Fundamentals of MAGNETICS

Revised October 6, 2008
Magnetic fields provide the fundamental mechanism by which energy is converted from one form to another by means of:

- motors (electrical energy ↔ mechanical energy)
- generators (mechanical energy ↔ electrical energy)
- transformers (electrical energy ↔ electrical energy)
Four basic principles:

1. A current flowing in a wire produces a magnetic field in the vicinity of the wire.
2. A wire in the presence of a time-varying magnetic field will induce a voltage across the wire (transformer action).
3. A current-carrying wire in the presence of a magnetic field experiences a force exerted on the wire (motor action).
4. A wire moving in the presence of a magnetic field has a voltage induced upon it (generator action).

Each of these will be considered in detail.
The fundamental *postulate* governing the electrical behavior of magnetic fields is given by Ampere’s law:

$$\oint_C \vec{H} \cdot d\vec{\ell} = I_{\text{enclosed}}$$

- $\vec{H}$: Vector Magnetic Field (amperes/meter)
- $d\vec{\ell}$: Vector Closed Path of Integration (meters)
- $I_{\text{enclosed}}$: Net Current enclosed by $C$
\[ \oint_C \vec{H} \cdot d\ell = I \]

Side view (along axis)

\[ \vec{H}, C, I \]
Right Hand Rule Convention
\[
\oint_C \vec{H} \cdot d\vec{\ell} = 2\pi r H(r) = I \Rightarrow H(r) = \frac{I}{2\pi r}
\]
We assumed that field produced by the wire exists in air. The strength of the magnetic field produced also depends on the material in which the field exists. Certain materials, called *ferromagnetic materials*, have the property of confining the field inside of it. Ferromagnetic materials are characterized by their *permeability*, $\mu$.

The for air (or free space):

$$\mu = \mu_o = 4\pi \times 10^{-7} \text{ Henrys/meter}$$
The ratio of the permeability for a given material to that of free space is known as the *relative permeability*,

\[ \mu_r = \frac{\mu}{\mu_o} \]

For modern ferromagnetic materials,

\[ 1000 < \mu_r < 8000 \]

For an idealized case, \( \mu_r \to \infty \)
Note the similarity:

From circuit theory, the magnetic energy stored by an inductor is

\[ W_m = \frac{1}{2} L |I|^2 \]

From electromagnetic theory, the magnetic energy stored by a magnetic field is

\[ W_m = \frac{1}{2} \mu |\vec{H}|^2 \]
The relationship that described the effect of the permeability $\mu$ (the material) has on the magnetic field $H$ is given by the relationship

$$\vec{B} = \mu \vec{H}$$

where $B$ is the magnetic flux density.

Units:

$$\vec{B} = \mu \left[ \frac{\text{Henry}}{\text{meter}} \right] \vec{H} \left[ \frac{\text{Ampere}}{\text{meter}} \right] = \mu \vec{H} \left[ \frac{\text{Ampere-Henry}}{\text{meter}^2} \right]$$

$$= \mu \vec{H} \left[ \frac{\text{Weber}}{\text{meter}^2} \right] = \mu \vec{H} \left[ \text{Tesla} \right]$$
$B$ is the magnetic flux density. If we integrate the density over an area we obtain the total magnetic flux $\phi$

$$\phi = \int_A \vec{B} \cdot d\vec{a} = \int_A \vec{B} \cdot \hat{n} da = \int_A \vec{B} \cdot \hat{n} dx dy$$
EXAMPLE

Mean path length $\ell_c$

Current $i$

$N$ turns

Cross-sectional area $A$

Magnetic Core

$\phi$

$\mu$

EEL 3211

© 2008, Henry Zmuda
EXAMPLE

\[ \oint \vec{H} \cdot d\vec{l} = I_{\text{enclosed}} \Rightarrow H \ell_c = Ni \Rightarrow H = \frac{Ni}{\ell_c} \]

\[ B = |\vec{B}| = \mu |\vec{H}| = \mu H = \frac{\mu Ni}{\ell_c} \]

\[ \phi = \int_A \vec{B} \cdot d\vec{a} = \frac{\mu Ni A}{\ell_c} \]
Circuit analogy for Magnetic circuits:

The simplifications and idealizations discussed imply that simple circuit models can be used to study magnetic circuits.

