \sqrt{\frac{k_F{}^2 N_F{}^2}{4k_A{}^2 N_A{}^2} \frac{V_T{}^2}{R_F{}^2} + 4k_A N_A \tau}$$

Substitute this into the expression for ω, using the upper sign to keep I_A positive (for cumulative compounding) as shown in the figure.

Compounded DC Motor

Cumulative Compounding

$$\omega = \frac{V_T - RI_A}{k_F N_F \dfrac{V_T}{R_F} + k_A N_A I_A}$$

$$I_A = -\frac{k_F N_F}{2k_A N_A}\frac{V_T}{R_F} + \sqrt{\frac{k_F{}^2 N_F{}^2}{4k_A{}^2 N_A{}^2}\frac{V_T{}^2}{R_F{}^2} + 4k_A N_A \tau}$$

$$\Rightarrow \omega = \frac{V_T - R\left(-\dfrac{k_F N_F}{2k_A N_A}\dfrac{V_T}{R_F} + \sqrt{\dfrac{k_F{}^2 N_F{}^2}{4k_A{}^2 N_A{}^2}\dfrac{V_T{}^2}{R_F{}^2} + 4k_A N_A \tau}\right)}{k_F N_F \dfrac{V_T}{R_F} + k_A N_A\left(-\dfrac{k_F N_F}{2k_A N_A}\dfrac{V_T}{R_F} + \sqrt{\dfrac{k_F{}^2 N_F{}^2}{4k_A{}^2 N_A{}^2}\dfrac{V_T{}^2}{R_F{}^2} + 4k_A N_A \tau}\right)}$$

Compounded DC Motor

Cumulative Compounding:

$$\omega = \frac{V_T - R\left(-\frac{k_F N_F}{2k_A N_A}\frac{V_T}{R_F} + \sqrt{\frac{k_F^{\,2} N_F^{\,2}}{4k_A^{\,2} N_A^{\,2}}\frac{V_T^{\,2}}{R_F^{\,2}} + 4k_A N_A \tau}\right)}{k_F N_F \frac{V_T}{R_F} + k_A N_A\left(-\frac{k_F N_F}{2k_A N_A}\frac{V_T}{R_F} + \sqrt{\frac{k_F^{\,2} N_F^{\,2}}{4k_A^{\,2} N_A^{\,2}}\frac{V_T^{\,2}}{R_F^{\,2}} + 4k_A N_A \tau}\right)}$$

Differential Compounding (change the sign of k_A:

$$\omega = \frac{V_T - R\left(+\frac{k_F N_F}{2k_A N_A}\frac{V_T}{R_F} + \sqrt{\frac{k_F^{\,2} N_F^{\,2}}{4k_A^{\,2} N_A^{\,2}}\frac{V_T^{\,2}}{R_F^{\,2}} - 4k_A N_A \tau}\right)}{k_F N_F \frac{V_T}{R_F} - k_A N_A\left(+\frac{k_F N_F}{2k_A N_A}\frac{V_T}{R_F} + \sqrt{\frac{k_F^{\,2} N_F^{\,2}}{4k_A^{\,2} N_A^{\,2}}\frac{V_T^{\,2}}{R_F^{\,2}} - 4k_A N_A \tau}\right)}$$

Compounded DC Motor