This idea is used extensively in the design of motors and generators to simplify what would otherwise be an extremely complicated electromagnetic problem.
Circuit analogy for Magnetic circuits:
Circuit analogy for Magnetic circuits:

\[ I = \frac{V}{R} \]

\[ \phi = \frac{\mu N_i A}{\ell_c} = (N_i) \left( \frac{\mu A}{\ell_c} \right) \]

\[ \Rightarrow \phi = \frac{\mathcal{F}}{\mathcal{R}} \]
### Circuit analogy for Magnetic circuits:

<table>
<thead>
<tr>
<th>Electrical Circuit</th>
<th>Magnetic Circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromotive Force (emf)</td>
<td>Magnetomotive Force (mmf)</td>
</tr>
<tr>
<td>$V$</td>
<td>$\mathcal{F} = Ni$</td>
</tr>
<tr>
<td>Current</td>
<td>Flux</td>
</tr>
<tr>
<td>$I$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Resistance</td>
<td>Reluctance</td>
</tr>
<tr>
<td>$R = \frac{V}{I}$</td>
<td>$R = \frac{\mathcal{F}}{\phi}$</td>
</tr>
</tbody>
</table>
Circuit analogy for Magnetic circuits:

Note the polarity – determined via a modified right-hand-rule
Jargon – Just as the conductance is the reciprocal of the resistance,

$$G = \frac{1}{R}$$

the *permeance, $\mathcal{P}$* is the reciprocal of the reluctance.

$$\mathcal{P} = \frac{1}{\mathcal{R}}$$

*This circuit analogy is always an approximation to the real system.*
1-5. A ferromagnetic core is shown in Figure P1-2. The depth of the core is 5 cm. The other dimensions of the core are as shown in the figure. Find the value of the current that will produce a flux of 0.003 Wb.

With this current, what is the flux density at the top of the core? What is the flux density at the right side of the core? Assume that the relative permeability of the core is 1000.
1-5. A ferromagnetic core is shown in Figure P1-2. The depth of the core is 5 cm. The other dimensions of the core are as shown in the figure. Find the value of the current that will produce a flux of 0.003 Wb.

With this current, what is the flux density at the top of the core? What is the flux density at the right side of the core? Assume that the relative permeability of the core is 1000.

\[ \text{mean path} \]

\[ 5 + 20 + 2.5 = 27.5 \]
\[ \phi = (Ni) \left( \frac{\mu A}{\ell_c} \right) \Rightarrow R = \frac{F}{\phi} \]

\[ \Rightarrow R = \frac{F}{\phi} = R_{\text{left}} + R_{\text{top}} + R_{\text{right}} + R_{\text{bottom}} \]
SOLUTION There are three regions in this core. The top and bottom form one region, the left side forms a second region, and the right side forms a third region. If we assume that the mean path length of the flux is in the center of each leg of the core, and if we ignore spreading at the corners of the core, then the path lengths are \( l_1 = 2(27.5 \text{ cm}) = 55 \text{ cm}, \) \( l_2 = 30 \text{ cm}, \) and \( l_3 = 30 \text{ cm}. \) The reluctances of these regions are:

\[
\mathcal{R}_1 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_0 A} = \frac{0.55 \text{ m}}{(1000)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.15 \text{ m})} = 58.36 \text{ kA \cdot t/Wb}
\]

\[
\mathcal{R}_2 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_0 A} = \frac{0.30 \text{ m}}{(1000)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.10 \text{ m})} = 47.75 \text{ kA \cdot t/Wb}
\]

\[
\mathcal{R}_3 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_0 A} = \frac{0.30 \text{ m}}{(1000)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.05 \text{ m})} = 95.49 \text{ kA \cdot t/Wb}
\]

The total reluctance is thus

\[
\mathcal{R}_{\text{TOT}} = \mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_3 = 58.36 + 47.75 + 95.49 = 201.6 \text{ kA \cdot t/Wb}
\]

and the magnetomotive force required to produce a flux of 0.003 Wb is

\[
\mathcal{F} = \phi \mathcal{R} = (0.003 \text{ Wb})(201.6 \text{ kA \cdot t/Wb}) = 605 \text{ A \cdot t}
\]

and the required current is

\[
i = \frac{\mathcal{F}}{N} = \frac{605 \text{ A \cdot t}}{500 \text{ t}} = 1.21 \text{ A}
\]
The flux density on the top of the core is

\[ B = \frac{\phi}{A} = \frac{0.003 \text{ Wb}}{(0.15 \text{ m})(0.05 \text{ m})} = 0.4 \text{ T} \]

The flux density on the right side of the core is

\[ B = \frac{\phi}{A} = \frac{0.003 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 1.2 \text{ T} \]
Magnetic cores often require “gaps”.

\[ R = \frac{l}{\mu A} \]

\[ \Rightarrow R_{\text{gap}} = \frac{l_g}{\mu_o A} \]
Fringing fields in a gap. Note that the cross-sectional area of the gap is greater than that of the metal.

\[ R_{\text{gap}} = \frac{\ell_g}{\mu_o A_{\text{eff}}} \]

\[ A_{\text{eff}} \geq A \]

Typically, \( A_{\text{eff}} \approx 1.05A \)
Model:

\[ \mathcal{F} = Ni \]

\[ \mathcal{R}_{\text{core}} = \frac{\ell_c}{\mu A} \]

\[ \mathcal{R}_{\text{gap}} = \frac{\ell_{\text{gap}}}{\mu_o A_{\text{eff}}} \]
1-6. A ferromagnetic core with a relative permeability of 2000 is shown in Figure 1-3. The dimensions are as shown in the diagram, and the depth of the core is 7 cm. The air gaps on the left and right sides of the core are 0.050 and 0.070 cm, respectively. Because of fringing effects, the effective area of the air gaps is 5 percent larger than their physical size. If there are 300 turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?
Mean path

0.37 m 0.37 m

0.37 m

0.37 m

0.37 m

0.37 m

300 turns

0.07 cm

0.05 cm

30 cm

7 cm

7 cm

7 cm

7 cm

7 cm

7 cm

7 cm

7 cm
\[ \mathcal{F} = Ni \]
\[ R_{\text{TOT}} = R_5 + \left( R_1 + R_2 \right) \parallel \left( R_3 + R_4 \right) \]
\[ R_{\text{TOT}} = R_5 + \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} \]

\[ R_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.11 \text{ m}}{(2000)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})} = 90.1 \text{ kA} \cdot \text{t/Wb} \]

\[ R_2 = \frac{l_2}{\mu_0 A_2} = \frac{0.0005 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})(1.05)} = 77.3 \text{ kA} \cdot \text{t/Wb} \]

\[ R_3 = \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.11 \text{ m}}{(2000)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})} = 90.1 \text{ kA} \cdot \text{t/Wb} \]

\[ R_4 = \frac{l_4}{\mu_0 A_4} = \frac{0.0007 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})(1.05)} = 108.3 \text{ kA} \cdot \text{t/Wb} \]

\[ R_5 = \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.37 \text{ m}}{(2000)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})} = 30.0 \text{ kA} \cdot \text{t/Wb} \]
The total reluctance is

\[ R_{\text{TOT}} = R_5 + \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} = 30.0 + \frac{(90.1 + 77.3)(90.1 + 108.3)}{90.1 + 77.3 + 90.1 + 108.3} = 120.8 \text{kA} \cdot \text{t/Wb} \]

The total flux in the core is equal to the flux in the center leg:

\[ \phi_{\text{center}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{R_{\text{TOT}}} = \frac{(300 \text{ t})(1.0 \text{ A})}{120.8 \text{kA} \cdot \text{t/Wb}} = 0.00248 \text{ Wb} \]

The flux density in the air gaps can be determined from the equation \( \phi = B A \):

\[ B_{\text{left}} = \frac{\phi_{\text{left}}}{A_{\text{eff}}} = \frac{0.00135 \text{ Wb}}{(0.07 \text{ cm})(0.07 \text{ cm})(1.05)} = 0.262 \text{ T} \]

\[ B_{\text{right}} = \frac{\phi_{\text{right}}}{A_{\text{eff}}} = \frac{0.00113 \text{ Wb}}{(0.07 \text{ cm})(0.07 \text{ cm})(1.05)} = 0.220 \text{ T} \]
Example – A Simple Synchronous Machine – Assume that the rotor and stator have infinite permeability. Find the air-gap flux $\phi$ and the flux density $B_g$. Use $I = 10$ A, $N = 1000$ turns, gap length $g = 1$ cm, and $A_g = 2000$ cm$^2$. 
Example – A Simple Synchronous Machine

\[
R_{\mu \rightarrow \infty} = \frac{l_c}{\mu A} = 0, \quad R_{\text{gap}} = \frac{l_{\text{gap}}}{\mu_0 A_{\text{gap}}}, \quad \phi_{\text{gap}} = \frac{F}{2R_{\text{gap}}} = \frac{NI\mu_0 A_{\text{gap}}}{2l_{\text{gap}}}
\]
Example – A Simple Synchronous Machine

\[ \phi_{\text{gap}} = \frac{\mathcal{F}}{2R_{\text{gap}}} = \frac{NI\mu_oA_{\text{gap}}}{2\ell_{\text{gap}}} \]

\[ = \frac{1000 \cdot 10 \cdot 4\pi \cdot 10^{-7} \cdot 0.2}{2 \cdot 0.01} = 0.13 \text{ Webers} \]

\[ B_{\text{gap}} = \frac{\phi_{\text{gap}}}{A_{\text{gap}}} = \frac{0.13}{0.2} = 0.65 \text{ Tesla} \]
Real Magnetic Materials
Recall from Slide 14:

\[ H = \frac{Ni}{\ell_c} = \frac{\mathcal{F}}{\ell_c} \]

and

\[ \phi = BA = \mu HA = \mu \frac{Ni}{\ell_c} A = \frac{\mu\mathcal{F}}{\ell_c} A \]
Saturation or Magnetization Curve

\[ \phi = \mu \mathcal{F} A \]
\[ \mu = \mu(\mathcal{F}) \]

The permeability \( \mu \) is constant only for air (\( \mu_0 \)).
Recall from Slide 14:

\[ H = \frac{Ni}{\ell_c} = \frac{\mathcal{F}}{\ell_c} \]

and

\[ \phi = BA = \mu HA = \mu \frac{Ni}{\ell_c} A = \frac{\mu \mathcal{F}}{\ell_c} A \]
The slope of this curve defines the permeability $\mu$ of a given material.

Since generators/motors rely on the flux to produce voltage/torque, we wish to produce as much flux as possible. In practice we operate near the knee of the curve but not in the saturated region.
Saturation or Magnetization Curve

The slope of this curve defines the permeability $\mu$ of a given material.

Since generators/motors rely on the flux to produce voltage/torque, we wish to produce as much flux as possible. In practice we operate near the knee of the curve but not in the saturated region. **Why?**
$B$ vs. $H$

\[
\frac{\mu}{\mu_o} \propto \frac{\Delta B}{\Delta H} \quad \text{vs. } H
\]

Linear Region

Saturated Region
The advantage of using a ferromagnetic material for cores in electric machines and transformers is that one gets many times more flux for a given magnetomotive force (mmf) with iron than with air. However, if the resulting flux has to be (approximately) proportional to the applied mmf, then the core must be operated in the unsaturated region of the magnetization curve.
Hysteresis Effects in Magnetic Materials

![Diagram showing hysteresis loop]
Hysteresis Effects in Magnetic Materials

\[ i \]

\[ t \]

\[ \phi, B \]

\[ F, H \]
Hysteresis Effects in Magnetic Materials
Hysteresis Effects in Magnetic Materials

\[ i \]

\[ \phi, B \]

\[ F, H \]
**Hysteresis Effects** in Magnetic Materials

A Typical Hysteresis Loop – Present flux in the core is determined not only by the applied mmf but also on the previous history of the flux in the core.
What causes hysteresis effects?

MODEL

The permanent magnetic moment of an atom comes about from:

- Electron orbit
- **Electron spin** (primary contributor in solids)
- Nuclear dipole moment

Ferr**oomagnetic** materials (Fe, Ni, Co, Cd., & alloys)
These tend to be high conductivity metals (like iron and steel) and also quite lossy.

On a macroscopic scale, a piece of ferromagnetic material is made up of magnetic domains, where each domain is in effect a small permanent magnet. These domains are randomly oriented.
What causes hysteresis effects?

With no externally applied mmf or magnetic field, individual magnetic domains are randomly aligned and the net flux is zero.

With an externally applied mmf or magnetic field, individual magnetic domains align and a net flux occurs.

When no further alignment is possible, saturation occurs.
First Major Effect

Faraday’s Law – Induced voltage due to a time-changing magnetic field.
Faraday’s Law – Induced voltage due to a time-changing magnetic field.
Faraday’s Law – Induced voltage due to a time-changing magnetic field.
Faraday’s Law – Induced voltage due to a time-changing magnetic field.

A look ahead: Transformer Action
Faraday’s Law: If flux passes through a turn of a coil of wire, a voltage will be induced in the turn of wire that is directly proportional to the rate of change of the flux.

\[ e_{\text{induced}} = -\frac{d\phi_{\text{applied}}}{dt} \]

- \( e_{\text{induced}} \) is the induced voltage
- \( \phi \) is the flux passing through the loop

\[ \frac{d\phi_{\text{applied}}(t)}{dt} > 0 \]
Lenz’s Law (explains the minus sign in Faraday’s Law)

First, a slight variant of the right-hand-rule:
**Lenz’s Law**: For a short-circuited loop, the applied change of flux produces a current that would cause a flux opposing the original flux change. This a statement of the conservation of energy.

\[
\text{Applied Flux: } \phi_{\text{applied}}(t) \\
\text{Induced Flux: } \phi_{\text{induced}}(t) \\
B_{\text{induced}}(t) \\
\frac{d\phi_{\text{applied}}(t)}{dt} > 0
\]

The induced current is such as to OPPOSE the CHANGE in applied field.
**Lenz’s Law:** The polarity of the induced voltage at the terminals of the coil is such that the current produced would cause a flux in a direction *opposing* the original flux.

An induced electromotive force generates a current that induces a counter magnetic field that opposes the magnetic field generating the current.
For \( N \) turns all with the same flux,

\[
e_{\text{induced}} = -N \frac{d\phi}{dt}
\]

Note how this is also a simple GENERATOR.
Flux Linkage:

\[ e_{induced} = N \frac{d\phi}{dt} = \frac{d\lambda}{dt} \]

Flux Linkage: \( \lambda = N \phi \)
In practice, the polarity of the induced voltage is determined from physical considerations. As a result, the minus sign in Faraday’s law is often omitted. The text (Chapman is from industry) omits the minus sign for the remainder of the book. This could be confusing.

\[ e_{\text{induced}} = N \frac{d\phi}{dt} \]

Text omits the minus sign.
Example – Consider a coil of wire (10 turns) wrapped around an iron core. If a flux has a peak value of 0.1 Weber and varies sinusoidally at a frequency of 60 Hz. What is the value and polarity of the induced voltage (flux is increasing in the direction shown).
Solution –

\[ \phi(t) = 0.1 \sin(120\pi t) \]

\[ e_{\text{induced}} = N \frac{d\phi}{dt} = 10 \times 0.1 \times 120\pi \cos(120\pi t) \]

\[ = 120\pi \cos(120\pi t) \]
Solution – (Note how this one-half of a transformer – the secondary)

\[ e_{induced} = 120\pi \cos(120\pi t) \]
Example – A Primitive AC Generator

\[ B \text{ – Field} \quad \text{(1 Tesla)} \]

Cross-Sectional Area \( A = 0.4 \text{ m}^2 \)

10-Turn Coil

Find the induced voltage at the terminals.

Axis of Rotation

Speed = 3600 rpm
Example – A Primitive AC Generator

\[ B = 1 \, \text{T} \]

\[ A = 0.4 \, \text{m}^2 \]

\[ \phi = \int_A \vec{B} \cdot d\vec{a} = B \, A \cos \alpha \]

Projected Area

\[ \alpha = \omega t \]

Angular Speed: 3600 rpm

\[
\omega \frac{\text{radians}}{\text{second}} = 3600 \frac{\text{revolutions}}{\text{minute}} \times \frac{2\pi \text{ radians}}{\text{revolution}} \times \frac{1}{60} \frac{\text{minute}}{\text{seconds}}
\]

\[ = 120\pi = 377 \frac{\text{radians}}{\text{second}} \quad [60 \, \text{Hertz}] \]
Example – A Primitive AC Generator

\[ B - \text{Field} \]
(1 Tesla)

\[ \mathbf{B} \cdot \mathbf{A} = |\mathbf{B}| |\mathbf{A}| \cos \alpha \]
Example – A Primitive AC Generator

\[ B = 1 \, \text{T} \]

\[ A = 0.4 \, \text{m}^2 \]

\[ \phi = \int_A \vec{B} \cdot d\vec{a} = B \, A \cos \alpha \]

\[ \alpha = \omega t \]

\[ \phi(t) = BA \cos \omega t \]

\[ \Rightarrow e_{induced}(t) = -N \frac{d\phi}{dt} = NBA \omega \sin \omega t \]

\[ = 1508 \sin 377t \, \text{(volts)} \]

What is the polarity of the induced emf?
Polarity of induced emf:

\[ e_{\text{induced}}(t) = -N \frac{d\varphi}{dt} = -N \frac{d(B \cdot \vec{A})}{dt} \]

Two questions:

is \( B \cdot \vec{A} \pm \)?

is \( \frac{d\varphi}{dt} \pm \)?
Polarity of induced emf: *A mathematical approach*

Convention for positive loop orientation
Polarity of induced emf: \( e_{\text{induced}}(t) = -N \frac{d \left( \vec{B} \cdot \vec{A} \right)}{dt} \)

\( |\theta| < 90^\circ \Rightarrow \vec{B} \cdot \vec{A} > 0 \)

\( \frac{d\phi}{dt} = \frac{d \left( \vec{B} \cdot \vec{A} \right)}{dt} > 0 \) (increasing)

\( |\theta| < 90^\circ \Rightarrow \vec{B} \cdot \vec{A} > 0 \)

\( \frac{d\phi}{dt} = \frac{d \left( \vec{B} \cdot \vec{A} \right)}{dt} < 0 \) (decreasing)
Example – A Primitive AC Generator

In this example, $\vec{B}$ is constant but $\vec{B} \cdot \vec{A}$ is changing with time.

$$\cos(\pi - \alpha) = -\cos\alpha$$
Example – A Primitive AC Generator

\[ \phi = \vec{B} \cdot \vec{A} = \frac{1}{2} |\vec{B}| |\vec{A}| \cos \alpha \]

\[ \vec{B} \cdot \vec{A} < 0 \]

\[ \frac{d\phi}{dt} = -\omega \left[ -|\vec{B}| |\vec{A}| \sin \omega t \right] \]

\[ = +\omega |\vec{B}| |\vec{A}| \sin \omega t \]

\[ \Rightarrow e_{induced} (t) = -N \frac{d(\vec{B} \cdot \vec{A})}{dt} \]

\[ = -\omega N |\vec{B}| |\vec{A}| \sin \alpha \]
Example – A Primitive AC Generator

\[ e_{induced}(t) = -\omega N |\vec{B}| |\vec{A}| \sin \alpha \]
Example – A Primitive AC Generator

\[ \phi = |\vec{B}| |\vec{A}| \cos \alpha \]

\[ N \frac{d\phi}{dt} = -\omega N |\vec{B}| |\vec{A}| \sin \omega t \]

Approach used in text:
Current direction opposes original change in flux.

decreasing

e_{induced}(t) = -\omega N |\vec{B}| |\vec{A}| \sin \alpha
Second Major Effect

Lorentz Force Law: A magnetic field induces a force on a current-carrying wire in the presence of the field.
Lorentz Force Law – Force exerted on charges

\[ \vec{F}_E = q\vec{E} \quad \text{of little interest here} \]

\[ \vec{F}_M = q\left( \vec{v} \times \vec{B} \right) \]
Right-Hand Rule for determining the direction of \( \vec{A} = \vec{B} \times \vec{C} \)
Right-Hand Rule for determining the direction of \( \vec{F} = q(\vec{v} \times \vec{B}) \)
Curl fingers as if rotating vector \( \mathbf{v} \) into vector \( \mathbf{B} \). Thumb is in the direction of force.

Point thumb in direction of velocity, fingers in magnetic field direction. Then palm direction is direction of force on charge.

\[
\mathbf{F} = q \mathbf{v} \times \mathbf{B}
\]

South pole of magnet

Force is in direction that thumb points.

Force direction is outward from palm.

North pole of magnet
Lorentz Force Law – a more useful form for our application

\[ \vec{F}_M = \vec{F} = q \left( \vec{v} \times \vec{B} \right) \quad (\vec{v} \text{ is the velocity of the charge } q) \]

\[ = q \left( \frac{\vec{l}}{t} \times \vec{B} \right) = \frac{q}{t} \left( \vec{l} \times \vec{B} \right) \]

\[ = i \left( \vec{l} \times \vec{B} \right) \]
Lorentz Force Law

\[ \vec{F} = i(\vec{\ell} \times \vec{B}) \]

\( \vec{F} \): force
\( i \): current
\( \vec{\ell} \): oriented conductor path length
\( \vec{B} \): magnetic flux density
Lorentz Force Law

\[ \vec{F} = i (\vec{l} \times \vec{B}) \]
Lorentz Force Law

\[ \vec{F} = i(\vec{l} \times \vec{B}) \]

\[ F = |\vec{F}| = |i||\vec{l}||\vec{B}| \sin \theta \]

Where \( \theta \) is the angle between vectors \( \vec{l} \) and \( \vec{B} \).
Lorentz Force Law

Recall Faraday’s Law: \( e_{induced} = \frac{d\phi}{dt} \)

and

\( \phi = B \cdot A \)

For constant \( B \) (with some hand-waving)

\[
\frac{d\phi}{dt} = B \frac{dA}{dt} = B \frac{d\ell x}{dt} = B\ell \frac{dx}{dt} = B\ell v
\]

What is the polarity?
Lorentz Force Law

\[ \frac{d\phi}{dt} = B\ell v \]

*What is the polarity?*
Lorentz Force Law

Recall: 
\[ \vec{F} = q\vec{E} \Rightarrow \frac{\vec{F}}{q} = \vec{E} = \frac{q(\vec{v} \times \vec{B})}{q} \]

\[ \vec{E} = \vec{v} \times \vec{B} \]

For constant $B$, 
\[ e_{induced} = \int_{\ell} \vec{E} \cdot d\ell = \ell \cdot (\vec{v} \times \vec{B}) \]
Lorentz Force Law

\[ e_{\text{induced}} = \ell \cdot (\vec{v} \times \vec{B}) \]
Lorentz Force Law – An intuitive approach

\[ \vec{F}_+ = q (\vec{v} \times \vec{B}) \]

\[ \vec{F}_- = -q (\vec{v} \times \vec{B}) \]

The direction for \( \vec{l} \) is such that \( \vec{l} \cdot (\vec{B} \times \vec{v}) \) is positive. (upwards in this example)
Example – A Primitive Generator

Revisit our previous example.

\[
\ell = \text{Coil End Length}
\]

\[
L = \text{Coil Side Length}
\]

\[
F_{\text{end}} = N i \ell B \sin (90^\circ + \alpha)
\]
Example – A Primitive Generator

$\ell = \text{Coil End Length}$

$L = \text{Coil Side Length}$

$F_{side}$

$F_{side} = NiLB$
Example – A Primitive Generator

*Axial view:*

\[ F_{side} = NiLB \]

\[ \alpha = \omega t \]
Torque

\[ \vec{T} = \vec{r} \times \vec{F} \]

\[ |\vec{T}| = |\vec{r}| |\vec{F}| \sin \theta \quad [Newton \text{ -- Meters}] \]
Example – A Primitive Generator

\[ F_{\text{side}} = NiLB \]

\[ \alpha = \omega t \]

\[
\left| \vec{T} \right| = 2 \frac{\ell}{2} \left| \vec{F} \right| \sin \alpha \\
= \ell NBiL \sin \alpha \\
= NBiA \sin \alpha, \quad A = \ell L \ (\text{loop area})
\]
Observations:

• As the coil turns, the forces on the ends will vary with position, but will be equal and opposite, thus creating no net force on the coil as a whole, nor any torque about the axis of rotation.

• The forces on the sides of the coil will also be equal and opposite in direction, and will not vary with position, but will exert a torque about the axis of rotation.

• Note that the torque resists motion in the given direction, which generates a current. In other words, mechanical energy is being converted to electrical energy as the coil is turned.
Example – A Primitive Generator

Mechanical Power: \( P_m = \omega |\vec{T}| = i\omega NBA \sin \omega t \)

Electrical Power: \( e_{\text{induced}}(t) i = i\omega N |\vec{B}||\vec{A}| \sin \alpha \)

Mechanical Power = Electrical Power

Much more on this later...